

Mr. Wingate's Arithmetick:
CONTAINING
A PLAIN and FAMILIAR
METHOD

For Attaining the
Knowledge and Practice
~~OF~~ OF ~~Arithmetic~~
Common Arithmetick.

Composed by EDMUND WINGATE,
of Grays-Inn, Esq; K

And, upon his Request, Enlarged in his Life-Time;
also since his Decease carefully Revis'd, and much
Improv'd; as will appear by the Preface and Table
of Contents. By JOHN KERSET, late Teacher
of the Mathematicks.

The Twelfth Edition.

With A New Supplement; Of Easy Contractions in
the necessary Parts of Arithmetick; Useful Tables of
Interest, and Flemish Exchanges; As also Practical
Mensuration.

By GEORGE SHELLEY, Writing-Master at the
Hand and Pen in Warwick-Lane, near St. Paul's

L O N D O N:

Printed for J. Philips, at the King's Arms;
J. Taylor, at the Ship; and J. Knapton, at the
Crown, in St. Paul's Church-Yard. 1708.

Mr. Wingate's Arithmetick:
CONTAINING
A PLAIN and FAMILIAR
METHOD

For finding the
Knowledge and Practice
OF
Common Arithmetick.

Composed by THOMAS WINGATE
of Grafton, Esq.

And by his Request, Enlarged in his late Time;
also made a Decade carefully Revised; and much
improved: As will appear by the Preface and Table
of Contents. B. 2. S. Y. late Teacher
of the Manners



The City of London

With 3 other Supplements; Of each containing in
the necessary parts of Arithmetick; Useful Tables of
Money, and Weight; As also Practical
Mensuration.

By GEORGE SHELLEY Writing-Master at the
Hall and Post in Strand Lane, near St. Pauls

L O N D O N

Printed for J. Innes, at the Key, near
St. Pauls, at the Ship; and J. Knap, at the
Crown, in St. Pauls Church Lane.

The Epistle Dedicatory.

TO THE
RIGHT HONOURABLE

THOMAS,

Earl of Arundel and Surrey,

Earl Marshal of

ENGLAND, &c.

Right Honourable,

THE Good Affection You bear
to all Kind of Learning, and
in Particular to the Mathe-
maticks, makes me adventure to Present
Your Lordship with this Tractate of
Arithmetick, because that Art, com-
pared with other Mathematical Sciences,
is as the Primum Mobile, in Respect of
the other inferiour Orbs: For as the Poets

The Epistle Dedicatory.

used in times past to say of Venus, Sine Cerere & Baccho, Friget Venus, so may I also confidently averr of them, without Arithmetick they are Poor, and without Motion. Presuming therefore that Your Lordship, loving the Art, cannot disaffect the Artist, nor his Intention to do Good in that Kind, I am bold to shelter this Treatise under Your Lordship's Protection, humbly intreating Your gracious Acceptation, and earnestly desiring for ever to remain,

Your Honour's,

In all Service,

Affectionately devoted,

EDM. WINGATE.

THE

St. Aubyn

1710

THE
PREFACE
OF
JOHN KERSEY.

ABOUT the Year 1629, our Learned Country-Man, EDMUND WINGATE, Esquire, Published a Treatise of *Arithmetick* divided into Two Books, the one entituled *Natural Arithmetick*, and the other *Artificial Arithmetick*; and in regard his principal Design in that Treatise was, to remove the Difficulties which ordinarily arise in the Practice of *Common Arithmetick*, by the Help of artificial or borrowed Numbers, called *Logarithms*, (whose proper Work is to perform *Multiplication by Addition*; *Division by Subtraction*, &c.)

The Preface.

he did then in his said First Book omit divers Pieces of *Common* or *Practical Arithmetick*, which, for the perfect and universal Understanding thereof, were necessary to have been inserted: But, after the first Impression of both those Books was spent, our said Author being importuned to take Care of the *Second Edition*, he promised his Assistance therein; yet his other necessary Employments not permitting him to pursue his said Purpose, he was pleased to impart his Thoughts concerning the same unto me, together with his Request, That I would peruse the said First Book, and supply it with such Pieces of *Practical Arithmetick*, which for the Reasons aforesaid were wanting in the First Edition.

In Pursuance of which Request, I have contributed my Talent towards Perfecting this Tractate, upon our Author's Foundation; partly in his Life-time, to his good Liking, and partly since his Decease, in several Editions committed to my Care to be prepared for the Press; wherein I have used my best Endeavours, as well to preserve this Book as a Monument of our said Author's Worth, as also to make it a complear Store-house of *Common Arithmetick*; from

The Preface.

from whence the Ingenious may be furnish'd with the Excellencies of that Art, in Reference both to common Affairs, as also to the practical Parts of the Mathematicks. And in order to those Ends, I have made these following Alterations and Additions; namely,

First, For the Ease and Benefit of such Learners, who desire only so much Skill in Arithmetick, as is useful in Accompts, Trade, and such like ordinary Employments; the Doctrine of whole Numbers, (which, in the first Edition, was intermingled with Definitions and Rules concerning broken Numbers, commonly called Fractions,) is now entirely handled apart. And to the end the full Knowledge of *Practical Arithmetick* in whole Numbers might more clearly appear, I have explained divers of the old Rules in the First Five Chapters, and framed anew the Rules of *Division*, *Reduction*, and the *Golden Rule*, in the Sixth, Seventh, Eighth, and Ninth Chapters; so that now Arithmetick in whole Numbers is plainly and fully handled before any entrance be made into the craggy Paths of Fractions, at the Sight whereof some Learners are so dis-

The Preface.

discouraged, that they make a stand, and cry out, *Non plus ultra*, There's no Progress farther.

Secondly, To assist such young Students as desire to lay a good Foundation for the attaining of a general Knowledge in the Mathematicks, I have in a familiar Method delivered the entire Doctrine of Fractions, both Vulgar and Decimal, which was omitted in the First Edition; and have also newly framed the Extraction of the Square and Cube Roots, in a Method which by Experience is found to be much easier than that commonly used heretofore, and is exactly suitable to the Construction or Composition of Square and Cube Numbers.

Lastly, I have added an *Appendix*, which is furnished with Variety of choice and delightful Knowledge in Numbers, both Practical and Theoretical. In all which Performances, I have earnestly aimed at Truth, Perspicuity, and exact Correction, both of the Text and Numbers; so that I hope this Book is now supplied with all things necessary to the full Knowledge and Practice of *Common Arithmetick*, the usefulness whereof is so generally known, that there will be no need of Arguments to excite any one that desires his

The Preface.

his own or the Publick Good, to be acquainted with so excellent an Art.

But if the more curious Artist, after he is well exercis'd in vulgar Arithmetick, desires farther Inspection into the Myſteries of Numbers, his beſt Guide is the admirable Art call'd *Algebra*; the Elements whereof I have expounded at large in a Treatiſe lately Publish'd.

JOHN KERSEY.

THE

THE TABLE of CONTENTS.

Where those CHAPTERS of Mr. *Wingate's*, that have been altered and framed anew by *John Kersey*, are distinguished by this Mark, , and those Chapters that have been entirely Composed by the said *J. K.* may be discovered by this Asterisk *.

The Doctrine of whole Numbers is contained in the First Fifteen Chapters, the Titles whereof are these following.

	Chap.	Pag.
 Concerning the Notation of Numbers.—	1	1
 Concerning English Moneys, } Weights, Measures, &c. }	2	8
Addition. —————	3	16
Subtraction. —————	4	23
Multiplication. —————	5	31
 Division. —————	6	38
 Reduction. —————	7	58
 The Rule of Three Direct. ———	8	69
 The Rule of Three Inverse. ———	9	82
 The double Rule of Three Direct. —	10	87
 The double Rule of Three Inverse. —	11	94
The Rule of Three compound of five Numbers.—	12	98
The Rule of Fellowship. ———	13	102
The Rule of Alligation. ———	14	108
The Rule of False. ———	15	125
		The

The Doctrine of Fractions, both Vulgar and Decimal, is contain'd in the Sixteen Chapters next following.

Concerning Vulgar Fractions.		Chap.	Pag.
* Notation.	_____	16	133
* Reduction.	_____	17	137
* Addition.	_____	18	151
* Subtraction.	_____	19	154
* Multiplication.	_____	20	160
* Division.	_____	21	164

Concerning Decimal Fractions.			
* Notation.	_____	22	167
* Reduction.	_____	23	173
* Addition.	_____	24	211
* Subtraction.	_____	25	214
* Multiplication.	_____	26	215
* Division.	_____	27	226

In Vulgar and Decimal Fractions.			
* The Rule of Three Direct.	_____	28	240
* The Rule of Three Inverse.	_____	29	245
* The Double Rule of Three.	_____	30	247
* The Rule of False.	_____	31	250

The Extraction of Roots is contained in the two Chapters next following.

* The Extraction of the Square Root.	_____	32	257
* The Extraction of the Cube Root.	_____	33	270

The Relation of Numbers in Quantity and Quality is contain'd in the two following Chapters.

The Relation of Numbers in Quantity.	_____	34	290
The Relation of Numbers in Quality,	}	35	295
whereof Arithmetical and Geometrical Proportion.			

The

THE CONTENTS OF THE APPENDIX.

	Chap.	Pag.
• Concerning Contractions in the Rule of Three. —————	1	309
• Rules of Practice by Aliquot Parts. —————	2	315
• Exchanges of Coins, Weights and Measures. ———	3	332
• Practical Questions about Tare, Tret, Loss, Gain, Barter, Factorship, and measuring of Tapestry. —————	4	360
• Interest of Money, with Tables to value Annuities, &c. at any Rate per centum from 4l. to 12l. and the manner of making those Tables. —————	5	368
• A Demonstration of the Rule of Three. —————	6	440
• A Demonstration of the Double Rule of Fellowship. —————	7	443
* A Demonstration of the Rule of Alligation, with it's Use in the Composition of Medicines. ———	8	446
• A Demonstration of the Rule of False. —————	9	461
* A Collection of subtil Questions to exercise all the Parts of Vulgar Arithmetick; to which also are added various practical Questions about the Mensuration of Superficial Figures and Solids, with the Gauging of Vessels. —————	10	475
• Sports and Pastimes. —————	11	528

Book I

A

TREATISE

OF

Common Arithmetick.

The First Book.

CHAP. I.

Concerning Notation of Numbers.

I. **A**RITHMETICK is the Art of Accompting by Number. As Magnitude, or Greatness, is the Subject of Geometry, so Multitude, or Number, is that of *Arithmetick*.

II. Number is that by which every *Number*. Thing is number'd; or that which answers this Question, How many? (unless it be answered by nothing :) So if it be asked, How many

many Days are in a Week? the Answer is, Seven, which is called Number.

The Characters by which Number is expressed. III. The Notes or Characters, by which Number is ordinarily expressed, are these, 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 nothing.

IV. These Notes or Characters are either significant Figures, or a Cypher.

V. The significant Figures are the first Nine; viz. 1, 2, 3, 4, 5, 6, 7, 8, 9. The First whereof is more particularly called an Unit or Unity, and the rest are said to be composed of Unities: So 2 is composed of two Unities, 3 of three Unities, &c.

VI. The Cypher is the last, which though of itself it signifies nothing, yet, being annexed after any of the rest, it encreaseth their Value: As will appear in the following Rules.

VII. Arithmetick hath Two Parts, Notation and Numeration.

VIII. Notation teacheth how to express, read, or declare the Signification or Value of any Number written; and also to write down any Number propounded, with proper Characters, in their due Places.

The Places or Degrees of any Number. IX. A Number is said to have so many Places, or Degrees, as there are Characters in the Number; viz. When divers Figures, whether they be intermix'd with a Cypher, or Cyphers, or not, are placed together, like Letters in a Word, without any Point, Comma, Line, or other Note of Distinction interposed,

poled, all those Characters make but one Number, which consists of so many places as there are Characters so placed to one another; so this number 205 consists of 3 places, and this 30600 of 5 places, &c.

X. Notation consists in the Knowledge of two things, viz. the Order of places, and the Value of every place in any Number.

XI. The Order of the places is from the right-hand towards the left: *The Order of Places in any Number.* So in this number 465, the figure 5 standeth in the first place, 6 in the second, and 4 in the third; likewise in this number 7560, a Cypher stands in the first place, 6 in the second, 5 in the third, and 7 in the fourth.

XII. The first place of a Number, *The Value of Places in any Number.* (which as before, is the outermost towards the right-hand) is called the place of Units or Unities; in which place any Figure signifieth its own simple Value: So in this Number 465, the figure 5 standing in the first place signifieth five Unities, or five.

XIII. The second place of a Number is called the place of Tens; in which place any figure signifieth so many Tens as the figure containeth Unities: So in this Number 465, the figure 5 in the first place signifieth simply 5, but the figure 6 in the second place signifieth six Tens, or sixty.

XIV. The third place of a Number is called the place of Hundreds; in which place any Figure signifieth so many Hundreds as there are Unities contain'd in the figure: So in this number 465, the Figure 4 in the third place signifieth four Hundreds: Wherefore if it be requir'd to read or pronounce this number 465, you are to begin on the left-hand; and

and according to the aforesaid Rules to pronounce it thus, Four hundred sixty five, likewise this number 315 is to be pronounced thus, Three hundred and fifteen; and this number 205, Two hundred and five; also this number 500, Five hundred. Whence it is manifest, that although a Cypher of it self signifies nothing, yet being placed on the Right hand of a Figure it encreaseth the Value thereof, by advancing such Figure to an higher place than that wherein it would be seated, if the Cypher were absent.

The true reading or pronouncing the value of any number written, as also the writing down any number propounded, depends principally upon a right understanding of the Three first places beforementioned, and therefore I shall advise the Learner to be well exercis'd therein, before he proceeds to the following Rules.

XV. The fourth place of a number is called the place of Thousands (that is, any number of Thousands under Ten thousand;) the fifth place tens of thousands; the sixth place hundreds of thousands; the seventh place Millions; (a Million being Ten hundred thousand;) the eighth place tens of Millions; the ninth place hundred of Millions; the tenth place thousands of Millions; the eleventh place tens of thousands of Millions; the twelfth place hundreds of thousands of Millions: And in that order you may conceive places to be continued infinitely from the Right hand towards the left, each following place being ten times the Value of the next preceding place; but to give Names to them would be both a troublesome and unnecessary Task.

XVI. The

Chap. I.

of Numbers.

XVI. From the Rules *above*, an *easy way* may be collected to read or express the Value of a Number propounded, viz. Let it be required to read or pronounce this Number, 521426341. First, Distinguish by a Comma, or Point, every three Places, beginning at the Right-hand, and proceeding towards the Left, so will the *above* said Number be distinguished into Parts, which may be called *Periods*, and stand thus, 521,426,341. Where you may note the first *Period* towards the Right-hand to consist of these Figures, 341; the second of these 426, and the third of these 521. Secondly, Read or pronounce the Figures in every *Period* as if they stood apart from the rest, so will the first *Period* be pronounced three hundred forty one, the second, four hundred twenty six, and the third five hundred twenty one. Thirdly, to every *Period* except the first towards the Right-hand, a peculiar Denomination or *Sirname* is to be apply'd, viz. The *Sirname* of the second *Period*, is *Thousands*; of the third, *Millions*; of the fourth, *Thousands* of *Millions*, &c. Therefore beginning to pronounce at the highest *Period*, which in this Example is the third, and giving every *Period* its due *Sirname*, the said Number will be pronounced thus, Five hundred twenty one Millions, Four hundred twenty six thousands, Three hundred forty one.

Note, When a Number is distinguished into *Periods*, as before, the highest *Period* will not always compleatly consist of three Places, but sometimes of one Place, and sometimes of two, nevertheless after such *Period* is pronounced as if it stood apart, the due *Sirname* is to be annexed; so this

number 3204689, after it is divided into *Periods* will stand thus, 3, 204, 689. and to be pronounced thus, *Three Millions, two hundred and four thousands, six hundred eighty nine.*

XVII. The *aforeſaid Rules* for the right pronouncing or reading of a Number which is written down, being well underſtood, will ſufficiently inform the Reader how to write down any number propounded to be written.

The Table of Notation.

The Order of Places.		The Values of Places.	
	Sec.		Sec.
Fourth Period,	Twelfth place	3	Hundreds of Thousand Millions.
	Eleventh place	2	Tens of Thousand Millions.
	Tenth place	1	Thousand Millions.
Third Period,	Ninth place	9	Hundreds of Millions.
	Eighth place	8	Tens of Millions.
	Seventh place	7	Millions.
Second Period,	Sixth place	6	Hundreds of Thousands.
	Fifth place	5	Tens of Thousands.
	Fourth place	4	Thousands.
First Period,	Third place	3	Hundreds.
	Second place	2	Tens.
	First place	1	Units.

Notation of Numbers by Latin Letters.

1	I.	21	XXI.
2	II.	30	XXX.
3	III.	40	XL.
4	IIII. or thus IV.	49	XLIX.
5	V.	50	L.
6	VI.	59	LVIII. or thus LIX.
7	VII.	60	LX.
8	VIII. or thus IIX.	89	LXXXIX.
9	VIII or thus IX.	100	C.
10	X.	200	CC.
11	XI.	300	CCC.
12	XII.	400	CCCC.
18	XVIII or thus IIXX	500	D. or thus IC.
19	XVIII or thus XIX	600	DC. or thus ICC.
20	XX.	700	DCC. or thus ICCC.

1000	CI ₂ . or thus M.
2000	CI ₂ . CI ₂ .
3000	CI ₂ . CI ₂ . CI ₂ .
5000	IC ₂ .
10000	CCIC ₂ .

50000	ICCC.	
100000	CCCICCC.	or thus CM.
500000	ICCCC.	
1000000	CCCCI.	CCCC.
1703.	MDCCLIII.	

C H A P. II.

Concerning English Money, Weights,
Measures, &c.

I. **T**HE Things expressed by Numbers are principally Money, Weight, Measure, Time, and Things accounted by the Dozen: Of the three first of these, there are infinite Kinds and Varieties according to the Diversity of the several Common-wealths in which they are used, all which here to produce were both endless and needless: wherefore we intend here to treat only of such Money, Weights, Measures, &c. as are used in this Nation, being indeed only necessary for our present Purpose.

II. The least Piece of Money used in England is a Farthing, from whence ^{of English} Money, this following Table is produced.

1 Farthing	}	make	}	1 Farthing.
4 Farthings				1 Penny.
12 Pence				1 Shilling.
20 Shillings				1 Pound.

English (or Sterling) Money, is ordinarily written down with Figures after this manner:

l.	s.	d.	f.
34	13	04	2
09	05	10	1
69	00	06	3
00	12	11	0
00	00	07	2

The

The first Rank of the said Numbers signifies Thirty four Pounds, thirteen Shillings, five Pence, two Farthings; the second Rank expresseth Nine Pounds, five Shillings, ten Pence, one Farthing; the third Rank, Sixty nine Pounds, no Shillings, six Pence, three Farthings, &c.

III. The smallest Weight used in England is a Grain, that is, the Weight of a Grain of Wheat, well dried, and gathered out of the middle of the Ear, where of Thirty two make another Weight, called a Penny-weight, and twenty Penny-weight make an Ounce Troy.

Vide Stat. de compositione Ponderum, Hen. III.

Here observe, That by the Statutes quoted in the Margent, the Weight of Two and thirty Grains of Wheat, make a Penny-weight, which Weight being once discovered by two and thirty such Grains, the said Penny-weight (being the twentieth Part of an Ounce Troy) is usually subdivided into four and twenty Parts only, called also Grains, as appears by the ensuing Table.

A Table of Troy Weights.

Troy Weight.

24 Grains of Wheat	} make	24 Artificial Grains.
24 Grains		1 Penny-weight.
20 Penny-weight		1 Ounce.
12 Ounces		1 Pound Troy.

Troy Weight is ordinarily written down with Figures after this manner:

lb.	p. w.	gr.
17	05	13
00	11	06
00	00	20

The first Rank of the said Numbers expresseth seventeen Pounds, five Ounces, thirteen Penny weight, thirteen Grains of *Troy-weight*; the second Rank, no Pounds, eleven Ounces, seven Penny weight, six Grains; and the third, no Pounds, no Ounces, five Penny weight, and twenty Grains.

Now this *Troy-weight* serveth only to weigh Bread, Gold, Silver, and Electuaries.

Malynes lex
Mercat. p. 49.
Malynes ibid.
pag. 252.

And here observe also by the way, That *Troy-weight* regulateth and prescribeth a Form how to keep the Money of *England* at a certain *Standard*, For about Two Hundred Years before the Conquest, *Osbright* a Saxon, being then King of *England*, caused an Ounce *Troy* of Silver to be divided into 20 Pieces, at the same time called Pence; and so an Ounce of Silver at that Time was worth no more than twenty Pence, or one Shilling eight Pence, which continued at the same Value until the time of *Henry* the sixth, who (in regard of the enhancing of Money in Foreign Parts) valued the same at thirty Pence; so that then there were accordingly thirty Pieces made out of the Ounce, and the old Pieces went then for Three half-pence, until the Time of *Edward* the Fourth, who valued the Ounce at forty Pence, and then the old Pieces went for two Pence a-piece. After this, *Henry* the Eighth valued the Ounce of sterling Silver at forty five Pence, which Value continued until Queen *Elizabeth's* Time, who valued the same old Pence at Three-pence the Piece; so that all Three-pences coined by the same Queen weighed but a Penny weight, and every Six-pence two Penny weight; and so in like manner the Shilling and other Pieces accord;

Chap II. *Weights, Measures, &c.*

accordingly; which made the Ounce *Troy* of Silver, to be valued at Sixty pence or Five shillings, as it now remains at this Day without Alteration.

IV. The Weights used by Apothecaries are derived from a Pound *Troy*, which is subdivided, as in the following Table:

A Table of Apothecaries Weights.

℥	A Pound <i>Troy</i>	} is equal unto }	12 Ounces.
ʒ	An Ounce		8 Drams.
ʒ	A Dram		ʒ Scruples.
ʒ	A Scruple		20 Grains.

So that if you were to express in Figures twelve Pounds, ten Ounces, five Drams, two Scruples, and sixteen Grains; also three Pounds, five Ounces, seven Drams, one Scruple, and two Grains, the ordinary way to write them down is briefly thus:

℥	3	3	3	gr.
12	—	10	—	5
03	—	25	—	7
			—	1
				02

V. Besides *Troy* weight before-mentioned, there is another kind of weight used in *England*, called *Avoirdupois* weight, a Pound whereof is equal unto 14 Ounces, Twelve penny weight *Troy*. This *Avoirdupois* weight serveth to weigh all kind of Grocery-wares also Butter, Cheese, Flesh, Tallow, Wax and every other thing, which beareth the Name of *Garbel*, and whereof issueth a refuse or waste.

VI. *Avoirdupois* weight is either greater or less.

VII. The greater is, when one hundred and twelve pounds *Avoirdupois* are considered as one intire weight

commonly called an Hundred weight, and then each Hundred weight is subdivided first into four Quarters, and each Quarter into eight and twenty Pounds: Again, each Pound into four Quarters, or (if you will be more exact) into sixteen Ounces, and if you please each Ounce into four Quarters. But ordinarily a Pound is the least Quantity that is taken notice of in Avoirdupois gross Weights.

A Table of Avoirdupois greater Weight.

28 Pounds	} make	{	A Quarter of 112 lb.
4 Quarters			an Hundred weight, or 112 lb.
20 Hundred			One Tun.

So that if you were to express by Figures eight Hundred, three Quarters, and five Pounds; likewise, seven Hundred, one Quarter, and seventeen Pounds; the ordinary way to write them down is briefly thus,

C.	q.	lb.
8	3	5
7	1	17

Avoirdupois lesser Weight. VIII. The lesser Avoirdupois weight is, when a Pound is the highest Name or Integer, each Pound being subdivided into sixteen Ounces, and each Ounce again into sixteen Drams, and if you please each Dram into four Quarters, as by the subsequent Table is manifest.

A Table of Avoirdupois lesser Weight.

4 Quarters of a Dram	} make	{	1 Dram.
16 Drams			1 Ounce.
16 Ounces			1 Pound.

So

Chap. II. *Weights, Measures, &c.* 13

So that if you were to express by Figures eighteen Pounds, twelve Ounces, five Drams, and three quarters of a Dram; likewise five Pounds, no Ounces, twelve Drams, and one quarter of a Dram; the ordinary way to write them down is briefly thus,

lb.	oz.	dr.	q.
18	12	05	3
5	00	12	1

IX. The Measures used in *England* are either of Capacity or Length.

X. The Measures of Capacity are those which are produced from Weight, and they are either Liquid or Dry.

XI. The Liquid Measures are those, in which all kind of Liquid substances are measured, and they are expressed in the Table following.

A Table of Liquid Measures.

1 Pound of Wheat?	make	1 Pint.
Troy weight }		1 Quart.
2 Pints		1 Pottle.
2 Quarts		1 Gallon.
2 Pottles		1 Firkin of Ale, Soap, Herring.
8 Gallons		1 Firkin of Beer.
9 Gallons		1 Firkin of Salmon or Eels.
10 Gallons and an half		1 Kilderkin.
2 Firkins		1 Barrel.
2 Kilderkins		1 Tierce of Wine.
42 Gallons		1 Hoghead.
63 Gallons		1 Pipe or But.
2 Hogheads		1 Tun of Wine.
2 Pipes or Buts		

XII. Dry

XII. Dry Measures are those, in which all kind of dry Substances are meted, as Grain, Sea-coal, Salt, and the like; their Table is this that follows:

A Table of dry Measures.

1 Pint	1 Pint.
2 Pints	1 Quart.
2 Quarts	1 Pottle.
2 Pottles	1 Gallon.
2 Gallons	1 Peck.
4 Pecks	1 Bushel land measure.
5 Pecks	1 Bushel water measure.
8 Bushels	1 Quarter.
4 Quarters	1 Chaldron.
5 Quarters	1 Wey.
2 Weys	1 Last.

Long Measures.

XIII. Long Measures are express'd in this Table following.

3 Barly Corns in length	1 Incb.
12 Inches	1 Foot.
3 Foot or 16 Nails	1 Yard.
3 Foot nine Inches	1 Ell.
6 Foot or 2 Yards	1 Fathom.
5 Yards and an half	1 Pole or Perch.
40 Poles or Perches	1 Furlong.
8 Furlongs	1 English Mile.

A L S O

16 Nails or 4 Quarters	1 Yard.
20 Nails or 5 Quarters	1 Ell English.
12 Nails or 3 Quarters	1 Flemish Ell or Aune.

Note, That a Yard is usually subdivided into 4 Quarters, and each Quarter into 4 Nails: And each Ell into 4 Quarters, but each Quarter of an Ell contains 5 Nails.

XIV. Su-

XIV. Superficial, or square Measures of Land, are such as are expressed in the Table following:

40 Square Poles, or Perches	} make	1 Rood, or Quarter of an Acre.
4 Roods		1 Acre.
640 Acres		1 Mile.

So that if you would express by Figures these Quantities of Land, viz. Thirty six Acres, three Roods, twenty Perches; also seven Acres, no Roods, thirty two Perches, the ordinary way to write them down is thus,

A.	R.	P.
36	3	20.
7	0	32.

XV. A Table of Time is this that follows: Time.

1 Minute	} make	1 Minute.
60 Minutes		1 Hour.
24 Hours		1 Day natural.
7 Days		1 Week.
4 Weeks		1 Month of twenty eight Days.
13 Months, 1 Day, and 6 Hours		1 Year very near.

But in ordinary Computations of Time, the whole Year consisting of Three hundred sixty five Days, is divided either into Twelve equal Parts or Months; each Month then containing thirty Days and ten Hours; or else into twelve unequal *Kalendar Months*, according to the ancient Verse:

Thirty Days hath September, April, June, and November;

February hath twenty eight alone, each of the rest hath thirty one.

Note,

Note, That every *Leap-year* (which happeneth once in four Years) containeth Three hundred sixty six Days, and in such Year, *February* containeth twenty nine Days.

XVI. Of Things accounted by the Dozen, a Gross is the Integer, consisting of twelve Dozen, each Dozen containing again twelve Particulars: so that if you would exprels in Figures, Seven Gross, four Dozen, and five Particulars; also four Dozen and eight Particulars, they may be briefly written thus:

Of Things
accounted by
the Dozen.

G.	D.	P.
7	04	05
0	04	08

C H A P. III.

Addition of Whole Numbers.

I. Concerning Notation of Numbers, and how thereby the Quantities of Things are usually exprels'd, a full Declaration hath been made in the preceding Chapters: Numeration ensueth, which comprehends all manner of Operations by Numbers.

II. In Numeration, the four primary or fundamental Operations (commonly called Species) are these, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

III. Addition is that by which divers Numbers are added together, to the end that their Summ, Aggregate, or Total, may be discovered.

IV. In Addition, place the Numbers given, one above another, in such sort, that like Places

OR

Chap. III. of whole Numbers.

17

or Degrees in each Number, may stand in the same Rank; that is, Units above Units, Tens above Tens, Hundreds above Hundreds, &c. So these Numbers 1213 and 462, being given to be added together, you are to order them as you see in the Margent.

Addition of Numbers of one Denomination.

1213
462

V. Having thus placed the Numbers, and drawn a Line under them, add them together, beginning with the Units first, and saying thus, 2 and 3 make 5, which write under the Rank of Units, then proceed to the second Rank, and say, 6 and 1 make 7, which write under the second Rank (being the Place of tens) again 4 and 2 make 6, which write under the third Rank. Lastly, write down 1, being all that stands in the fourth Rank; So the Summ of the said given Numbers is found to be 1675, and the Operation will stand as in the Margent.

1213
462

1675

In like manner, the Numbers 2315, 7423, and 141, being given to be added together, their Summ will be found to be 9879, and the Operation thereof will stand as you see in the Example.

2315
7423
141

9879

VI. When the Summ of the Figures of any of the Ranks amounts unto ten, or any Number of tens, without any Excess, write down a Cypher under that Rank; but when the Summ of any Rank exceeds ten, or any number of tens, write down the Excess under such Rank, and for every ten contained in the Summ of any Rank, reserve an Unit or 1 in your Mind, and add such Unit or Units to the Figures

Figures of the next Rank towards the Left-hand; so the Numbers 4937, 9878, and 394, being given to be added together, the Operation will be thus, *viz.* Beginning

4937	with the Rank of Units, I say, 4, 8 and 7 make 19, wherefore I write down 9, the Excess above Ten, and carrying 1 in mind instead of the ten contained in the said 19, I say, 1 and 9 (9 being the lowermost Figure of the second Rank) make 10, which added to 7 and 3, the other Figures of the same Rank, the whole Summ of them is 20, wherefore setting down a Cypher under the Line in that Rank (because the Excess above the two Tens is nothing) I carry 2 to the third Rank, and say, 2 and 3 (3 being the lowermost Figure of the third Rank) make 5, which being added to 8 and 9 (the other Figures of the same Rank) the Summ of them is 22, wherefore writing down 2 (being the Excess above the two tens) under the Line in the third Rank, I carry 2 in mind (because there were two tens in 22) to the fourth Rank, and say, 2 and 9 make 11, which added to 4 makes 15, this 15, because it is the Summ of the last Rank, I write totally down under the Line, on the Left-hand of the Figures before subscribed; so the Summ of the three Numbers given, is found to be 15209, as in the Example.
9878	
394	
15209	

down 9, the Excess above Ten, and carrying 1 in mind instead of the ten contained in the said 19, I say, 1 and 9 (9 being the lowermost Figure of the second Rank) make 10, which added to 7 and 3, the other Figures of the same Rank, the whole Summ of them is 20, wherefore setting down a Cypher under the Line in that Rank (because the Excess above the two Tens is nothing) I carry 2 to the third Rank, and say, 2 and 3 (3 being the lowermost Figure of the third Rank) make 5, which being added to 8 and 9 (the other Figures of the same Rank) the Summ of them is 22, wherefore writing down 2 (being the Excess above the two tens) under the Line in the third Rank, I carry 2 in mind (because there were two tens in 22) to the fourth Rank, and say, 2 and 9 make 11, which added to 4 makes 15, this 15, because it is the Summ of the last Rank, I write totally down under the Line, on the Left-hand of the Figures before subscribed; so the Summ of the three Numbers given, is found to be 15209, as in the Example.

Addition of Numbers of divers Denominations. VII. When Numbers given to be added, do express Things of divers Denominations; first write them down orderly (according to the Examples in Chap II.) then after a Line is drawn under them all, begin to add the Numbers

of the least Denomination, and if the sum of them amounts to one Integer, or many Integers of the next greater Denomination, with some excess of the less Denomination, write down that excess, or a Cypher when there is no excess, under the line, so as it may stand under the least Denomination, and keep the said Integer or Integers in mind, to be added to those of the next greater Denomination, on the Left hand: But when the Sum of the Numbers of the least Denomination amounts not to one Integer of the next greater Denomination, set down the Sum it self under the Line; then add the Integer or Integers kept in mind (when any happens) to the Numbers of the next greater Denomination on the Left hand, and proceed to add them, as also those of every greater Denomination, in like manner as above is directed, until you come to the Numbers of the greatest (or highest) Denomination, which are to be added according to the foregoing Rules V. and VI. of this Chapter.

So these severall sums 24*l.*—13*s.*—5*d.*—3*f.*
Also 12*l.*—0*s.*—8*d.* and 5*l.*—18*s.*—2*f.*
being propounded to be added, their total Sum is
42*l.*—12*s.*—2*d.*—1*f.* For having

written them down orderly according to the second Rule of the Second Chapter, and drawn a Line underneath, I begin with the Farthings first, and say, Two Farthings and three Farthings make five

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>
Farthings, that is, one Penny	24	13	05	3
with a Farthing over and	12	00	08	0
above; wherefore setting	05	18	00	2
down 1 under the Deno-				
mination of Farthings, I	42	12	02	1

carry

carry one Penny to the Denomination of Pence, then I say, 1, 8, and five Pence make 14 Pence, which contain one Shilling and two Pence, wherefore writing two under the Denomination of Pence, I likewise carry 1 Shilling to the Denomination of Shillings; Then adding the said 1 Shilling unto 18 Shillings and 13 Shillings, the sum will be found 1 Pound and 12 Shillings, wherefore setting down 12 under the Denomination of Shillings, I carry 1 Pound in mind unto the Denomination of Pounds, saying, 1 Pound in mind, together with 5, 2, and 4 Pounds which stand in the first rank of Pounds, make 12 Pounds, wherefore (according to the sixth Rule of this Chapter) I write 2, the excess above 10, underneath the said first rank of Pounds, and carry 1 in mind for the said 10 to the second rank of Pounds; then saying in like manner, 1 in mind, together with 1 and 2 which stand in the second rank of Pounds make 4, which I write underneath the line, that done, I find the total of the three sums propounded to be 42*l.*—12*s.*—and 2*d.*—1*f.*

In like manner 3*lb.*—05*oz.*—19*p.w.*—15*gr.* Also 2*lb.*—0*oz.*—3*p.w.*—7*gr.* Also 0*lb.*—10*oz.*—6*p.w.*—0*gr.* And 0*lb.*—9*oz.*—0*p.w.*—17*gr.* being given to be added together, their sum will be found 7*lb.*—01*oz.*—9*p.w.*—15*gr.* and the Work will stand thus,

<i>lb.</i>	<i>oz.</i>	<i>p.w.</i>	<i>gr.</i>
3	05	19	15
2	00	03	07
0	10	06	00
0	09	00	17
7	01	09	15

Note,

Note, In adding together the Numbers in the last Example, it must be remembred that twenty four Grains make one Penny-weight; twenty Penny-weight one Ounce; and twelve Ounces one Pound *Troy* (as before declared in the third Rule of the second Chapter;) And then you are to proceed according to Rule VII. of this Chapter.

More Examples of Rule VII. are these following which presuppose the Learner to be well exercis'd in the Tables of Chap. 2. that he may readily know what Integers are to be carried from every lesser Denomination to the next greater.

Addition of English Money.

lb.	s.	d.	f.	l.	s.	d.
230	17	10	1	0	13	05
175	12	11	3	0	17	08
952	05	00	0	0	00	10
009	00	08	1	0	1	03
506	13	00	2	0	15	06
974	09	06	3	2	17	08

Addition of Troy Weight.

lb.	oz.	p.w.	gr.	oz.	p.w.	gr.
23	07	16	13	536	13	16
17	10	15	07	208	11	10
325	06	19	20	063	10	05
49	11	07	12	099	00	12
417	00	19	04	907	15	19

Addition of Avoirdupois Weight.

C.	q.	lb.	lb.	oz.	dr.
235	3	13	14	13	12
576	1	17	05	10	14
628	2	15	12	00	06
412	0	10	06	09	05
1852	3	27	39	02	05

Addition of Measures of Length.

Fards.	q.	nails.	Ells.	q.	nails.
26	3	2	15	3	2
13	1	3	10	1	3
12	0	1	09	0	1
29	1	1	12	2	1
81	2	3	53	3	2

Addition of Superficial Measures of Land.

Acres.	Roods.	Per.	A.	R.	P.
136	3	13	240	2	17
513	1	26	500	3	13
212	2	10	249	1	36
517	0	00	006	0	10
1379	3	09	996	3	36

C H A P. IV.

Subtraction of whole Numbers.

I. SUBtraction is that by which one number is taken out of another, to the end that the remainder or difference, between the two numbers given may be known.

II. The Number out of which the Subtraction is to be made, must be greater, or at least, equal with the other. As you may Subtract 4347 or 9478 out of 9478, so can you not subtract 9478 out of 4347.

Subtraction of numbers of one denomination.

III. In subtraction rank the two given numbers one under the other as in Addition, with this caution, that the number placed uppermost may exceed, or at least be equal unto the other: So if the number 4347 be given to be subtracted from 9478, I order them as in the Margin: Then proceeding to the Subtraction, I say 7 taken out of 8, there remains 1, which I place in the same rank under the Line. 9478
In like manner 4 being taken out of 7, the remainder is 3, which likewise I set under the line in the next

9478
4347
5131

rank; again taking 3 from 4, the remainder is 1, which I likewise place under the third rank; lastly, subtracting 4 from 9, there will remain 5, which I subscribe under the fourth rank; so the whole operation being finished, I find, that if 4347 be taken out of 9478, the remainder is 5131, or (which is the same) the difference between the numbers 9478 and 4347 is 5131, as in the Example.

In like manner if 106 be subtracted from 2856 the remainder will be found 2750; for after the numbers are orderly ranked, I begin at the place of Units, and say, 6 from 6, there remains nothing; wherefore I subscribe 0, then proceeding to the second rank, I say, if 0 (or nothing) be taken from 5, there will remain 5, which I also subscribe under the line; again 1 from 8, there remains 7: lastly, 0 from 2, there remains 2. See the Work in the Margin.

IV. When any of the figures of the number given to be subtracted is greater than the upper figure out of which it is to be subtracted, you must borrow 10 of the next rank towards the left hand, and add the said 10 to the said upper figure, then from the sum of such Addition subtract the lower figure and set down the remainder: In this case the figure of the next rank which is to be subtracted; must be esteemed an unite greater than it is; wherefore keeping one in your mind add it to the next figure of the number given to be subtracted, and deducting all out of the figure above it, proceed in like sort till you have finished the whole operation. Example, Let it be required to subtract 374 out of 8023. Having ranked them as before, I say 4 out of 3, that cannot be, wherefore borrowing 10 of the next rank and adding the same to the said 3, I say 4 out of 13, there remains 9; then writing 9 under the line, and carrying 1 in my mind, I say 1 and 7 make 8, 8 out of 2 that cannot be, but 8 out of 12 (12, because 10 being borrowed, and added to 2, makes 12) there remains 4, which I subscribe under the line;

$$\begin{array}{r} 2856 \\ 106 \\ \hline 2750 \end{array}$$

$$\begin{array}{r} 8023 \\ 374 \\ \hline 7649 \end{array}$$

line ; again 1 in my mind being added to 3 makes 4, 4 out of nothing, that cannot be, but 4 out of 10 there remains 6, which I likewise subscribe under the line ; lastly 1 in mind being taken out of 8 there remains 7. Thus you see that the remainder after 374 is subtracted from 8023 is 7649. Note diligently, that as often as 10 is borrowed, 1 must be kept in mind to be added to the figure standing in the next place of the lower number, and the sum of such Addition must be subtracted from the upper place ; but if it happen that there is no figure in the next place of the lower number, then the 1 in mind must be subtracted from the upper place, (as in the last rank of the last Example.) *Another Example.* Let it be required to subtract 92 from 62801. Having placed the greater number uppermost, and the lesser orderly underneath, I begin at the place of Units, and say, 2 from 1 I cannot take, but 62801
borrowing 10, and adding it to the 92
said 1, I say 2 from 11 there re-
mains 9, which I subscribe under the 62709
line ; then I proceed and say, 1 in mind with 9, makes 10, 10 out of 0 I cannot take, but borrowing 10, I say 10 out of 10 and there remains 0, wherefore I subscribe 0 under the line ; again, 1 in mind out of 8, there remains 7 ; then because there are no more Figures in the lower number, I say 0 out of 2 there remains 2 ; lastly, 0 out of 6 there remains 6 ; therefore I conclude that 62801 exceeds 92 by 62709.

V. If the numbers propounded have divers denominations, place them as before, and beginning with

Subtraction of numbers of divers denominations.

the least denomination first, subtract the lower number from the upper when it may be subtracted, and place the remainder underneath; but if it happen that the lower number cannot be taken out of the upper, you must borrow an Integer of the next greater denomination on the left hand; which Integer, after it is converted into the same denomination with the said upper number, must be added to it: Then from the sum of such Addition, you are to subtract the lower number, and write down the remainder, keeping 1 (that is the Integer borrowed) in your mind, to be added to the next place of the number given to be subtracted, as before: So
 $90l. — 14s. — 10d. — 3f.$ being subtracted from
 $124l. — 11s. — 7d. — 1f.$ the remainder is $33l. — 16s. — 8d. — 2f.$ For beginning with the Farthings, I say, 3 Farthings out of

l.	s.	d.	f.	
124	11	07	1	1 Farthing I cannot take, where-
90	14	10	3	fore borrowing 1 Penny (that
33	16	08	3	is an Integer of the next great-
				er denomination) and having
				converted this Penny into 4

Farthings, I add them to the aforesaid 1 Farthing; so the sum is 5 Farthings, out of which subtracting 3 Farthings, there remains 2 Farthings, which I place underneath the denomination of Farthings; then I proceed to the next denomination, and say 1 penny which I borrowed and 10d. make 11d. this 11d. out of 7d. I cannot take, wherefore borrowing 1 Shilling or 12d. and adding 12d. to the said 7d. the sum is 19d. from which I subtract the said 11d. so there remains 8d. which I subscribe under the denomination of Pence; again 1 Shilling which I borrowed being added to 14s. makes

makes 15s. which I cannot subtract out of 11s. and therefore I borrow 1 Pound or 20s. which being added to the said 11s. makes 31s. from which subtracting 15s. there remains 16s. which I subscribe under the denomination of shillings; then carrying 1 Pound which I borrowed to the lower place of Pounds, I say 1 in mind with 0 makes 1, which taken out of 4, there remains 3; again 9 out of 2, I cannot take, but 9 out of 12 (10 being borrowed and added to the said 2, according to the fourth Rule of this Chapter) and there remains 3. Lastly, 1 (for the 10 that was borrowed) being taken out of 1, there remains nothing; and so at last I find, that if A. being indebted to B. in 124—11s. — 7d. — 1f. hath paid in part thereof 90l. — 14s. — 10d. — 3f. there remains yet undischarged 33l. — 16s. — 8d. — 2f.

VI. When many numbers are given to be subtracted from a number propounded, you must first add those numbers together, according to the Rules of the third Chapter, and then the sum found is to be subtracted from the number first propounded. *Example*, A. being indebted unto B. in 3240l. paid thereof at one time 700l. at a second payment 1236. and at a third 305l. the Question is how much of the debt remained undischarged? First, I add together the several sums paid, and find the total to be 2241l. this I subtract from 3240l. so there remains 999l. undischarged, as you see by the Operation in the Margin.

Subtraction of many numbers from one number

3240	The Debt
700	} Payments
1236	
305	
2241	Total paid
999	rest unpaid

The Debt l. s. d.
 500—00—00

Payments { 340—12—06
 13—18—03
 17—16—10

Paid in all 372—07—07

Rest unpaid 127—12—05

---10d. the Question is how much was in arrear? Here if the operation be prosecuted as before, it will appear that there was 127*l.*—12*s.*—05*d.* unpaid: See the Work in the Margin.

*The proof of
 Addition and
 Subtraction.*

VII. Addition is proved by Subtraction, and Subtraction by Addition. For having added divers numbers together, if you subtract one of them out of the sum, the remainder must be equal to all the rest, as you may observe by the Example following, viz. supposing

236

452

29

217

934

698

934

236

698

these 4 numbers are given to be added viz. 236, 452, 29, 217. and that their sum is found to be 934 (by the Rules of the third Chapter) it is required to prove whether the said sum be true or not; to perform this, I draw a line under the uppermost number 236, to separate it from the rest, and seek the sum of all the numbers given, except that uppermost, which sum I find to be 698. Then I subtract the said uppermost number 236 from 934, (the total sum of all the numbers first found) and because the remainder 698 is the same with

Another Example of
 like nature. A. being
 indebted to B. in 500*l.*
 paid in part thereof
 at one Payment 340*l.*
 —12*s.*—06*d.* at a
 a second Payment 13*l.*
 ---18*s.*—3*d.* at a
 third 17*l.*—16*s.*

with t
 perm
 bers n
 In
 tion,
 ber g
 must
 numb
 the
 made
 be ful
 9478
 der i
 if 5+
 2347,
 the n
 made

exper
 31.
 was fi
 M
 follow

Rec.

aid

est

roof

with the sum of all the numbers, excluding the uppermost, I conclude that the sum of all the numbers first found was truly computed.

In like manner is Subtraction proved by Addition, for if you add the remainder and the number given to be subtracted together, the sum must be equal to the

number out of which the Subtraction is made, so if 4347 be subtracted from 9478 the remainder is 5131, for if 5131 be added to

Example 1.

out of 9478

subtr. 4347

Rest 5131

Proof 9478

Example 2.

l. s. d.

24—13—07

19—15—08

04—17—11

24—13—07

4347, the sum is 9478, which is the same with the number out of which the Subtraction was made. Again, if a Servant receive 24*l.*—13*s.*—07*d.* and lay out or disburse 19*l.*—15*s.*—08*d.* there must remain in his hands—4*l.*—17*s.*—11*d.* for this being added to 19*l.*—15*s.*—08*d.* which was the Money he expended, the sum will be equal to 24*l.*—13*s.*—07*d.* (being the Money wherewith he was first charged.)

More Examples of Subtraction are these that follow.

Subtraction of English Money.

	l.	s.	d.	f.		l.	s.	d.	f.
Rec.	3090	—10	—07	—1		24	—00	—00	—0
aid	0099	—14	—08	—3		05	—17	—11	—3
Rest	2990	—15	—10	—2		18	—02	—00	—1
Proof	3090	—10	—07	—1		24	—00	—00	—0

Sub-

Subtraction of Troy Weight.

lb.	oz.	p.w.	gr.	oz.	p.w.	gr.	
Boug.	352	10	13	15	205	13	19
Sold.	019	11	16	18	118	16	20
Rest.	332	10	16	21	86	1	23
Proof	253	10	13	15	205	13	19

Subtraction of Avoirdupois Weight.

C.	q.	lb.	lb.	oz.	dr.
Bought	256	2	25	13	12
Sold	079	3	00	14	13
Rest.	176	2	24	14	15
Proof	256	2	25	13	12

Subtraction of Superficial Measures of Land.

Acres, Roods, Perch.	A.	R.	P.
Bought	780	2	35
Sold	090	3	36
Rest	689	2	39
Proof	780	2	35

Questions to exercise Addition and Subtraction.

Qu. 1. Two Persons, A. and B. owe several debts the lesser debt being that of A. is 3045^l. the difference of their debts is 104^l. what is the debt of B?
 Ans. 3149^l.

Quest.

Quest. 2. Two Persons A. and B. are of several Ages, the Age of the elder, being that of A. is 70, the difference of their Ages is 19, what is the Age of B? *Answer,* 51.

Quest. 3. What number is that which being added to 168 maketh the sum to be 205? *Ans.* 37.

Quest. 4. The sum of two Numbers is 517, the lesser is 40, what is the greater? *Ans.* 477.

Quest. 5. A certain Person born in the Year of our Lord 1616, desired to know his Age in the Year 1676. what was his Age? *Ans.* 60.

Quest. 6. The greater of two numbers is 130, their difference is 49; what is the lesser number? *Ans.* 81.

CHAP. V.

Multiplication of whole Numbers.

Multiplication teacheth how by two Numbers given to find a third, which shall contain either of the numbers given so many times as the other contains 1 or unity.

II. Of the two numbers given in Multiplication, one (which you will) is called the Multiplicand, and the other the Multiplier, (or both are called factors.)

III. The number sought, or arising by the multiplication of the two numbers given, is called the Product, the Fact, or the Rectangle: so if 5 be given

given to be multiplied by 3 or 3, by 5, the product is 15, that is 3 times 5 or 5 times 3 makes 15: and here 5 may be called the Multiplicand, and 3 the Multiplier, or 3 may be called the Multiplicand, and 5 the Multiplier; and as 3 (one of the two numbers given) containeth 1 or unity thrice, so 15 the product containeth 5 (the other given number) thrice; likewise as 5 (one of the given numbers) contains unity 5 times, so 15 (the product) contains 3 (the other given number) 5 times.

IV. Multiplication is either single or compound.

*Single Multipli-
cation.*

V. Single Multiplication, is, when the Multiplicand and Multiplier consist each of them of one onely figure as in the last Example. In like manner if you multiply 9 by 5, the product is 45, this is likewise single Multiplication: Now the several varieties of single Multiplication are well express'd in the Table following, usually called *Pythagoras's Table*.

The Table of Multiplication.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The use of the Table is this, having one figure given

given to be multiplied by another to know the product of them, find the multiplicand in the top of the *Table*, and the multiplier in the first Column thereof towards the left hand; this done, in the angle of position just against those two figures you shall find the product. So 9 being given to be multiplied by 5 I find 9 in the top of the *Table*, and 5 in the first Column towards the left hand, then carrying my Eye from 5 in a right line equidistant to the upper side or top line of the *Table*, until I come to that square which is directly under 9, I find 45, which is the product required. The particular varieties of this *Table* ought to be learned by heart, (that is, a Man must be able to give the Product of any single multiplication, without the least pause or stay) before he can readily work compound multiplication, as will further appear hereafter.

VI. Compound Multiplication, is when the multiplier and multiplicand, either one or both, consists of more figures than one.

Compound Multiplication.

VII. In compound Multiplication, when the numbers given do end with significant figures, place them as in Addition and Subtraction. So 134 being given to be multiplied by 2, place them thus: Then proceeding to the multiplication
 say thus: 2 times 4 is 8, which write under the line in the rank of your multiplying figure; again, say 2 times 3 is 6, which likewise write under the line in the next rank: Lastly, 2 times 1 is 2, which being likewise written down under the line in the next rank, the product is discovered to be 268, and the Work will stand as in the Margin.

134

2

268

VIII. When

VIII. When the Multiplier consists of more figures than one, as many figures as it hath, so many several products must be subscribed under the line, which at last being added into one sum, gives you the total product of all. So 1232 being given

$$\begin{array}{r}
 1232 \\
 \times 23 \\
 \hline
 3696 \\
 2464 \\
 \hline
 28336
 \end{array}$$

$$\begin{array}{r}
 1321 \\
 \times 123 \\
 \hline
 3963 \\
 2642 \\
 1321 \\
 \hline
 162483
 \end{array}$$

to be multiplied by 23, the operation thereof will stand thus, for 1232 being multiplied by 3, (according to the last Rule) the product is 3696. Again, 1232 being multiplied by 2, the product is 2464, which several products after they are placed in their due order, (that is, the first figure arising in each product under his respective multiplying figure) and added together, produce 28336, the product required: In like manner 1321 being given to be multiplied by 123, the product is 162483, and the operation will stand as you see in the Margin.

IX. When the product of any of the particular figures exceeds ten, place the excess under the line as before, and for every ten that it so exceeds, keep one in mind to be added to the next Rank.

Example, 3084 being given to be multiplied by 36, the work will stand thus; for 6 times 4 being 24, I write 4 under the line, and reserve 2 in mind for the two tens; then I say 6 times 8 is 48, unto which if I add 2 kept in mind, the whole is 50, wherefore subscribing 0 in the next rank under the line 0 (because there is no excess of 50 above 5 tens) I reserve 5 in mind for the 5 tens; again, I say 6 times

$$\begin{array}{r}
 3084 \\
 \times 36 \\
 \hline
 18504 \\
 0252 \\
 \hline
 111024
 \end{array}$$

nothing

nothing is nothing, to which adding 5 that I kept in mind, the whole will be but 5, which I likewise subscribe under the line in the next rank; again 6 times 3 is 18, which (in regard 3 is the last figure of the multiplicand) I write wholly down; so that the particular product arising from the multiplying figure 6 is 18504. In like manner proceeding with the multiplying figure 3, the particular product arising will be 9252. Lastly, these several products being placed in due order, and added together (after the manner of the 8th Rule of this Chapter) will give 111024, which is the total product arising from the multiplication of 3084 by 36, and the operation will stand as in the Margin. After the same manner if 5073 be given to be multiplied by 256, the product will be found to be 1298688, and the operation will stand as you see in the example.

$$\begin{array}{r}
 5073 \\
 \times 256 \\
 \hline
 30438 \\
 25365 \\
 10146 \\
 \hline
 1298688
 \end{array}$$

X. When the two numbers given to be multiplied, do one or both of them end with a Cypher or Cyphers towards the right hand, multiply the significant figures in both numbers, one by the other, neglecting such Cyphers, and when the multiplication of the significant figures is finished, annex on the right hand of the number produced by the multiplication the Cypher or Cyphers with which one or both of the numbers first given did end, so will the whole give you the true product demanded: *Example*, 43100 being given to be multiplied by 15000 the product will be found for omitting the Cyphers which stand

$$\begin{array}{r}
 43100 \\
 \times 15000 \\
 \hline
 2155 \\
 431 \\
 \hline
 646500000
 \end{array}$$

in

in the last places towards the right hand as well in the multiplicand as the multiplier, I multiply the significant figures 431, by the figures 15 (according to the former Rules,) so there will arise 6465, to which annexing on the right hand all the Cyphers before omitted, the true product will be 64650000: more Examples hereof are these following.

$$\begin{array}{r}
 43125 \\
 \times 1500 \\
 \hline
 215625 \\
 64687500 \\
 \hline
 64687500
 \end{array}$$

XI. When in the multiplier Cyphers are included between significant figures, multiply by the said significant figures, neglecting such Cyphers or Cypher, but observe diligently to set the particular products of the significant figures in their due places according to the 8th Rule of this Chapter. So if 56324 be given to be multiplied by 20006, I first multiply the whole multiplicand 56324 by 6, and place the product orderly underneath the line, then passing over the three Cyphers, I multiply 56324 by 2 and place 8 (which is the first excess of this particular product) directly under the multiplying figure 2, and the rest in their order, so at last the true product will be found 1126817944, and the Work will stand as you see in the Example.

More

More Examples hereof are these that follow.

$$\begin{array}{r} 3094 \\ 104 \\ \hline 12376 \\ 3094 \\ \hline 321776 \end{array}$$

$$\begin{array}{r} 23765 \\ 10302 \\ \hline 47530 \\ 71295 \\ 23765 \\ \hline 244827030 \end{array}$$

Note, That one of the principal *cautions* to be observed in Multiplication, is the due placing of the particular Products arising by each multiplying Figure; and that may be performed either by taking care to place the first Figure or Cypher which riseth in each Product under the respective multiplying Figure; or at least the first place arising in the second Product must stand under the second place of the first Product, and the first place of the third particular Product, under the third place of the first, &c.

XII. When a number is given to be multiplied by a number that consists of 1 (or an unit) in the first place towards the left hand, and a Cypher or Cyphers on the right hand of such an unit (such are 10, 100, 1000, 10000, &c.) the multiplication is performed by annexing the Cypher or Cyphers of the multiplicator at the end (to wit on the right hand) of the multiplicand; so if 326 be given to be multiplied by 10, the Product is 3260; if by 100, the Product is 32600; if by 1000, the Product is 326000; in like manner if 170 be multiplied by 10, the Product is 1700; if by 100, 17000, &c.

XIII. When more numbers than two are given to be multiplied one by the other, that kind of Multiplication

Continual Multiplication.

is called *Continual*, and is thus performed, viz. first multiply any two of the Numbers given one by the other, then multiply the Product by another of the Numbers given, and this Product by the Fourth Number given (if there be so many) and in that order till every one of the given

Numbers hath been made a Multiplier, so the last Product is the true Product required. Example, If 4, 18, and 22 were given to be multiplied continually, first 18 multiplied by 4 produceth 72, which multiplied by 22 (the third Number) produceth 1584, the last Product, or Number required. See the Work in the Margin. The Proof of Multiplication is by Division, as will appear by the next Chapter.

18
4
—
72 Prod. 1.
22
—
144
144
—
1584 Prod. 2.

CH A P. VI.

Division by whole Numbers.

I. **D**ivision is that by which we discover, how often one Number is contained in another; or (which is the same) it sheweth how to divide a Number propounded into as many equal parts as you please.

II. In Division there are always three remarkable Numbers which are commonly called by these Names, the *Dividend*, the *Divisor*, and the *Quotient*.

III. The *Dividend* is the Number given to be divided into equal Parts.

IV. The

IV. The Divisor is the Number by which the Dividend is to be divided; that is, it is the number which declareth into how many equal parts the Dividend must be divided.

V. The Quotient is the number arising from the division and sheweth one of the equal parts required; so if 15 were given to be divided by 5, or into 5 equal parts, the number arising, or one of the equal parts will be 3, for 5 is found three times in 15: And here 15 is the *Dividend*, 5 the *Divisor*, and 3 the *Quotient*.

VI. Division being the hardest Lesson in Arithmetick, must be heedfully intended by the Learner, for whose ease I shall use my utmost endeavours to make the way smooth by Rules and Examples, beginning with the easiest first, which will be in that case when the Divisor consists of one figure only; for Example, Let it be required to divide 192, by 8, or 192 pounds into 8 equal parts or shares; here 192 is the *Dividend*, 8 is the *Divisor*, and the *Quotient* or one of the equal parts is sought.

Division by a single figure.

VII. Place a crooked line at each end of the *Dividend*, that on the left hand serving for the place of the *Divisor*, and that on the right for the *Quotient*; then if the *Divisor* be a single figure, subscribe a point under the first figure of the *Dividend* towards the left hand, if such first Figure be either equal unto, or greater than the *Divisor*; but if such first figure be less than the *Divisor*, put a point under the next place of the *Dividend*; which number so distinguished by the point may be called the *Dividual*; so in the example

What the Dividual is.

8) 192 (

D 2

given

given in the 6th Rule, 192 being the *Dividend* and 8 the *Divisor*, I subscribe a Point under 9, not under 1, because it is less than the *Divisor*. This done, the *Dividual*, or Number whereof the Question must be asked, is 19.

VIII. Having thus prepared the Numbers, ask how often the *Divisor* is contained in the *Dividual*, and write the Number which answers the Question in the *Quotient*; then multiply the *Divisor* by the Number placed in the *Quotient*, and subscribe the Product underneath the *Dividual*. Lastly, having drawn a Line under the Product, subtract it from the *Dividual*, and subscribe the remainder orderly underneath the Line. So demanding

8) 192 (2 how many times the *Divisor* 8 is found
 16 · in the *Dividual* 19, the Answer is two
 — times, wherefore I write 2 in the *Quo-*
 3 *tient*; then multiplying the *Divisor* 8

by 2 (the Number placed in the *Quotient*) the Product is 16, which I subscribe orderly under the *Dividual* 19; and after a line is drawn underneath the Product 16, I subtract it from the *Dividual* 19, and place the remainder 3 underneath the Line.

IX. Put another point under the next place of the *Dividend* towards the Right hand, and bring down the Figure or Cypher standing in that place to the remainder; that is, set it next after it, so the whole will be a new *Dividual*: Thus a point being placed under 2 which stands in the next place

8) 192 (2 of the *Dividend*, I write 2 next after
 16 · (to wit on the Right hand of) the re-
 — mainder 3, so is 32 a new *Dividual*, or
 32 Number whereof the Second Question

must be asked, and the Work will stand as you see in the Example.

X. A

X A new *Dividual* being set apart, renew the question, and proceed according to the 8th Rule of this Chapter. Thus demanding how often the *Divisor* 8 is found in the *Dividual* 32, the answer is four times, wherefore I write 4 in the *Quotient*, then multiplying the *Divisor* 8 by 4 (the Figure last placed in the *Quotient*) the product is 32, which I subscribe under the *Dividual* 32, and after a

$$\begin{array}{r} 8 \overline{) 192} \quad (24 \\ \underline{16} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

line is drawn underneath, I subtract the Product 32 from the *Dividual* 32, and there being no remainder, I subscribe 0 under the line, so the whole Work being finish'd, the *Quotient* is found to be 24, and the operation stands as you see in the Example; wherefore I conclude, if 192 pounds be equally divided amongst 8 Persons, the share of each Person will be 24 Pounds.

A second Example. Let it be required to divide 936 Pounds into 9 equal parts; having distinguished the first *Dividual* by a point, (according to the 7th Rule of this Chapter) I demand how often the *Divisor* 9 is found in the *Dividual* 9 and finding it once contained in it, I write 1 in the *Quotient*; then multiplying the *Divisor* 9 by 1, the Product is 9, which I subscribe under the *Dividual* 9, after this, a line being drawn under the Product 9, I subtract it from the *Dividual* 9, and there being no remainder, I place a 0 underneath the line, as you see in the Example.

$$\begin{array}{r} 9 \overline{) 936} \quad (1 \\ \underline{9} \\ 0 \end{array}$$

Again, placing a point under 3 which stands in the next place of the *Dividend*, I transcribe the said 3 next after the remainder 0 for a new *Dividual*, then asking

$$\begin{array}{r} 9 \overline{) 936} \quad (10 \\ \underline{90} \\ 03 \end{array}$$

how often the *Divisor* 9 is contained in the *Divi-*
dual 3, and not finding it once contained therein,
 I write 0 in the *Quotient*, and now because the
 Product which ought to arise from the Multiplica-
 tion of the *Divisor* by 0 (the Cypher last placed
 in the *Quotient*) amounts to 0, the *Dividual* 3, out
 of which that Product should have been subtracted,
 remains the same without Alteration; wherefore
 after a Point is subscribed under Six, the next place

9) 936 (104

$$\begin{array}{r}
 9 \\
 \hline
 036 \\
 \hline
 36 \\
 \hline
 0
 \end{array}$$

of the *Dividend*, I annex 6 to the
Dividual 3, so there will be a new
Dividual, to wit 36, then demand-
 ing how often the *Divisor* 9 is
 found in the *Dividual* 36, the An-
 swer will be 4 times; wherefore I
 place 4 in the *Quotient*, and multi-
 plying the *Divisor* 9 by 4, the Pro-
 duct is 36, which I subscribe under, and subtract
 from the *Dividual* 36, so the remainder is 0, thus the
 whole Work being finisht, the *Quotient* is found to be
 104, as you see in the *Example*; wherefore I conclude,
 if 936 l. be divided equally amongst 9 Persons, the
 share of each will be 104 l. In like manner if 296163
 be divided by 7, the *Quotient* will be 42309.

The Substance of
 Division by what
 Method soever.

The whole Work of Division is
 briefly contained in this following
 Verso.

Dis quot, multiplica, subduc, transferque secundum.

Or thus,

First you must ask how oft, in *Quotient* Answer make:
 Then Multiply, Subtract, a new *Dividual* take.

A Compendious
 way of dividing
 by a single Figure.

XI. When in the Division the
Divisor consists of a single Figure
 only, the *Quotient* may be written
 down,

Chap. VI. *by whole Numbers.*

43

down, and all the operation performed in mind, without writing down any part thereof, so 82506 being given to be halved, or divided into two equal parts, the Work will be thus: The *Divisor* 2 is found in 8 four times; 2)82506(41253 in 2 once; in 5 twice, and there will remain 1, which 1 being supposed to stand before (to wit, on the Left hand of) the Cypher, makes 10, then I say 2 is found in 10, 5 times; and last of all in 6, 3 times; so that the true *Quotient* or one half of the given Number 82506 is found to be 41253.

In like manner, if 82506 be given to be divided by 3, or into 3 equal parts, the Work will be thus, the *Divisor* 3 is 3)82506(27502 found in 8 twice, and there will remain 2, which 2 being supposed to stand before (to wit, on the Left hand of) the following 2, makes 22; then I say 3 is found in 22, 7 times; in 15, 5 times; in 0 not at all, and lastly, in 6 twice; so that the true *Quotient* or one of the 3 equal parts required is 27502. After the same manner may Division be wrought by any single Figure, without much charge to the Memory.

Note, Here the *Learner* may ask what shall be done with the last *Remainder*, if any happen, when the *Division* is finished? For a full

A Note, concerning the Remainder after the Division is ended, if any happen.

Answer to this, I refer the *Reader* to the *Note* in the Fifth Rule of the Seventh Chapter; yet I shall here propound an Example where the said case happens, viz. Let it be required to divide 351 by 8, or 351 Pounds equally amongst 8 Persons; now if the operation be prosecuted according to the former Rules, the *Quotient* will be found to be 43, and after the *Division*

$$\begin{array}{r} 7 \\ 8 \overline{) 351} \quad (43 \\ \underline{32} \\ 31 \\ \underline{24} \\ 7 \end{array}$$

D 4

is finish'd, there will remain 7, that is, each Person must have 43 Pounds, and there will be an overplus of 7 Pounds, which must be also divided equally among the 8 Persons, but that cannot be done till the 7 Pounds be reduced into Shillings, and then those Shillings must be divided by 8 to give every Person his due share of the Shillings contained in the said 7 Pounds; again, if there yet remain any surplufage of Shillings, they must be reduced to Pence, which must also be divided by 8, to give every Person his due share of Pence: So that when this Question is fully answered, each Persons share will appear to be 43*l*.—17*s*.—6*d*. But how the before-mentioned *Reduction* is performed, will be made manifest in the fifth Rule of the next Chapter.

Division by two or more figures, the first and easiest Method.

XII. When the Divisor consists of two, three, or how many places soever, the operation is more difficult than the former, but depends upon the same grounds, and therefore the Learner being well vers'd in the proceeding method of dividing by a single figure, will the more readily understand these that follow, which are two, whereof the first is the easier, but the latter more expeditious, and that which indeed is principally to be aimed at: For an example of the former, let it be required to divide 4112772 by 708, or (which is the same) to divide 4112772 into 708 equal parts.

First, A *Table* is to be made to shew at first sight any *Multiple* or *Product* of the *Divisor*, it being taken twice, thrice, or any number of times under ten, so having first written down the *Divisor* it self 708, and drawn a line on the right hand thereof, I place 1 on the right hand of the line directly against

gainst the *Divisor*; then underneath the *Divisor* 708, I subscribe the double thereof, which is 1416, and place the figure 2 directly against the said double, to wit, on the other side of the line. Again, adding 1416 (to wit the double of the *Divisor*) to the *Divisor* itself 708, the sum is 2124 for the

The *Divisor*. 708

Multiples of the <i>Divisor</i> .	1416	2
	2124	3
	2832	4
	3540	5
	4248	6
	4956	7
	5664	8
	6372	9

triple of the *Divisor*; this triple I subscribe under the double, and place 3 on the other side of the line right against the triple. Again adding 2124 (the triple of the *Divisor*) to the *Divisor* 708, I find 2832 for the quadruple of the *Divisor*, which quadruple I subscribe under the Triple, and proceeding in like manner, at last the table is finish'd, which really shews the *Divisor*, with the duple, triple, quadruple, quintuple, sextuple, septuple, octuple, and nonuple of the *Divisor*.

Now for a proof of the said Table, adding the last number thereof, to wit, 6372 (which was found to be nine times the *Divisor*) to the *Divisor* 708, I find the sum to be 7080, which (by the 12th Rule of the fifth Chap.) is evident ten times the *Divisor*; wherefore I conclude that the Table is true, in regard that the last number thereof is derived from all the superior numbers.

The Table of Multiples or Products of the *Divisor* being thus prepared, write down the *Dividend* on the right hand of the *Divisor*; then distinguish by a point so many of the foremost places of the *Dividend* towards the left hand, as are either equal in value (being consider'd apart) to the *Divisor*, or which

708	1	4112772	(58	9
1416	2	3540	...	
2124	3	5727		
2832	4	5664		
3540	5	6372		
4248	6	6372		
4956	7	0		
5664	8			
6372	9			

which being greater, yet come nearest to the Value thereof, thus I subscribe a point under 2, thereby setting apart 4112, being the fewest of the foremost places which will contain the Divisor 708, so is 4112 the *Dividual* (or Number whereof the first Question must be asked;) then demanding how often the Divisor 708 is contained in the *Dividual* 4112, the Answer will be found by the Table to be five times, for looking in the Table I cannot find the *Dividual* exactly, but I see that 6 times the Divisor is the next greater than the *Dividual* 4112, and five times is the next lesser, wherefore I write 5 in the *Quotient*, and the Number in the Table which stands against 5, to wit, 3540, I subscribe under the *Dividual* 4112, then having drawn a Line underneath, I subtract 3540 (which is five times the Divisor) from the *Dividual* 4112, and subscribe the Remainder 572 underneath the Line; that done, I put a Point under the next place of the *Dividend* towards the Right hand, and because the Figure 7 stands in that place, I transcribe 7 next after the Remainder 572, so there is 5727 for a new *Dividual*.

Then demanding how often the Divisor 708 is contained in the *Dividual* 5727, the Answer will be found by the Table to be 8 times, for looking in the Table I find that 9 times the Divisor is the next greater, but 8 times is the next lesser than the *Dividual*, wherefore I write 8 in the *Quotient*, and the

the number in the *Table*, which stands against 8, to wit, 5664 I subscribe under, and subtract from the *Dividend* 5727, placing the remainder 63 underneath the line.

Again, I put a point under the next place of the *Dividend*, where I find the figure 7, and therefore transcribing 7 next after the remainder 63, the new *Dividend* will be 637, then demanding how often the *Divisor* 708 is contained in the *Dividend* 637, and not finding it once contained therein, I write 0 in the *Quotient*, and since in this case (that is when a Cypher answers the Question) the *Dividend* remains the same without alteration, the figure or cypher standing in the next place of the *Dividend* is to be transcribed after the *Dividend* for a new *Dividend*, so writing 2 next after 637, the new *Dividend* 6372, wherefore demanding how often the *Divisor* 708 is contain'd in 6372, I find by the *Table* it is contain'd in it 9 times, wherefore writing 9 in the *Quotient*, and placing the number which stands against 9 in the *Table*, to wit, 6372 under the *Dividend* 6372, and subtracting it from the *Dividend* there will remain 0. Wherefore I conclude 4112772 be divided by 708, or into 708 equal parts, the true *Quotient* or one of the equal parts required is 5809. *Divisor* 188 | 1) 20304 (108

In like manner if 20304 be divided by 188, that is into 188 equal parts, the *Quotient* arising or one of those equal parts will be 108, and the operation will stand as you see.

Multiples of the Divisor	188	1)	20304	(108
	376	2		
	564	3	188	
	752	4	1504	
	940	5	1504	
	1128	6	0	
	1316	7		
	1504	8		
	1692	9		

The preceding Method of *Division* by the help of a *Table* of the *Multiples* or *Products* of the *Divisor*, as it is most easie, so in some Cases namely, where the *Divisor* is great, and a *Quotient* of many places is required, (as in calculating *Tables* of *Interest*, *Astronomical Tables*, and such like) it excels all other ways of *Division*, both in respect of *Certainty* and *Expedition*, but for common Practice it is too tedious, and therefore I shall proceed to the choicest practical Method.

XIII. I now come to the last and principal Method of *Division*, when the *Divisor* consists of many Places, which to such as have the *Table* of *Multiplication* by Heart will not be difficult. For Example, Let 56304 be a Number given to be divided by 184, that is, into 184 equal parts, and the *Quotient* or one of the equal parts is required.

First, Distinguish by a point (as before) so many of the foremost places of the *Dividend* towards the Left hand, as are either equal in value (when they are consider'd apart) to the *Divisor*, or else, which being greater, yet come nearest unto it, thus I subscribe a Point under the Figure 3, thereby setting apart 563, being the fewest of the foremost places which will contain the *Divisor*; so is 563 the *Dividual*, or Number whereof the first Question must be asked. Having thus prepar'd the Numbers, I demand how often the *Divisor* 184 is contained in the *Dividual* 563; and since to answer this Question and such like, there is a necessity of tryal, it will be requisite to shew how this tryal may fitly be made; first there-

fore

more compare the Number of places in the *Dividual* with the Number of Places in the *Divisor*, and when the Number of Places is the same in both, let me be asked how often the first or extream Figure of the *Divisor* towards the Left hand is contained in the first Figure of the *Dividual* towards the same Hand; so here demanding how often 1 is contained in 5, the Answer is 5 times; whence I infer that the *Divisor* 184 is not contained oftner than 5 times in the *Dividual* 563 (for 6 times 184 is manifestly greater than 563) but whether it be contained 5 times in it or not, Examination must be made either by multiplying (in some by place) the *Divisor* 184 by the said 5, and comparing the Product with the *Dividual*, 563; or else thus, saying 5 times (to wit the 1 in the *Divisor*) is contained in 5, to wit, the first Figure of the *Dividual* 563, 5 times, but then 8 the following Figure of the *Divisor*, cannot be found 5 times in 6, the following Figure of the *Dividend*, and consequently the *Divisor* 184 is not contained 5 times in the *Dividual* 563; wherefore I make another tryal to see whether it may be contained 4 times in it or not, saying 4 times 1 is 4, which is found in 5, and there will remain 1; but when 4 times 8, which is 32, cannot be had in 16 for the 1 before remaining being supposed to stand on the Left hand of 6 maketh 16) hence I conclude again, that the *Divisor* 184 is not contained 4 times in the *Dividual* 563; wherefore I make another tryal to see whether it may be contained 3 times in it or not, saying 3 times 1 is 3, which is found in 5, and there will remain 2; again, 3 times 8 is 24, which is found in 26 (for the 2 before remaining being supposed to stand before the 6 in the

the

the *Dividual* makes 26) and there will remain 2. Lastly, three times 4 is 12, which is likewise found in 23, (for the 2 remaining in the 26 being supposed to stand before the 3 in the *Dividual* makes 23) where- by I see that the *Divisor* 184 is contained 3 times in the *Dividual* 563, wherefore I write 3 in the *Quotient*, and proceeding according to the 8th Rule of this Chapter, I multiply the *Divisor* 184 by 3 (the figure placed in the *Quotient*) so the *Product*, is 552, which I sub-

scribe orderly underneath the *Divi- dual* 563, then having drawn a line underneath the said *Product*, I subtract it from the *Dividual*, and sub- scribe the remainder which is 11 under the line.

Again, according to the 9th Rule of this Chapter, I bring down 0 which stands in the next Place of the *Dividend*, to the remainder 11, so there is 110 for a new *Dividual*, then demanding how often the *Divisor* 184 is found in the *Dividual* 110, and not finding it once contained in it, I write 0 in the *Quotient* (which is to be done as often as the Que- stion is answered by nothing;) now because the *Product* arising from the multiplication of the *Di- visor* by 0 (the Cypher last placed in the *Quotient*) amounts to 0, the *Dividual* 110

out of which that *Product* should be subtracted, remains the same without alteration; where- fore after a point is sub- scribed under 4 the following place of the *Dividend*, I annex

4 to the last *Dividual* 110, and there will be a new *Di- vidual*, to wit, 1104; and here the Question at large is to know how often 184 is found in 1104: but to lessen

the

the trial, because the *Dividual* consists of one place more than is in the *Divisor*, it must be asked how often the first Figure of the *Divisor* on the Left hand is contained in the two foremost places of the *Dividual* towards the Left hand, *viz.* I demand how often 1 is contained in 11, and although it may be had 11 times, yet I need never begin the trial above 9 times, therefore I make trial with 9, saying 9 times 1 is 9, which is found in 11, and there will remain 2; but then 9 times 8 which is 72 cannot be found in 20 (20 because the 2 remaining being supposed to stand before 0 in the *Dividual* makes 20) therefore I make trial with 8, saying 8 times 1 is 8, which is found in 11, and there will remain 3, but then 8 times 8 cannot be had in 30 (30 because the 3 remaining being supposed to stand before the 0 or Cypher makes 30) therefore I make trial with 7, saying 7 times 1 is 7, which is found in 11, and there will remain 4; but then 7 times 8 cannot be had in 40, therefore I make trial with 6, saying 6 times 1 is 6, which is found in 11, and there will remain 5; then 6 times 8 is 48, which is found in 50, and there will remain 2; lastly, 6 times 4 is 24, which is found in 24, whereby at length I see that the *Divisor* 184 is contained 6 times in the *Dividual* 1104, wherefore I write 6 in the *Quotient*, and proceeding according to the 8th Rule of this Chapter, I multiply the *Divisor* 184 by 6 (the Figure last placed in the *Quotient*) so the *Product* is 1104, which being subscribed under, and subtracted from the *Dividual* 1104, the remainder is 0, so at last I conclude that the *Quotient* sought is 6.

Note,

Note, If the Figure assumed for the *Quotient* holds good upon tryal, as aforelaid, by two or three of the foremost places of the *Dividual*, it will for the most part hold throughout the *Dividual*; but this must be a perpetual Rule, that whensoever the *Product* of the multiplication of the *Divisor* by the figure placed in the *Quotient* happens to be greater than the *Dividual*, from which it ought to be subtracted, such *Product* must be struck out of the Work, and a lesser Figure is to be placed in the *Quotient*.

For a second *Example*, Let it be required to divide 15114220 by 2987, or into 2987 equal parts.

First, The *Divisor* 2987 being greater than 1511, (to wit, the four foremost places of the *Dividend*) I set a point under 4, thereby setting apart 15114 for a *Dividual*; then because the *Dividual* consists of one place more than the *Divisor*, I ask how often 2 (the first Figure of the *Divisor* towards the left hand) is contained in 15, (the two foremost places of the *Dividual*) and finding the answer to be 7 times, I infer thence that the *Divisor* 2987 cannot be contained more than 7 times in the *Dividual* 15114; but whether it will be contained 7 times in it or not, examination must be made, either by multiplying 2987 by 7 (in some by-place) and by comparing the *Product* with the *Dividual* 15114, or else by the manner of tryal before delivered in the last *Example*: So at length it will be discovered that the *Divisor* 2987 will not be found above 5 times in the *Dividual* 15114; wherefore (according to the 8th Rule of this Chapter) writing 5 in the *Quotient*, and multiplying 2987 by 5,

I sub.

k L
olds
e of
the
this
Pro-
gure
the
ted,
and
vide
11,
nd)
114
s of
Di-
(the
to-
on-
bre-
wer
987
Di-
ed 7
ei-
ace)
dual
eli-
dif-
und
fore
ing
5,
sub.

chap. VI.

by whole Numbers.

53

subscribe the product of that Multiplication, which is 14935, under the *Dividual* 15114, then drawing a line underneath the said Product, and subtracting from the *Dividual* 15114, I subscribe the remainder 179 under the line.

Again (according to the 9th Rule of this Chapter) bring down 2, the next 2987) 15114220 (50
see of the Dividend, to 14935

the said Remainder 179, the new Dividual will

1792; that done, asking how often the *Divisor* 2987 is contained in the *Dividual* 1792, and not finding it once contained in it, I write 0 in the quotient; and here because the question is answered by 0, the next place of the *Dividend*, to wit 2, be brought down to 2987) 15114220 (5060

Dividual 1792, so the *Dividual* is 17922.

then renewing the Question, and proceeding as before, at length the Division being finish'd, the quotient will be found 50 exactly, without any remainder; but if any Re-

mainder had hapned after the subtraction of the last product, it must have been prosecuted according to the note before given in the Example at the latter end of the 11th Rule of this Chapter.

In like manner if 1208939550 be divided by 299, or into 2999 equal Parts, the *Quotient*, or one of those equal parts, will be found 60450, and the Work will stand as you see here.

E

This

19999) 1208939550 (60450

19994

89995

79996

99995

99995

00

This latter method of Division is to be preferred before any of the common ways of dividing by dashing out of figures where the Steps of the Division are so

confounded (besides the burden upon the memory, by a promiscuous Multiplication and Division) that if any error happen, it can hardly be corrected without beginning the Work anew; But in the way before explained, the particular Multiplications, Subtractions, and Remainders, which belong to every figure of the Quotient, are so distinctly and clearly express'd, that if an error happen, the Work may easily be reformed.

XIV. So often as the question is repeated in Division, so many places there must be in the Quotient (which may be discovered by the number of Points placed under the *dividend*) and so many times is one and the same kind of operation repeated, the substance whereof is contained in the Verse before-mentioned at the end of the tenth Rule of this Chapter.

XV. When the *Divisor* consists of 1 or an unit in the extream place towards the left hand, and nothing but Cyphers towards the right, the division is performed by cutting off with a line so many places of the *Dividend* towards the right hand as the *Divisor* hath Cyphers; so the figures which

How the number of places in the Quotient may be discovered.

A Compendious way of dividing by 10, 100, 1000, &c.

which stand on the left hand of the line, give the Quotient, and those cut off to the right (if they be significant figures) are to be proceeded with as a surplusage or overplus remaining, according to the Note at the end of the eleventh Rule of this Chapter. So if 4720 ℓ . were given to be divided equally amongst 10 persons, the share of each would be 472 ℓ . also if the said 4720 ℓ . were to be divided equally amongst 100 persons, the share of each would be 47 ℓ . and there would be a surplusage or remainder of 20 ℓ . to be also subdivided amongst them, after the said 20 ℓ . are converted into shillings according to the fifth Rule of the next Chapter. Lastly, if the said 4720 ℓ . were to be divided amongst 1000 persons, the share of each would be 4 ℓ . and there would be a remainder of 720 ℓ . to be also divided as aforesaid. See the form of the Work in the Margin.

XVI. When the *Divisor* consists of any significant figure or figures in the first or foremost place or places towards the left hand, and nothing but a Cypher or Cyphers towards the right, cut off

Another Compendium in Division.

by a line so many places of the *Dividend* towards the right hand as the *Divisor* hath Cyphers towards the right; then divide the figures of the *Dividend*, which stand on the left hand of the line, by the figures in the *Divisor* which remain, when the said Cypher or Cyphers are omitted, remembering after the division is finished, to write down next after the last remainder the places of the *Dividend* which were first cut off: So if 36732 were given to be

divided by 20, the *Quotient* will be 1836, and there will remain 12, viz. if you cut off one place from the *Dividend* towards the Right hand (because the *Divisor* ends with one Cypher) and then di-

vide the rest, to wit, 3673 by 2 (according to the 11th Rule of this Chapter) there will arise in the *Quotient* 1836, and the last remainder after such Division is finish'd, will be 1, unto which if 2 (the Figure first cut off from the *Dividend*) be annexed, the Total remainder is 12.

In like manner if 7456787, were given to be divided by 304000, the *Quotient* will be 24, and there will remain 160787; viz. If you cut off 3 places from the *Dividend* towards the Right hand

(3 places, because the *Divisor* ends with 3 Cyphers) and then divide 756 by 304, there will arise in the *Quotient* 24, and the last remainder, after such Division is

finish'd, will be 160, unto which if 787 (the Places first cut off from the *Dividend*) be annex'd, the Total Remainder or Surplusage is 160787, which is to be proceeded with, as is directed in the Note at the latter end of the Eleventh Rule of this Chapter.

XVII. Division and Multiplication do interchangeably prove one another; for

The Proof of
Multiplication
and Division.

in Division if you multiply the *Divisor* by the *Quotient*, the Product will be equal to the *Dividend*: So in the

Example of the 13th Rule of this Chapter, if 184

the

Chap. VI. *by whole Numbers.* 57

the *Divisor* be multiplied by 306 the *Quotient*, the *Product* is 56304, which is the same with the *Dividend*; but when, after the whole Division is finished, any Figures remain of the last Subtraction, add them likewise to the *Product*: So in the last *Example* of the 16th *Rule* of this Chapter, the *Divisor* 304000 being multiplied by the *Quotient* 24, produceth 7296000, unto which if you add the Number remaining, to wit, 160787, the Summ is 7456787, which is the same with the *Dividend*. Again in Multiplication, if the *Product* be divided by the *Multiplicator*, the *Quotient* will give you the *Multiplicand*, or if the *Product* be divided by the *Multiplicand*, the *Quotient* will give you the *Multiplicator*: So in the First *Example* of the 9th *Rule* of the last Chapter, if the *Product* 111024 be divided by the *Multiplicand* 3084, the *Quotient* gives the *Multiplicator* 36.

There is also of Multiplication a *Common proof* argued from the *Multiplicand*, the *Multiplicator* and the *Product* by casting away Nines, but by that way of Proof (though rightly used) a *false Product* will be affirmed to be true: *Example*, If 3462 be multiplied by 786, the true *Product* is 2721132; but if I say 4953132 or 3153132 is the *Product* (or many others which may be given) the Proof by Nines will confirm them to be true *Products*, though they are false, as will be evident to such as know the *Rule*, which I mention here only to set a brand upon it, that it may be avoided by all lovers of Truth.

C H A P. VII.

Reduction.

IF Orasmuch as in *Money*, there are diversities of kinds, viz. in *England*, *Pounds*, *Shillings*, *Pence*, and *Farthings*; also divers kinds of *Weights*, *Measures*, &c. as hath been fully declared in the second Chapter; and because it is oftentimes required to find how many pieces of one kind of *Money* are equal in value to a given number of another (and so likewise of *Weights*, *Measures*, &c.) it will be convenient in this place to shew how that is performed, since thereby the *Rules* of *Multiplication* and *Division* before delivered will be exercis'd. This kind of operation is called *Reduction*.

He Reduction is either descending or ascending.

III. Reduction descending is, when some Integers of a Number of greater denomination being given, it is required to find how many Integers of a lesser denomination are equal in value to that given number of the greater: As when it is required to find how many *shillings* are contained in 30*l*. Likewise how many *pence* in 320*s*, or how many *hours* in 365 *days*, &c.

IV. Reduction ascending is, when some Integers of a number of lesser denomination being given, it is required to find how many Integers of a greater denomination are equal in value to that given number of the lesser: As when it is required to find how many *pence* are contained in 500 *farthings*: Likewise how many *shillings* in 348 *pence*: Or how many *days* in 864 *hours*, &c.

V. *Reduction descending* is performed by *Multi-
plication*, for if the given number of
Integers of a greater denomination
be multiplied by a number, which
expresseth how many Integers of the
lesser are equal to one of the Integers
given, the Product is the number of Integers of
the lesser denomination required.

*Reduction de-
scending is per-
formed by Multi-
plication.*

So 230*l.* of English Money will be reduced in-
to 4600*s.* for if 230 be multiplied by 20 (the num-
ber of *shillings* which are equal to 1 *pound*) the
product is 4600; in like manner
4600*s.* will be reduced into
55200*d.* for if 4600 be multi-
plied by 12 (the number of *pence*
contained in 1 *shilling*) the pro-
duct is 55200. Also 55200
pence being multiplied by 4 (be-
cause 4 *farthings* make a *penny*)
are reduced into 220800 *Far-
things*, as by the operation in the
Margin is Evident.

$$\begin{array}{r}
 230 \text{ pounds.} \\
 \underline{20} \\
 46000 \text{ shillings} \\
 \underline{12} \\
 92 \\
 46 \\
 \hline
 55200 \text{ pence.} \\
 \underline{4} \\
 220800 \text{ Farthings}
 \end{array}$$

The like method is to be
observed in *Weights, Measures,*
&c. So 345 *Ounces Troy* are
reduced into 6900 *penny weights*,
and 6900 *penny weights* to
165600 *Grains*, as by the
operation in the *Margin* you
may see.

$$\begin{array}{r}
 345 \text{ Ounces.} \\
 \underline{20} \\
 6900 \text{ penny W.} \\
 \underline{24} \\
 276 \\
 \underline{138} \\
 165600 \text{ Grains.}
 \end{array}$$

Note. By this Rule the Learner
is furnished with Skill to resolve
that case in Division, when the
dividend is less then the *divisor* :

Compare this with the
Note upon the last Ex-
ample of the 11th Rule
of the 6th Chapter.

Example. Let it be required to divide 7 Pounds of English Money equally amongst 8 Persons; here it is evident that the *Dividend* 7 is less than the *Divisor* 8; that is, the number of Pounds is less than the number of Persons, and consequently each share must be less than a Pound; so that in effect it is required to find how many *Shillings* and *Pence* belong to each Person for his share: First, therefore reduce the 7 Pounds into *Shillings*, which will be 140, these divided by 8 give 17 *Shillings* to each Person, and there will yet be a remainder of 4 *Shillings* to be also equally divided into 8 parts, but these 4 *Shillings* must be first reduced into *Pence*, which will be 48, then dividing 48 by 8, the *Quotient* will give 6 *Pence* more to every Person: So at last it appears that if 7 Pounds of English Money be equally divided into 8 parts, the entire *Quotient* (or one of the equal shares) will be 17 *Shillings* and 6 *Pence*.

In like manner, if 354 Pounds of English Money be given to be divided equally amongst 125 Persons, the share of each will be found to be 2 Pounds, 16 *Shillings*, 7 *Pence*, 2 *Farthings*, and somewhat more, but the parts of a *Farthing* being of no moment (and not properly to be handled in this place) are neglected.

Compare these two Examples with the last Example of the Eleventh Rule of the sixth Chapter.

In *Reduction* descending, the Learner may receive help by the subsequent *Tables*.

1. Of English Moneys.

Pounds	Multiplied by	20	Produce	Shillings.
Shillings		12		Pence.
Pence		4		Farthings.

2. Of Troy Weight.

Pounds	Multiplied by	12	Produce	Ounces.
Ounces		20		Penny Weights.
Penny Weight		24		Grains.

Also in Apothecaries Weights.

Ounces Troy	Multiplied by	8	Produce	Drams.
Drams		3		Scruples.
Scruples		20		Grains.

3. Of Averdupois Weights.

Hundred W.	by	4	Produce	Quarters.
Quarters		28		Pounds.
Pounds		16		Ounces.
Ounces		16		Drams.

4. Of Liquid Measures.

Hogsheads	by	63	Produce	Gallons.
Gallons		2		Pottles.
Pottles		2		Quarts.
Quarts		2		Pints.

5. Of

5. Of Dry Measures:

Quarters	8	Bushels.
Bushels	4	Pecks.
Pecks	2	Gallons.
Gallons	2	Pottles.
Pottles	2	Quarts.
Quarts.	2	Pints.

6. Of Long Measures.

Englifo Miles	8	Furlongs.
Furlongs	220	Yards.
Yards	3	Feet.
Feet	12	Inches.
Inches	3	Barley Corns.

Also,

Yards	4	Quarters.
Ells	5	Quarts of Yards.
Quarters	4	Nails.

7. Of Superficial Measures of Land.

Acres	4	Roods.
Roods	40	Perches or Poles.

8. Of Time.

Weeks	7	Days.
Days	24	Hours.
Hours	60	Minutes.

To reduce Integers of divers Denominations into the lowest of those Denominations.

VI. Integers of divers Denominations may be reduced into the last of those Denominations according to the fifth Rule foregoing, by descending orderly to the next inferior Denomination,

nation; and adding to each Product such Integers (if there be any) which are of the same name.

So 12 Pounds 13 shillings, and 10 pence may be reduced into 3046 pence in this manner, viz. 12 l. multiplied by 20 (because 20 s. make one l.) produce 240 shillings, unto which adding 13 s. the sum is 253 Shillings. Again, 253 s. multiplied by 12 (because 1 Shilling is equal to 12 Pence) produce 3036 Pence, unto which if 10 pence be added, the sum is 3046 pence, as by the operation in the Margin is manifest.

l.	s.	d.
12	13	10
20		
240		
add 13		
253	shillings.	
12		
506		
add 10		
3036		
add 10		
3046	pence.	

But after that general Method is well understood, the Work of the last Example, and such like may be contracted thus; viz. To convert 12 Pounds, 13 shillings, 10 pence, all into pence, First 12 multiplied by 0, (which stands in the Units place of 20) produceth 0, but instead of 0, I write down 3 under the line (to wit, the three that stands in the units place of the 13 shillings in the sum propounded;) Then I proceed to multiply 12 by 2, saying twice 2 is 4, to which adding 1 for the ten in the said 13 shillings, it makes 5, which I set on the left hand of 3 before written; Lastly, twice 1 is 2, which I set on the left hand of 5; And so 12 pounds, 13 shillings and 10 pence are converted into 253 shillings.

l.	s.	d.
12	13	10
20		
253	Shillings.	
12		
516		
add 10		
253		
3046 pence		

It remains to multiply the said 253 by 12 (because 12 Pence makes 1 Shilling) and to add 10 to the Product, which may be done thus; First, twice 3 is 6, to which adding 10 (to wit, 10 Pence in the Summ first propounded) it makes 16, wherefore (according to the Rule of Multiplication) I set 6 under the Line, and keep one in mind; Again, twice 5 with 1 in mind making 11, I write down 1, and keep 1 in mind; likewise twice 2 and 1 in mind making 5, I write down 5; Then 253 multiplied by 1 makes 253, which I set orderly under 516; Lastly, those two Products added together make 3046, which is the Number of Pence contained in 12 *l.*—13 *s.*—10 *d.* as before was found out by the general Method.

So 35 Ounces, 16 Penny Weights, and 12 Grains Troy will be reduced into 17196 Grains.—

VII. *Reduction ascending* is performed by Division, for if the Number of Integers given be divided by such a Number of the same Integers, as are equal to one of the Integers required, the Quotient is the Number of Integers sought.

So 220800 Farthings being divided by 4 (the Number of Farthings in a Penny, give 55200 Pence in the Quotient: In like manner if 55200 Pence be divided by 12 (the Number of Pence in a Shilling) the Quotient is 4600 Shillings. Lastly, 4600 Shillings being divided by 20 (because 20 *s.* make a Pound sterling) the Quotient is 230 Pounds sterling, which are equal to 220800 Farthings first given. The Operation is as followeth.

4) 220800 (55200 (46010 (2301.

48

72
72

00

In like manner, 34268 Grains Troy will be reduced to 54 11 Ounces, 7 Penny Weight, and 20 Grains. This kind of Reduction may be made the easier to the Learner by the following Tables.

1. Of English Money.

Farthings	} Divi. by	{ 4 12 20	} give	Pence.
Pence				Shillings.
Shillings				Pounds.

2. Of Troy Weights.

Grains	} Divi. by	{ 24 20 12	} give	Penny Weights.
Pennyweight				Ounces.
Ounces				Pounds Troy.

Also in Apothecaries Weights.

Grains	} Divi. by	{ 20 3 8	} give	Scruples.
Scruples				Drams.
Drams				Ounces Troy.

3. Of Averdupois Weight.

Drams	} Divided by	{ 16 16 28 4	} give	Ounces.
Ounces				Pounds.
Pounds				Quarters of C.
Quarters				Hund. Weight.

3. Of Liquid Measures.

Pints	} Divided by	{ 2 }	give	Quarts.
Quarts				Pottles.
Pottles				Gallons.
Gallons				Hogheads.

5. Of Dry Measures.

Pints	} Divided by	{ 2 }	give	Quarts.
Quarts				Pottles.
Pottles				Gallons.
Gallons				Pecks.
Pecks				Bushels.
Bushels				Quarters

6. Of Long Measures.

Barly Corns	} Divided by	{ 3 }	give	Inches.
Inches				Feet.
Feet				Yards.
Yards				Furlongs.
Furlongs				English Miles

Also,

Nails	} Divided by	{ 4 }	give	Quarters of Yards.
				Yards.
Quarters				Ells English.

Of Superficial Measures of Land.

Perches or Poles	} Divided by	{ 40 }	give	Roods or Quarters of Acres.
Roods				Acres.

8 Of Time.

Minutes	} Divided by	{ 60 }	give	Hours.
Hours				Days.
Days				Weeks.

Note,

Note. That if after Division is finisht in Reduction ascending there be any remainder, it is of the same denomination with the Dividend.

Note also that Reduction descending and ascending do mutually prove one another, by inverting the Question; for as in 56 Pounds sterling, there will be found 53760 Farthings, by Reduction descending; So for Proof thereof, 53760 Farthings will be reduced to 56 Pounds, by Reduction ascending.

Questions to exercise Reduction.

1. In 257l. how many shillings? *Answer*, 5140.
2. In 3076l. how many shillings? *Answer*, 61520.
3. In 902 shillings how many pence? *Ans* 10824.
4. In 2179 shillings how many farthings? *Answer*, 104592.
5. In 49l. ——— 13s. ——— 7d. how many pence? *Answer*, 11923.
6. In 2053l. ——— 14s. ——— 9d. ——— 2f. how many farthings? *Answ.* 1971590.
7. In 354lb. of Troy weight how many grains (of Goldsmiths weight?) *Answer*, 2039040.
8. In 300 English miles how many yards? *Answer*, 528000.
9. In 1 English mile, how many Barley corns length? *Answ.* 190080.
10. In 560 Acres how many Perches? *Answer*, 89600.
- 11 In 225 Acres, 3 Roods, and 30 Perches, how many Perches? *Answ.* 36150.
12. In 11923 pence how many pounds? *Answer*, 49l. ——— 13s. ——— 7d.

13. In 5764684 farthings how many pounds?
Ans. 6004l.—17s.—7d.

14. In 234678 Perches, how many Acres? *Answer*, 1466 Acres, 2 Roods, and 38 Perches.

15. In 525960 minutes of an hour, how many days? *Ans.* 365 days and 6 hours (or 1 year very near.)

16. In 19080 Pints, how many Hogheads? *Ans.* 20.

17. In 34678 grains of Apothecaries Weight how many Ounces Troy? *Ans.* 72 Ounces, 1 Dram, 2 Scruples, and 18 Grains.

18. In 106735 Pints of Wheat, how many Quarters? *Ans.* 208 Quarters, 3 Bushels, 2 Pecks, 1 Gallon, 1 Pottle, 1 Quart, 1 Pint.

19. In 3969301 Barley-corns length, how many Miles? *Ans.* 20 Miles, 7 Furlongs, 12 Yards, 2 Feet, 4 Inches, and 1 Barley-corn's length.

20. In 1900800 Barley-corns length, how many Miles? *Ans.* 10.

C H A P. VIII.

Of the Rule of Three Direct.

TH E Rule of Three is so called, because by three Numbers known or given, it teacheth to find a fourth unknown; it is also called the Golden Rule for the excellency thereof; Lastly, it is called the Rule of Proportion for the reason hereafter declared.

I The Rule of Three is either single or compound.

II The single Rule is, when three terms or numbers are propounded, and a fourth proportional unto them is demanded. *The Rule of Three.*

III Four Numbers are said to be proportionals, when the first containeth the second, or is contained by the second in the same manner as the third containeth the fourth, or is contained by the fourth: So these 4 Numbers are said to be Proportionals, 8, 4, 12, 6, for as 8 containeth 4 twice, so doth 12 contain 6 twice, and therefore 8 is said to have such proportion to 4 as 12 hath to 6; likewise these are Proportionals, 4, 8, 6, 12. For as 4 is the half of 8, so is 6 the half of 12; and therefore 4 is said to have such proportion to 8 as 6 hath to 12.

IV The Terms or Numbers of the Rule of Three (to wit the three Numbers given, and the fourth sought) consist of two difference denominations, viz. two of the three given terms have one name, and the other given term with the term

The divers denominations of the Rule of Three.

F

required.

required have another: So this Question being demanded, If four Students spend 19 Pounds in certain Months, how much Money will serve 8 Students for the same time, and at the same rate of Expence? Here Students and Pounds are the two denominations of the terms in the Question, viz. 4 and 8 (being two of the terms propounded) have the denomination of Students, and 19 the other term given, together with the term required, have the denomination of Pounds.

VI. In the Rule of Three, two of the three given terms imply a supposition, and the third moves a Question: So in the aforementioned Question a supposition is made, that 4 Students spend 19 Pounds, and a Question is moved with the Number 8, to wit, how many Pounds will 8 Students spend.

VII. In the Rule of Three, the Numbers given must be so ranked, that the known Number, or term upon which the Question is moved, must possess the third place in the Rule; also of the other two, that which hath the same denomination with the third, must be in the first place: Lastly, the other known term, which is of the same denomination with the fourth term sought (or answer of the question) must possess the second place: so in the question before-mentioned, the terms 4, 19, and 8, are to be thus placed, viz. 8 is the term upon which the question is moved, and therefore to possess the third place in the Rule; 4 is of the same denomination with 8 viz. of Students, and therefore to be in the first place: Lastly, 19 being of the same denomination with the term sought, viz. of Money, is to be in the
second

The right ordering of the terms given.

second place; And so they will be placed in the Rule thus,

Students. *Pounds.* *Students.*

If 4 ————— 19 ————— 8

That is to say, If 4 Students spend 19 Pounds, what will 8 Students spend? And here for the better discerning of the term or number upon which the Question is moved, you may observe, that for the most part it is the known number in the question which immediately followeth these or such like words; viz. *How many? How much? What will? How long? How far? &c.*

VIII. The Rule of Three is either Direct or Inverse.

IX. The Rule of Three Direct is, when the sense or tenour of the Question requireth that the fourth unnumber sought must have such proportion to the second, as the third number hath to the first; so in the aforementioned Question, If 4 Students spend 19 Pounds, how many Pounds will 8 Students spend at the same rate of expence? It is evident that the thing required is to find a number which may have such proportion to 19, as 8 hath to 4; that is, as 8 is the double of 4, so ought the fourth number to be the double of 19; for if 19 Pounds be required to maintain 4 Students a certain time, as much more must needs be required for the maintenance of 8 Students the same time, and therefore in this case we may say in a direct proportion, as 4 is to 8, so is 19 to a number which ought to be as much more as 19.

*The Rule of
Three Direct.*

How to Work the Rule of Three Direct, the three given terms being single numbers.

X. In the direct Rule of Three, if you multiply the second term by the third, or (which is all one) the third term or the second, and then divide the Product by the first, the Quotient will give the fourth term or fourth proportional required. So in the question before propounded, if you multiply 19 by 8, the Product is 152, which if you divide by 4, the Quotient will give you 38 the fourth term demanded, and the Work will stand thus.

Stud. l. Stud. l.
If 4—19—8—(38

8

4)152(38 pounds

12

32

32

0

A second Example may be this. If 8 Yards cost 9 Pounds, how much will 3 Yards cost?
Answer, 3 l. ——— 7 s. ———
6 d.

This Question being stated according to the seventh Rule of this Chapter, will stand as here you see; then multiplying (as before) the second term 9 by the third term 3, the Product is 27, which being divided by the first term 8, the Quotient is 3 Pounds, and there is a remainder of three Pounds, which must be reduced into sixty Shillings, and after those Shillings are divided by 8, and the rest of the Work prosecuted according to the Note

y. l. y. l. s. d.
8—9—3 (3:7:6.

3

8)27(3 pounds

24

3 the remaind.

20

8)60(7 shillings.

56

4 the remaind.

12

8)48(6 pence.

Chap. VIII. *of Three Direct.* 73

Note at the latter end of the 11th Rule of the 6th Chapter, at length the entire *Quotient* or answer of the Question is 3*l.* — 7*s.* — 6*d.*

A third Example, If 51 ounces of Silver Plate be sold for 13 Pound sterling, what is the price of 1 Ounce of that Plate?

Ans. 5*s.* — 1*d.* and somewhat

more. The operation is thus:

After the three known terms of this Question are rightly ordered, they will stand as

here you see in the Example;

then multiplying the second

term 13 by the third term 1,

the Product will be also 13,

(for multiplication by one

makes no alteration;) which

13 being divided by 51, af-

ter the manner of operation

delivered in the Note upon the 5th Rule of the 7th

Chapter, the entire *Quotient* or Answer of the Que-

stion will at length be found to be 5*s.* — 1*d.* and

somewhat more, but the surplussage being less than

a farthing, is omitted as useless.

Example 4. What must be paid to a Labourer for

his Wages for 27 Weeks at the rate of 4*s.* for 1

Week? *Answer,* 5*l.* — 8*s.*

After the three given Terms are rightly placed

in the Rule, they will stand *Week. Stil. Week.*

as you see in the Example; 1 — 4 — 27

then multiplying the third

term 27 by the second term 4,

the Product is 108, which

I should divide by the first term 1, but in regard

F 3

Division

$$\begin{array}{r}
 \text{oz.} \quad \text{l.} \quad \text{oz.} \\
 51 - 13 - 1 \\
 \hline
 13 \\
 20 \\
 \hline
 51 \overline{) 260} (5 \text{ shill.} \\
 255 \\
 \hline
 5 \\
 12 \\
 \hline
 51 \overline{) 60} (1 \text{ penny.} \\
 51 \\
 \hline
 9
 \end{array}$$

division by 1. makes no alteration, the *Quotient* is also 108, so that the fourth term sought is 108 Shillings, which being reduced to Pounds, according to the seventh Rule of the seventh Chapter, give 5 l. 8 s. for the answer of the Question.

XI. In the Rule of Three, if after the Question is stated according to the seventh Rule of this Chapter, any of the three given terms be a compound term consisting of divers denominations, as Pounds, Shillings, and Pence; or Weeks, Days, Hours, &c. such compound term must first be reduced into the lowest of those denominations (by the 6th Rule of the seventh Chapter) to the end that the three given terms may be three single Numbers; also of these three single numbers the first and third must always be of one and the same denomination: For if it happen that they express things of different names, such of the two which hath the greater name (or denomination) is to be reduced into the same name with the lesser (by the 5th Rule of the seventh Chapter :) These preparations being observed, the rest of the Work is to be prosecuted according to the tenth Rule of this Chapter. *Example*, What will 48 Ounces, 17 Penny-Weight, and 20 Grains of Silver-Plate amount unto at the rate of 5 s. — 6 d. the Ounce? *Answer*, 13 l. — 8 s. — 10 d. — 3 f. very near.

Chap. VIII.

of Three Direct.

75

This Question being stated according to the seventh Rule of this Chapter, will stand in the Rule as you see in the Example, to wit, If 1 Ounce cost 5 s. — 6 d. what will 48 oz. —

oz.	s.	d.	oz.	p.w.	gr.
1	5	6	48	17	20
20	12	20			
20	66	977			
24		24			
480		3928			
		1954			

23468 Grains.

17 p. w. — 20 gr. cost? Here because the third term is compounded of divers denominations, it must be reduced into the lowest of those denominations, to wit, Grains; so by the sixth Rule of the seventh Chapter, there will be found 23468 Grains for the third term: Likewise because the second term 5 s. — 6 d. is a compound term, whose lowest name is Pence, it must be reduced into Pence (by the aforesaid Rule;) so there will be found 66 Pence for the second term: Moreover because the first term hath the name Ounce, and the third term the name Grain, the first term 1 Ounce must be converted into 480 Grains (which are equal to 1 Ounce;) then will the three terms or single numbers stand in the Rule, as here you see viz.

gr.	pence.	gr.
480	66	23468

If 480 Grains cost 66 Pence, how many Pence will 23468 Grains cost? Now proceeding according to the tenth Rule of this Chapter, there will arise in the Quotient 3226 Pence, besides a remainder of 408, which being reduced to 1632 Farthings, and

those divided by the first term 480 the quotient will be 3 Farthings, so that the entire quotient is 3226 Pence, 3 Farthings, and somewhat more (but the parts of a Farthing being of no moment, may be neglected.) Lastly, the said 3226 Pence being reduced according to the seventh Rule of the seventh Chapter, give 13*l.* — 8*s.* — 10*d.* — 3*f.* so that 13*l.* — 8*s.* — 10*d.* — 3*f.* and somewhat more, will be the Answer of the Question.

XII. For the proof of the Direct Rule of Three, multiply the fourth term by the first, which done, if that Product be equal to the Product of the second term multiplied by the third, the Work is right, otherwise it is erroneous: So in the first Example, 38 the fourth term, being multiplied by the first term 4, the Product is 152, which is also the Product of 19 multiplied by 8. But if it happen that after the fourth term, or answer of the Question is found in the same denomination with the second term, there is yet a remainder, such remainder must be added to the Product of the first term, multiplied by such fourth term, and then the sum must be equal to the Product of the second and third terms (the second term consisting of the same denomination with the fourth:) so in the last Example the fourth term is 3226, and there happens to be a remainder of 408, which being added to the Product of the Multiplication of the said 3226 by the first term 480, gives 1548888, which is the same with the Product of the third term 23468 multiplied by the second term 66, as will appear when work'd.

XIII. When

XIII. When the first of the three given numbers in the Rule of three Direct, is 1 or unity, the Question may oftentimes be answered more speedily than by the Rule of Three, even by those who have but little skill in Arithmetick, as will partly appear by the following Examples, viz.

A compendious operation in the Rule of three direct, when the first term is 1 or unity.

1. At 17s. — 9d. the yard, what will 84 yards cost? *Answer*, 74l. — 11s. For Reason sheweth that 84 yards must (at the said rate) cost 84 Angels, 84 Crowns, 84 half Crowns, and 84 Three pences, all which being computed and added together, will give the full value of 84 yards, Viz.

	l.	s.	d.
84 Angels make	42	00	00
84 Crowns	21	00	00
84 half Crowns	10	10	00
84 Three Pences	1	01	00
Sum	74	11	00

2. At the rate of 9s. the Bushel of Wheat, what will 51 Quarters amount unto? *Answer*, 183 l. 12s. — 0d.

It

It is evident that the price of 1 Quarter (which consists of 8 Bushels) will be 8 Angels wanting 8 Shillings : Therefore,

	l.	s.	d.
from 8 Angels, to wit,	8	00	00
Subtract	0	08	00
remains the price of 1 Quarter	3	12	00

Then the value of 51 Quarters, at the rate of 3*l.* 12*s.* 0*d.* the Quarter, may be found in manner following, *viz.*

	l.	s.	d.
51 times 3 <i>l.</i> or 3 times 51 <i>l.</i> is	51	00	00
	51	00	00
	51	00	00
51 Angels make	25	10	00
51 Shillings doubled make	5	02	00
the price of 51 Quarters	183	12	00

3. What is a Chest of Sugar worth, that weigheth neat weight (the Tare being subtracted) 7 C. 3 q. 7 lb. at the rate of 6*l.* 3*s.* 4*d.* for 1 C? Answer, 48*l.* 3*s.* 6*d.* 2*f.*

Tare is that wherein any thing is put, as a Bag for Pepper, a Chest for Sugar.

7. times

times 6 pounds make	42	00	00
times 3 Shillings	1	01	00
Groats	0	02	04
The half of 6 <i>l.</i> --- 3 <i>s.</i> --- 4 <i>d.</i> for } 2 <i>qu.</i> is	3	01	08
The half of 3 <i>l.</i> --- 1 <i>s.</i> --- 8 <i>d.</i> for } 1 <i>qu.</i> is	1	10	10
The fourth part of 1 <i>l.</i> --- 10 <i>s.</i> --- 10 <i>d.</i> (because 7 <i>l.</i> is a fourth part of 28 <i>l.</i> or of 1 <i>qu.</i>) is	0	07	08
	48	03	06

Practical Rules of this nature cannot be completely understood without some skill in Fractions, as will hereafter appear in the second Chapter of the Appendix: And therefore I shall conclude this Chapter with the following Questions, whose Answers are annexed to them, and may be found out by the Preceding Rules; but the operations are purposely omitted, and left as an exercise for the Learner.

Questions to exercise the Rule of Three direct.

1. If 17 yards of Cloth cost 19*l.* 2*s.* 6*d.* what will 35 yards cost at that rate? *Answer*, 39*l.* 7*s.* 6*d.*
2. If 35 yards cost 39*l.* 7*s.* 6*d.* how many yards may be bought at that rate for 19*l.* 2*s.* 6*d.*? *Answer*, 17 yards.
3. If 35 yards cost 39*l.* 7*s.* 6*d.* what are 17 yards worth at that rate? *Answer*, 19*l.* 2*s.* 6*d.*
4. If 17 yards be sold for 19*l.* 2*s.* 6*d.* how many yards will 39*l.* 7*s.* 6*d.* buy at that rate? *Answer*, 35 yards.

5. What

5. What must I pay for the carriage of 17 hundred weight, 3 quarters and 11 pounds *Averdupois*, at the rate of 7 shillings the hundred weight? *Answ.* 6*l.*—4*s.*—11*d.*—1*farth.*

6. If 6*l.*—4*s.*—11*d.*—1*farth.* be pay'd for the carriage of 17 hundred weight, 3 quarters, and 11 pounds, what was paid for the carriage of 1 pound weight? *Answ.* 3 Farthings.

7. What must I pay for 39 ounces, 7 penny weight, and 18 grains of white Plate at the rate of 5*s.* and 5*d.* the ounce? *Answ.* 1*ol.*—13*s.*—4*d.* and three quarters of a farthing.

8. What must 1*l.* (or 20*s.*) pay towards a Tax, when 326*l.*—6*s.*—8*d.* is assessed at 41*l.*—16*s.*—2*d.*—3*f*? *Answ.* 2*s.*—6*d.*—2*f.*

9. What will the Interest of 876*l.*—17*s.*—6*d.* amount unto for 1 year at the rate of 6*l.* for 100*l.* for the same time? *Answ.* 52*l.*—12*s.*—3*d.*

10. If 3 yards in length of English measure be equal to 4 ells Flemish; how many Flemish ells are contained in 120 yards English? *Answer* 160 Flemish ells.

11. If 4 Flemish ells in length, be equal to 3 English yards; how many English yards in 300 Flemish ells? *Answ.* 225 English yards.

12. If 3 ells in length of English measure, be equal to 5 Flemish ells, how many Flemish ells in 120 English ells? *Answ.* 200 Flemish ells.

13. If 5 Flemish ells in length, be equal to 4 English ells; how many English ells in 145 Flemish ells? *Answ.* 87 English ells.

14. If 3 ounces of Silk weight, be equal to 4 ounces of Venice weight; how many ounces Venice are equal to 60 ounces of Silk weight? *Answer* 80 ounces Venice.

15. A Merchant delivered at *London* 120*l.* sterling, to receive 207*l.* Flemish at *Amsterdam*; what was 1*l.* sterling valued at in Flemish Money?

Ans. 1*l.* — 14*s.* — 6*d.*

16. If a Bill of Exchange be accepted at *London* for payment of 400*l.* sterling, for the value deliver'd at *Amsterdam*? at 1*l.* — 13*s.* — 6*d.* Flemish for 1*l.* sterling, how much Flemish money was deliver'd at *Amsterdam*? *Ans.* 670*l.* Flemish.

17. When the Exchange from *Antwerp* to *London* is at 1*l.* — 4*s.* — 7*d.* Flemish for 1*l.* sterling; how much sterling must I pay at *London* to receive 236*l.* Flemish at *Antwerp*? *Ans.* 192*l.* sterling.

18. A Merchant deliver'd at *London* 370*l.* sterling by Exchange for *Roan* at 74*d.* sterling for 50*s.* Tournois; how much Tournois ought he to receive at *Roan*? *Ans.* 60000*s.* Tournois.

19. In 370 Ducats, at 4*s.* — 2*d.* the Ducat; how many French Crowns at 6*s.* — 2*d.*? *Ans.* 250 Crowns; For if 74*d.* give 1 Crown, 18500*d.* (or 370 Ducats) will give 250 Crowns.

20. In 516 Dollers, at 4*s.* — 5*d.* the Doller; how many Guineas at 1*l.* — 1*s.* 6*d.* the piece? *Ans.* 106 Guineas. For if 258*d.* give 1 Guinea, 27348*d.* or 516 Dollers, will give 106 Guineas.

C H A P. IX.

Of the Inverse Rule of Three.

THE Rule of Three Inverse is, when the fourth term required ought to proceed from the second term, according to the same rate or proportion that the first proceeds from the third: So this Question being propounded, If 8 Horses will be maintained 12 days with a certain quantity of Provender, how many days will the same quantity maintain 16 Horses? Here, as 8 is half 16, so ought the fourth term required to be half 12; for if certain bushels of Provender serve 8 Horses 12 days; 16 Horses will eat up as much Provender in half that time: And therefore you cannot say here in a direct proportion (as before)

horses days horses in the Rule of Three direct
 8 — 12 — 16 as 8 to 16, so is 12 to another number which ought to be in that case as great again as 12; but contrariwise by an *inverted proportion*; beginning with the last term first, as 16 is to 8, so is 12 to another number, which ought to be in this case half 12. And by the due observation of this definition, together with that of the Rule of Three Direct (propounded in the ninth Rule of the eighth Chapter) when any Question belonging to the single Rule of Three is propounded, you may readily discern by which of those Rules it ought to be resolved; for if the three terms given look for a fourth

Chap
four
in th
direc
inver
by th

II.
given
Rule.
accor
eight
term
cond
by th
fourth
thus i
multi
vide b
requir

III.
a quest
to the
to the
directi
the sen
sion c

fourth

fourth in a direct proportion as they stand ranked in the Rule, you must resolve the Question by the direct Rule; contrariwise when the proportion is inverted or turned backwards, it ought to be resolv'd by the Inverse Rule of Three, which here followeth.

II. In the inverse Rule of Three, after the three given terms are rightly placed in the Rule, and reduced (if there be need) according to the eleventh Rule of the eighth Chapter, multiply the first term by the second, or (which is the same) the second term by the first, and then divide the Product by the third term, so the Quotient will give you the fourth term required, or answer of the question; thus in the question premised in the last Rule, if you multiply 12 by 8, the product is 96, which if you divide by 16 the Quotient gives you 6, the fourth term required, as by the subsequent operation is manifest.

How to work the Inverse Rule of Three.

$$\begin{array}{ccccccc}
 \text{horses} & & \text{days} & & \text{horses} & & \text{days} \\
 8 & \text{---} & 12 & \text{---} & 16 & \text{---} & 6 \\
 & & 8 & & & & \\
 \hline
 & & 16) 96 & (6 & & & \\
 & & 96 & & & & \\
 \hline
 & & 0 & & & &
 \end{array}$$

III. For the more ready discovering, whether a question propounded belongs to the Rule of Three Direct, or to the Rule Inverse, observe the directions following, viz. 1. By the sense and tenour of the question consider whether more be

How to discern whether a question in the Rule of Three is to be resolved by the Rule Direct or by the Rule Inverse.

required,

required or less; that is, whether the Number sought must be greater or less than the second term. Secondly, esteeming the first and third terms as extremes in respect of the second, this will be a general Rule; namely, When more is required, the Lesser extremum is the *Divisor*; but when less is required, the greater extremum is the *Divisor*. Lastly, the *Divisor* being found out, it will be apparent whether the Rule be Direct or Inverse, for when the *Divisor* is the first term, it is a Rule Direct; but when the *Divisor* is the third term, the Rule is Inverse. Another Example of the Rule Inverse may be this: If 12 Mowers do Mow certain Acres in 4 days, in what time will 23 Mowers perform

M. D. M.

$$\begin{array}{r}
 12 - 4 - 23 \\
 4 \\
 \hline
 23) 48 \text{ (2 days)} \\
 46 \\
 \hline
 2 \\
 24 \\
 \hline
 23) 48 \text{ (2 hours)} \\
 46 \\
 \hline
 2
 \end{array}$$

the same Work? *Answer* 2 days 2 hours, and somewhat more. Here, the 3 known terms being rightly placed in the Rule, will stand as you see in the Example; and since it is evident that 23 Men will require less time than 12 Men to finish the same Work, therefore (by the Rule foregoing) the greater of the two extremum Numbers 23 and 12 must be the *Divisor*; and because the *Divisor* 23 stands in the third place, this Question is to be wrought by the Rule Inverse; wherefore multiplying the first term 12 by the second term 4, the Product is 48, which being divided by the first term 23, the *Quotient* gives 2 days, and there is a remainder

Ch
ren
and
hou
sub
four
days
on A
my
(the
how
requ
what
after
oper

IV.
this, 1
the fou
qual to
multip
is true
ple of
equal

remainder of 2 days, which being reduced to hours, and those divided by 23, the Quotient will be 2 hours, and there is yet a remainder of 2 hours to be subdivided into 23 parts if you please; so that the fourth term sought, or answer of the Question is 2 days, 12 hours, and somewhat more, as from m^o.
 Again, Take this for a third Example, If I lend my Friend 356 pounds for one year and 35 days (the year being supposed to consist of 365 days) how long time ought he to lend me 500 pounds to requite my courtesy? *Answer*, 1284 days and somewhat more, there being a remainder, to wit, 400, after the Division is finish'd, as by the subsequent operation is manifest.

$$\begin{array}{r} \text{l.} \qquad \text{y.} \qquad \text{D.} \qquad \text{l.} \\ 356 \text{ --- } 1 : 35 \text{ --- } 500 \end{array}$$

$$\begin{array}{r} 365 \\ \text{add } 35 \\ \hline \end{array}$$

$$\begin{array}{r} \text{multiply } 356 \\ 356 \end{array}$$

$$500 \overline{) 142400} \text{ (284 days.}$$

IV. The proof of the Inverse Rule of Three is this, Multiply the third term by the fourth, then if this Product be equal to the Product of the first term multiplied by the second, the Work is true, otherwise erroneous; so in the Example of the second Rule, the Product of 16 and 6 equal to the Product of 8 and 12. But if it

The proof of the Rule of Three Inverse.

G happen

happen that after the fourth term, or answer of the Question, is found in the same denomination with the second term, there is yet a remainder, such remainder must be added to the Product of the third term multiplied by the fourth, and then the sum must be equal to the Product of the first and second terms (such second term being of the same particular denomination with the fourth: So in the last Example, the fourth term is 284 days, and there remains 400 after the division is finish'd, this 400 being added to the Product of the Multiplication of the third term 500 by the fourth term 284, gives 142400, which is equal to the Product of the first term 356, multiplied by the second term 400 days.

C H A P.

C H A P. X.

The double Golden Rule Direct, performed by two single Rules.

THE Compound Golden Rule is, when more than 3 terms are propounded.

II. Under the Compound Golden Rule, is comprehended the double Golden Rule, and divers Rules of plural proportion.

III. The double Golden Rule is, when five terms being propounded, a sixth proportional unto them is demanded: as in this Question, If 4 Students spend 19 pound in 3 months, how much will serve 8 Students 9 months? Or this, if 9 Bushels of Provender serve 8 Horses 12 days, how many days will 24 Bushels last 16 Horses.

IV. The five terms given in this Rule consist of two parts, *Viz.* A supposition expressed in the three first terms; and a demand propounded in the two last: So in the first Example of the last Rule, this Clause (if four Students spend 19 pounds in 3 months) is the supposition, and this (how much will serve eight Students nine months)

The parts into which the terms of the same Rule are distributed.

months) is the demand: Likewise in the other Example of the same Rule, this clause (if nine Bushels of Provyender serve 8 Horses 12 days) is the supposition, and this (How long, or how many days will 24 Bushels last 16 Horses) is the demand propounded.

V. Here for ranking the terms propounded in their due order, first observe amongst the terms of supposition, which of them hath the same denomination with the term required, then refer-

The right ordering of the terms.

ving that term for the second place, write the other two terms of supposition one above another in the first place; And lastly, the terms of demand likewise one above another in the third place of the Rule, in such sort that the uppermost may have the same denomination with the uppermost of those in the first place. Example, If 4 Students spend 19 pounds in 3 months, how much will serve 8 Students, 9 months? Here the three terms of supposition are 4, 19, and 3, and of these terms 19 hath the same denomination with the term required, viz. of Pounds, for you are to enquire how much Money is requisite for the maintenance of 8 Students 9 months, wherefore reserving 19 for 4 — 19 the second place I write 4 and 3 one above another thus; then drawing a line upon the right hand of 4, I write 19 in the second place; this done, the Work will stand as in the Margin: Last of all, the terms of demand being 8 and 9, and 8 having the denomination of Students, I place it in the same line with 4 and 19, and write 9 under

under it; and this performed, the terms in this Question rank themselves as followeth;

Viz. Thus,

$$\frac{4}{3} \frac{12}{9} \frac{16}{8}$$

Or Thus,

$$\frac{3}{4} \frac{9}{12} \frac{8}{16}$$

In like manner, if the second Question of the third Rule of this Chapter were propounded; the terms thereof ought to be disposed

Thus,

$$\frac{8}{9} \frac{12}{10} \frac{16}{24}$$

Or Thus,

$$\frac{9}{8} \frac{10}{12} \frac{24}{16}$$

VI. Questions belonging to the double Golden Rule may be resolved by two single Rules of Three, or by the Golden Rule Compound of five Numbers.

VII. When Questions of this nature are resolved by two single Rules, the proportions are as followeth: *The Proportions of the double Golden Rule, when it is performed by two single Rules.*

I. As the uppermost term of the first place, is to the middle term; So is the uppermost term of the last place to a fourth Number:

G 3

II. As

II. As the lower term of the first place is to that fourth number; so is the lower term of the last place to the term required.

So in this Example before recited,

$$\begin{array}{r} 4-19-8 \\ 3 \quad 9 \end{array}$$
 using tacitly the lower term of the first place as a common number in the first proportion, *say, thus,*

I. If 4 Students spend 19 pounds (in three months) what will serve 8 Students the same time?

Or thus, If 4 Students spend 19 pounds, what will 8 spend?

Which Rule of Three will be discovered to be direct (by the third Rule of the ninth Chapter?) therefore the fourth proportional proceeding from the said three given Numbers 4, 19, and 8 is 38 (by the 10th Rule of the 8th Chap. aforegoing.) Again to find the term required, using tacitly the uppermost term of the third place as a common Number in this last proportion, *say as followeth.*

II. If in three months 38 pounds are spent (by 8 Students) how much will serve them for 9 months?

Or thus, If 2 give 38, what will 9 yield?

Which Rule of Three will likewise be discovered to be direct (by the third Rule of the 9th Chapter;) therefore the fourth proportional proceeding from the said 3 Numbers, 3, 38, and 9, you shall likewise find (by the 10th Rule of the 8th Chap. before recited) to be 114, for 38 being multiplied by 9, the Product is 342. which divided by 3, yields you in the *Quotient* 114: So that I conclude, if four Students spend nineteen Pounds in three months, 114 Pounds will serve 8 Students

dents 9 months; as you may further observe by the Work following:

4 — 19 — 8 — (38	3 — 38 — 9 — (114
8	9
4) 152 (38	3) 342 (114
12	3
32	04
32	3
0	12
	12
	0

In like manner if two single Rules of Three be formed (according to the preceding 7th Rule) out of the five numbers given in the last mentioned Question, the same being ranked according to the latter manner of ordering the said numbers in the fifth Rule, each of the said two Rules of three will be a Rule direct, and the same answer of the question, to wit, 114 pounds will be discovered, as you may see by the subsequent operation.

3 — 19 — 9 — (574	4 — 57 — 8 — (114
9	8
3) 171 (57	4) 456 (114
15	4
21	05
21	4
0	16
	16
	0

VIII. The double Golden Rule is either Direct or Inverse.

IX. The double Golden Rule Direct is, when both the single Rules do each of them look for a fourth term in a direct proportion: As in the Example of the seventh Rule, where each of the two single Rules of Three is a Rule Direct.

For another Example take this, If the Carriage of 8 C. weight 128 miles, cost 48 Shillings, for how much may I have 4 C. weight carried 32 miles after the same rate? The terms of this Question according to the fifth Rule of this Chapter, rank themselves in this Order:

128 — 48 — 32
8 — 4 —

Now taking tacitly the lower term of the first place as a common Number, I form the first Rule of Three according to the seventh Rule, saying,

I. If the carriage of a certain weight (to wit, 8 C.) 128 miles, will cost 48 Shillings, what will the carriage of the same weight 32 miles cost?

Here it is easie to discern, that the fewer miles any weight is carried, the less money will pay for the carriage of that weight; therefore the fourth Number sought by the said Rule of three must be less than the second Number 48: And forasmuch as by the third Rule of the ninth Chapter, when less is required, the greater extreame (whether it be the first or third Number) must be the Divisor; therefore the first Number 128 is the Divisor, and consequently the Rule of Three above propounded is a Rule direct, wherefore finding out the fourth Num-

ber

Chap. X.

of Three Direct.

93

ber by the tenth Rule of the eighth Chapter, to be 12 Shillings, I proceed to the second proportion, and say,

II. If the carriage of 8 C. (32 miles) cost 12 Shillings, how much must I give to have 4 C. carried the same distance?

And here likewise finding a fourth Number to be looked for in a Direct proportion, I discover that fourth, by the said tenth Rule of the eighth Chapter, to be 6s. which is the term demanded, and the answer to the Question propounded: So that at last I conclude, if the carriage of 8 C. 128 miles cost 48s. the carriage of 4 C. 32 miles will cost 6s. according to the same rate: See the whole Work.

$$\begin{array}{r} 128 \text{ --- } 48 \text{ --- } 32 \\ 8 \text{ --- } 4 \text{ --- } (6) \end{array}$$

$$\begin{array}{r} 128 \text{ --- } 48 \text{ --- } 32 \text{ --- } (12) \end{array}$$

$$32$$

$$96$$

$$144$$

$$128) 1536 (12$$

$$128$$

$$256$$

$$256$$

$$0$$

$$8 \text{ --- } 12 \text{ --- } 4 \text{ --- } (6)$$

$$4$$

$$8) 48 (6$$

$$48$$

$$0$$

CHAP.

C H A P. XI.

The double Golden Rule Inverse, performed by two single Rules.

TH E Double Golden Rule Inverse is, when one of the single Rules looks for a fourth term in an inverted proportion : As in the last Example propounded in the fifth Rule of the last Chapter. For if you rank the terms of that Question, according to the said fifth Rule, thus.

$$\begin{array}{ccccc} 8 & \text{---} & 12 & \text{---} & 16 \\ 9 & & & & 24 \end{array}$$

And then work by two single Rules of Three formed according to the seventh Rule of the last Chapter, you shall find by the third Rule of the ninth Chapter, that the first of the said two Rules of Three will be inverse, and the latter direct ; for saying first, if 8 Horses be maintained 12 days (by 9 Bushels of Provender) how many days will 16 Horses be kept by so much Provender ? Here the answer 6 days will be found out by the Rule of Three inverse : Secondly, saying, if 9 Bushels of Provender be eaten up (by 16 Horses) in 6 days, in how many days will 24 Bushels be spent ? Here the answer 16 days will be found out by the Rule of Three direct.

But if you order the given terms of the same Question, thus,

$$\begin{array}{ccc} 9 & \text{---} & 12 & \text{---} & 24 \\ 8 & & & & 16 \end{array}$$

And then work by two single Rules of Three formed according to the seventh Rule of the last Chapter, you shall find by the third Rule of the ninth Chapter, that the first of the said two Rules of Three will be direct, and the latter Inverse; for saying, first, If 9 Bushels of Provender will last 12 days (to maintain 8 Horses) how many days will 24 Bushels serve the same number of Horses: The answer 32 days will be found out by the Rule of Three direct. Secondly, saying, If 8 Horses will be maintained 32 days (by 24 Bushels of Provender) how long will 16 Horses be kept by the same quantity of Provender? Here the answer 16 days will be found out by the Rule of Three Inverse.

Wherefore whensoever a Question belonging to the double Rule of Three is severed into two single Rules of Three (according to the preceding Rules) if one of them happens to be a Rule Inverse, that double Rule is called the double Rule Inverse.

Now the Resolution of the Question propounded being ranked after the first manner, is as followeth.

$$\begin{array}{r} 8 \text{ --- } 12 \text{ --- } 16 \\ 9 \text{ --- } 24 \text{ --- } (16) \end{array}$$

And then work by two single Rules of Inverse
 formed as follows: $8 \text{ --- } 12 \text{ --- } 16$ (6)
 Chapter. You shall find the third Rule of the
 ninth Chapter, that the 8 of the said two Rules
 of Three will be directed by the last Inverse 6
 for saying that, if a Horse of Provender will last
 12 days (to maintain 2 Horses) how many days
 will 24 Horses serve the same number of Provender?
 The answer 24 days will be found out by the Rule
 of Three direct, secondly saying the 12 Horses will
 be maintained 12 days by 2 Horses (of Provender)
 how long will 24 Horses be kept by the same quan-
 tity of Provender? Here the answer 6 days will
 be found out by the Rule of Three Inverse.
 Wherefore whenever a Question belonging to
 the double Rule of Three is levered into two single
 Rules of Three (according to the preceding Rules)
 if one of them happens to be a Rule Inverse, that
 double Rule is called the double Rule Inverse.
 Now the Resolution of the Question proposed
 being ranked after the first manner, is as fol-

Again, The Resolution of the same Question,
 being ranked after the last manner, is this,

$$\begin{array}{r} 9 \text{ --- } 12 \text{ --- } 24 \\ 8 \text{ --- } 16 \text{ --- } (16) \end{array}$$

$$\begin{array}{r} 9 \text{ --- } 12 \text{ --- } 24 \text{ --- } (32) \\ 12 \end{array}$$

THE Golden Rule compound of five num-
bers is, in the terms being ranked, as be-
fore, indicated of the double terms we use their Pro-
ducts, and find the term required
by one single Rule of three.

Here, where the Question propounded ought
to be performed by the double Rule
first, multiplying the terms of the
first place, the one by the other,
take their Product for the first
term, the middle number for the se-
cond, and the Product of the two
last terms for the third: thus, 8 being
found by the Rule of Three direct, a fourth pro-
portional unto those three, that fourth term is
found is the number 61 (61) 62 (61) to this Que-
stion being again propounded, it is evident that
96 in 3 months, now months will serve 8 Students
3 months; and the terms being ranked
as before, we have

$$\begin{array}{r} 96 \\ 96 \\ \hline 0 \end{array}$$

So that at last I say, if 9 Bushels of Provender
serve 8 Horses 12 days, 24 Bushels will last 16
Horses 16 days, which is the Resolution of the
Question propounded.

C H A P XII.

*The Golden Rule compounded of five.
Numbers.*

I. **T**HE Golden Rule compound of five numbers is, when the terms being ranked, as before, instead of the double terms we use their Products, and then proceed to find the term required by one single Rule of three.

II. Here when the Question propounded ought to be performed by the double Rule direct, multiplying the terms of the first place, the one by the other, take their Product for the first term, the middle number for the second, and the Product of the two last terms for the third term; this done, having found by the Rule of Three direct, a fourth proportional unto those three, that fourth term so found is the number you look for; so this Question being again propounded, if 4 Students spend 19 *l.* in 3 months, how much will serve 8 Students 9 months? and the terms thereof being ranked as before, viz. thus,

$$\begin{array}{ccccc} 4 & \text{---} & 19 & \text{---} & 8 \\ 3 & & & & 9 \end{array}$$

The Product of 4 multiplied by 3 is 12, and the Product of 8 multiplied by 9 is 72; wherefore I say, As 12 to 19, so 72 to the term required, which I find by the single Rule of Three direct to be 114.

Ch
So
114
Stu
follo

In
(in the
riage o
carriage
thereun

Chap. XII. of five Numbers. 99

So that if 4 Students spend 19l. in three months, 114l. will be requisite for the maintenance of 8 Students 9 months; see the whole operation, as followeth.

$$\begin{array}{r}
 4 \text{ --- } 19 \text{ --- } 8 \\
 3 \text{ --- } 9 \text{ --- } (114 \\
 \hline
 12 \qquad \qquad 72 \\
 \qquad \qquad 19 \\
 \hline
 \qquad \qquad 648 \\
 \qquad \qquad 72 \\
 \hline
 12)1368(114 \\
 \underline{12} \qquad \qquad \qquad \\
 \qquad 16 \qquad \qquad \qquad \\
 \qquad 12 \qquad \qquad \qquad \\
 \hline
 \qquad \qquad 48 \\
 \qquad \qquad 48 \\
 \hline
 \qquad \qquad 0
 \end{array}$$

In like manner this being the Question as before (in the last Rule of the tenth Chapter) if the carriage of 8 C. 128 miles, cost 48s. What will the carriage of 4 C. 32 miles stand me in? the Answer thereunto will be 6s. as appears by the Work.

$$128 \text{ ---}$$

$$\begin{array}{r}
 128 \quad 48 \quad 32 \\
 8 \quad 128 \quad 4 \\
 \hline
 1024 \quad 384 \quad 128 \\
 \quad 96 \\
 \quad 48 \\
 \hline
 1024) 6144 \text{ (6 Shillings)} \\
 \underline{6144} \\
 0
 \end{array}$$

III. When the Question propounded ought to be resolved by the double Rule Inverse, having multiplied the double terms across, that is, the uppermost term of the first place by the lower of the last and the uppermost of the last place by the lower of the first, write each Product under the lower term by which it is produced: And then if the Inverse proportion be found in the uppermost line, using those Products as single terms, proceed to find the term required by the single Rule of Three direct: But in case you find the Inverse proportion in the lower line, perform the Work by the single Rule of Three Inverse.

So in the Example above-mentioned if 9 Bushels of Pravender serve 8 Horses 12 days, how long will 24 Bushels last 16 Horses? Here

8—12—16 if you rank the terms thus, you shall
 9 24 find the Inverse proportion in
 8 the first line, as is observed in the last
 1 Chapter: And therefore having subscribed the
 Products

Chap. XIII. The Rule of Three compound. 101

Products according to the direction given you in this Rule, I proceed to satisfy the demand of this Question by the single Rule of Three direct, as appears by the Work following.

$$\begin{array}{r}
 8 \text{ ——— } 12 \text{ ——— } 16 \\
 9 \qquad \qquad \qquad 24 \text{ — } (16 \\
 \hline
 144 \qquad \qquad \qquad 192 \\
 \qquad \qquad \qquad 12 \\
 \qquad \qquad \qquad \hline
 \qquad \qquad \qquad 384 \\
 \qquad \qquad \qquad 192 \\
 \qquad \qquad \qquad \hline
 \qquad \qquad \qquad 144) 2304 (16 \\
 \qquad \qquad \qquad 144 \\
 \qquad \qquad \qquad \hline
 \qquad \qquad \qquad 864 \\
 \qquad \qquad \qquad 864 \\
 \qquad \qquad \qquad \hline
 \qquad \qquad \qquad 0
 \end{array}$$

But the terms of this Question being ranked thus, the Inverse proportion is found in the lower line, as you may observe likewise by the last Chapter : Whereupon in this case to resolve the Question, I proceed by the single Rule of Three Inverse, as appears by the Work hereunto annexed : Howsoever therefore you work the Question, you shall find the term required to be 16 ; so that at last I conclude, as before in the last Chapter, If 9 Bushels of Provender serve 8 Horses 12 days, 24 Bushels, will last 16 Horses 16 days.

H

9 —

$$\begin{array}{r}
 9 \text{ --- } 12 \text{ --- } 24 \\
 8 \text{ --- } 16 \text{ --- } 16 \\
 \hline
 192 \qquad 144 \\
 12 \\
 \hline
 384 \\
 192 \\
 \hline
 144) 2304 (16 \\
 \underline{144} \\
 864 \\
 \underline{864} \\
 0
 \end{array}$$

CHAP. XIII.

The Rule of Fellowship.

I THE Rules of plural Proportion are those, by which we resolve Questions that are discoverable by more golden Rules than one, and yet cannot be performed by the double golden Rule mentioned before in the three last Chapters. Of these Rules there are divers kinds and varieties, according to the nature of the Question propounded, for here the terms given are sometimes four, sometimes five, sometimes more, and the terms required sometimes more than one, &c.

II. Two

II. Two particular Rules of plural Proportion are these, the Rule of Fellowship and the Rule of Alligation.

III. The Rule of Fellowship is that, by which in Accompts amongst divers Men (their several stocks together with the whole Gain or Loss being propounded) the Gain or Loss of each particular Man may be discovered: As in this Example. *A* and *B* were Sharers in a parcel of Merchandize, in the Purchase of which, *A* laid out 7 *l.* and *B* 11 *l.* and they having sold this Commodity, find that their clear Gains amount to 54 *s.* Now here the Question to be resolved by this Rule is, what part of that 54 *s.* accrews to *A*, and what to *B*, according to the rate of the several Summs or Stocks which they adventured? Again, *A*, *B*, and *C*, freight a Ship from the *Canaries* for *England*, with 108 Tuns of Wine, of which *A* had 48, *B* 36, and *C* 24, the Mariners meeting with a Storm at Sea, were constrained for the safety of their Lives, to cast 45 Tun thereof over-board; here the Question to be resolved is, How many of the 45 Tun each particular Merchant hath lost, according to the Rate of his Adventure?

The Rule of Fellowship.

IV. The Rule of Fellowship is either single or double.

V. The single Rule is, when the Stocks propounded do continue in the Adventure (or common Bank) equal times, to wit, one Stock as long time as another.

VI In the single Rule of Fellowship, take the Total of all the Stocks for the first Term, the whole Gain or Loss,

How to work the single Rule.

hath relation to a particular time: Example, *A*, *B* and *C*, hold a Pasture in common, for which they pay 45 *l. per Annum* In this Pasture *A* had 24 Oxen went 32 days; *B* had 12 there 48 days, and *C*, fed 16 Oxen there 24 days; now the Question to be resolved by this Rule is, what part each of these Tenants ought to pay of the 45 *l. rent*? and here you may observe, that the stocks propounded are double Numbers, viz. each stock of Oxen hath reference to a particular time; for the respective stock of *A*. is 24 Oxen, and its particular time is 32 days; again, the stock of *B*. is 12 Oxen, and the respective time is 48 days: And lastly, the stock of *C*. is 16 Oxen, and its peculiar time is 24 days, which as you see are double Numbers.

VIII. In the double Rule of Fellowship, multiply each particular stock by its respective time, and take the total of their Products for the first term, the whole gain or loss for the second, and the said particular Products of the double Numbers for the third term: This done, repeating, as before the Rule of Three, so often as there are Products or the double Numbers; the fourth terms produced upon those several operations. are the Numbers you look for: So in the Example of the last Rule, the Product of 24 and 32 is 768, the Product of 12 and 48 is 576, and the Product of 16 and 24 is 384, the sum of these Products is 1728, which is the first term in the Question, then 45 *l. the rent*, is the second term, and 768 the first Product, is the third term of the first proportion. Wherefore I say, as 1728 to 45 *l. so* 768 to another Number, which I find by the di-

*How to work
the double
Rule.*

rect Rule of *Three* to be 20*l.* viz. the part of the Rent that *A.* ought to pay: Then for the second proportion I say, as 1728 to 45 *l.* so 576 to 15 *l.* which is the part that *B.* ought to pay: And lastly, as 1728 to 45 *l.* so 384 to 10*l.* viz. the part that *C.* must pay.

$$\begin{array}{r} 768 \\ 576 \\ 384 \end{array} \left. \vphantom{\begin{array}{r} 768 \\ 576 \\ 384 \end{array}} \right\} 1728 \text{ --- } 45 \text{ --- } \left\{ \begin{array}{r} 768-20 \\ 576-15 \\ 384-10 \end{array} \right.$$

A second Example of the eighth Rule: Three Merchants, *A.* *B.* and *C.* enter Partnership, and agree to continue in a joint Adventure 16 months; *A.* puts into the common stock at the beginning of the said Term 100 Pounds, at 8 months end he takes out 40 Pounds, and 4 months after such taking out he puts in 140 Pounds. *B.* puts in at first 200 Pounds, at 6 months end he puts in 50 Pounds more, and 4 months after the putting in of the 50 Pounds, he takes out 100 Pounds. *C.* puts in at first 150 Pounds, at 4 months end he takes out 50 pounds, and 8 months after such taking out puts in 100 Pounds. Now at the end of the said 16 months, they had gained 357 Pounds, the Question is, How much of the said gain belongs to each Merchant for his share?

In Questions of this Nature, two things are principally to be observed: 1. The whole time of Partnership. 2. The respective time belonging to each Man's particular stock; so here, it is evident that the whole time is 16 months, and the particular stocks and times belonging to each Merchant will be as followeth, viz.

A. Had

Chap. XIII. *The Rule of Fellowship.* 107

A. Had 100*l*. in the common stock for 8 months, therefore 100 multiplied by 8 produceth _____ } 800

Also 60*l*. for 4 months, therefore 60 multiplied by 4 produceth _____ } 240

Also 200*l*. for 4 months, therefore 200 multiplied by 4 produceth _____ } 800

The Total of the Products of Money and Time for A, is _____ } 1840

B. had 200*l*. in the common stock for 6 months, therefore 200 multiplied by 6 produceth _____ } 1200

Also 250*l*. for 4 months, therefore 250 multiplied by 4, produceth _____ } 1000

Also 150*l*. for 6 months, therefore 150 multiplied by 6 produceth _____ } 900

The Total of the Products of Money and Time for B, is _____ } 3100

C. had 150*l*. in the common stock for 4 months, therefore 150 multiplied by 4 produceth _____ } 600

Also 100*l*. for 8 months, therefore 100 multiplied by 8 produceth _____ } 800

Also 200*l*. for 4 months, therefore 200 multiplied by 4 produceth _____ } 800

The Total of the Products of Money and Time for C, is _____ } 2200

Then adding the said three totals together, to wit, 1840, 3100 and 2200, the sum is 7140 wherefore proceeding as in the last Example, I say by the Rule of three direct, as 7140 is to the total gain 157

H 4

Pounds;

pounds; so is 1840 to 92 pounds the gain of *A*; again, As 7140 is to 357; so is 3100 to 155 the gain of *B*: Lastly, as 7140 is to 357; so is 2200 to 110 the gain of *C*.

IX. The Rule of Fellowship is proved
The Proof. by addition of the terms required, whose sum ought to be equal to the second term in the Question, otherwise the whole work is erroneous: So in the first *Example* of the sixth Rule foregoing, 21 *s.* and 33 *s.* being added together, are equal to 54 *s.* the *second term* in that Question. Likewise in the last *Example* of the same Rule, as also in the first *Example* of the last Rule, the *sum* of 20, 15, and 10, the terms required, are equal to 45, the *second term* propounded.

C H A P. XIV.

The Rule of Alligation.

I. **T**HE Rule of Alligation is that, by which we resolve Questions, that concern the mixing of divers Simples together.

II. Alligation is either Medial or Alternate.

III. Alligation Medial is, when having the several quantities and rates of divers simples propounded, we discover the mean rate of a mixture compounded of those simples. So 10 Bushels of Wheat at 4 *s.* or (which is all one) 48 *d.* the Bushel; 40 Bushels of Rye at 3 *s.* or 36 *d.* the Bushel; and 50 Bushels of Barley at 2 *s.* or 24 *d.* the Bushel; being mixed with

*Alligation
Medial.*

with 20 Bushels of Oats at 12 *d.* the Bushel, the Rule of *Alligation Medial* sheweth you the mean price of that Misting.

IV. In Alligation medial, first sum the given quantities, then find the total value of all the simples : This done, the proportion will be as followeth.

*The Operations
and Proportions
of the same Rule.*

As the sum of the quantities is to the total value of the simples :

So is any part of the mixture propounded to the required mean rate or price of that part.

Repeating again the premised Example of the third Rule, I demand how much one Bushel of that Misting is worth? Now the sum of 10, 40, 50, 20, (the given quantities) is 120 Bushels, and the value of the 10 Bushels of Wheat at 48 *d.* the Bushel amounts to 480 *d.* for 48 being multiplied by 10, the Product is 480 : Again the value of the 40 Bushels of Rye at 36 *d.* the Bushel, is 1440 *d.* The value of the 50 Bushels of Barley at 24 *d.* the Bushel is 1200 *d.* And the value of 20 Bushels of Oats at 12 *d.* the Bushel is 240 *d.* All these values being added together, their total is 3360 *d.* I say then by the Rule of *Three direct*, if 120 Bushels give 3360 *d.* what will 1 Bushel yield? The Rule presently answers me 28 *d.* Whereupon I conclude, that a Bushel of that Misting may be afforded for 28 *d.* that is, 2 *s.* 4 *d.* which is the Resolution of the *Question propounded*.

$$120 \text{ ————— } 3360 \text{ ————— } 1 \text{ ————— } 28$$

In like manner if it be demanded what 3 Bushels or a Quarter of that Misting is worth, the *Answer* will be 224*d.* which being divided by 12, and by that means reduced into *shillings* is 18*s.* 8*d.*

$$120 \text{ --- } 3360 \text{ --- } 8 \text{ --- } 224$$

V. In *Alligation Medial*, the trial of the Work is by comparing the total value of the *The Proof.* several Simples with the value of the whole mixture: For when those sums accord, the operation is perfect: So in the first Example of the last Rule

The Value of	{	10 Bushels of Wheat at 4 <i>s.</i> the	7 <i>s.</i> 6 <i>d.</i>
		Bushel is	2—0—0
		40 Bushels of Rye at 3 <i>s.</i> the	
		Bushel is	6—0—0
		50 Bushels of Barley at 2 <i>s.</i> the	
	{	Bushel is	5—0—0
		And 20 Bushels of Oats at 12 <i>d.</i>	
	{	the Bushel is	1—0—0

All which amount to ——— 14—0—0 which is likewise the value of 120 Bushels at 28*d.* or 2*s.* 4*d.* the Bushel, for that also amounts to 14*l.*

VI. *Alligation Alternate* is, when having the several rates of divers Simples given, we discover such quantities of them, as are necessary to make a mixture, which may bear a certain rate propounded.

Example: A Man being determined to mix 10 Bushels of Wheat at 4*s.* or 48*d.* the Bushel, with Rye

Chap. XIV.

Alligation.

III

Rye of 3s. or 36d. the Bushel, with Barley of 2s. or 24d. the Bushel, and with Oats of 1s. or 12d. the Bushel; the Rule of *Alligation Alternate* will discover unto you how much Rye, how much Barley, and how much Oats he ought to add unto the 10 Bushels of Wheat, in such sort that the mixture of them altogether may bear a certain rate or price propounded.

VII. In Questions of *Alligation Alternate*, you must rank the terms in such sort, that

the given rate of the mixture may represent the Root, and the several rates of the Simples may stand as Branches issuing from that Root: So the Example of the last

The right ordering of the Terms.

Rule being propounded, I demand how much Rye, Barley and Oats, ought to be added to the 10 Bushels of Wheat, that the mixture of all together may bear the rate or price of 28d. or 2s. 4d. the Bushel: And therefore drawing a line of connexion, I place 28d. the given rate of the mixture, upon the left Hand thereof, by it self, representing the *Root*, and likewise write the other rates propounded, viz. 48d.

36d. 24d. and 12d. one above another upon the right hand of that line of connexion, which rates are conceived to Issue from 28d. as branches from the *Root*, the fabrick hereof appears plainly in the Margin.

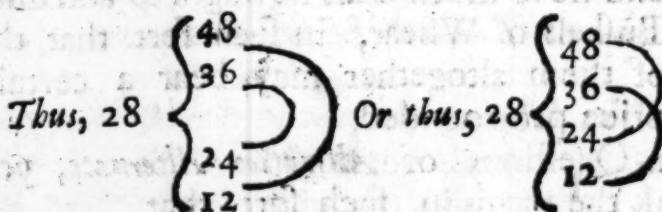
$$\left. \begin{array}{l} 48 \\ 36 \\ 28 \\ 24 \\ 12 \end{array} \right\}$$

VIII. Having ranked the terms in their due order, link the branches together by certain Arches, in such sort, that one that is greater than the *Root* or rate of the mixture, may always be coupled with another

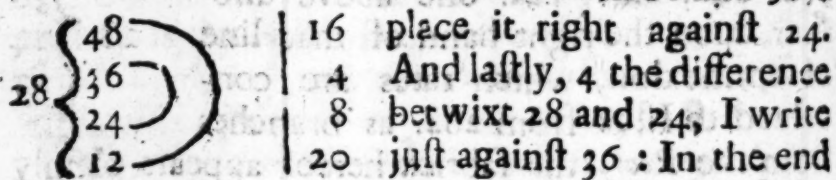
How to couple the Terms.

ther

ther that is less than the same: So in the premised Example, 48 may be linked with 12, and 36 with 24, or otherwise 48 may be coupled with 24, and 36 with 12, and then the Work will stand



IX. Having alligated the branches, and found the differences betwixt them and the Root, write the differences of each branch just against his respective yoke-fellow. So the branches of the Example aforegoing being linked after the first manner, and the difference between 28 and 48 (by the third or fourth Rule of the fourth Chapter of this Book) being 20, I place 20 just against 12, the respective yoke-fellow of 48. Again, 16, being the difference betwixt 28 and 12, I write it just against 48. In like manner 8 being the difference between 28 and 36, I



place it right against 24. And lastly, 4 the difference betwixt 28 and 24, I write just against 36: In the end the whole *Fabrick* of the Work (as the Branches are thus linked) will stand as in the Example.

But

Cha
Bu
man

For
and
the i
the p
ticula
X.
bran
ough
linke
you
48 an
linke
differ
both
differ
likew
Again
I pla
and
differ
36 I
again
24 an
forme
as in
2.

But the branches being linked after the other manner, the Work will be thus disposed:

	48		4
28	36		16
	24		20
	12		8

For in this case 48 hath 24 for his yoke follow, and the respective *Comrade* of 36 is 12; and here the interchangeable placing of the *difference* (as in the premised Examples) is that which is more particularly termed *Alternation*.

X. When one branch is linked to divers other branches, and not to one alone, the *differences* ought to be as often transcribed, as it is so diversly linked. So in the premised Example, you may (if you please) conceive 12 to be coupled both with 48 and 36; likewise 24 may be conceived to be linked with the same 48 and 36; wherefore the *difference* betwixt 28 and 12 being 16, I write it both just against 48 and 36. In like manner the *difference* between 28 and 24 being 4, I write it likewise over against the same numbers 48 and 36. Again, 20 being the *difference* betwixt 28 and 48, I place it just against 24 and 12; and 8 being the *difference* between 28 and 36 I write it likewise over against the same numbers 24 and 12; All this performed, the whole frame of the work will stand as in the *Margin*.

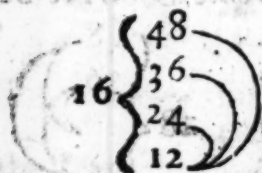
	48		16
	36		4
28	24		20
	12		8

2. Take this for another Example; It is required

red to mix 10 Bushels of Wheat at 48 *d.* the Bushel with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, and with Oats of 12 *d.* the Bushel, and the Question now is, How much Rye, Barley, and Oats ought to be added to the 10 Bushels of Wheat, that the entire mixture may be afforded at 16 *d.* the Bushel? Here the Branches of this Question (according to the Eighth Rule of this Chapter) ought to be linked *thus*,



And as for the *Alternation* of the Differences, it is evident (by the present Rule) that the difference between 16 and 12 being 4, ought to be thrice transcribed, *viz.* first just against 48, then against 36, and last of all against 24. Again, 32 the difference betwixt 16 and 48, as also 20 the difference between 16 and 36; and lastly, 8 the difference betwixt 16 and 24, ought all to be placed just against 12.



4
4
4
32. 20. 8.

3. I determining to mix 10 Bushels of Wheat at 48 *d.* the Bushel, with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, and with Oats

Cha
of r
each
who
Wor
goin

4.
at 48
with
the B
fires t
Oats
that t
afford
thus p
aforeg
of the

5. I
Carect's
Carect's
that 19

of 12 *d.* the Bushel, desire to know how much of each I ought to take, that I might afford the whole mixture at 40 *d.* the Bushel: Here the whole Work being ordered according to the Rules aforegoing, it will stand as followeth,

$$\begin{array}{r|l}
 48 & 4. \ 16. \ 28. \\
 40 \left\{ \begin{array}{l} 36 \\ 24 \\ 12 \end{array} \right. & 8 \\
 & 8 \\
 & 8
 \end{array}$$

4. A Man intending to mix 10 Bushels of Wheat at 48 *d.* the Bushel, with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, with Pease of 16 *d.* the Bushel, and with Oats of 12 *d.* the Bushel, desires to know how much Rye, Barley, Pease, and Oats he ought to add to the 10 Bushels of Wheat, that the whole mass of Corn so mixed might be afforded at 20 *d.* the Bushel. This *Question*, being thus propounded, the terms thereof (by the Rules aforegoing) may be *Alligated*, and the differences of the terms *Alternated*, as followeth.

$$\begin{array}{r|l}
 48 & 4 \\
 20 \left\{ \begin{array}{l} 36 \\ 24 \\ 16 \\ 12 \end{array} \right. & 4 \\
 & 4.8. \\
 & 28. \ 16. \ 4. \\
 & 4
 \end{array}$$

5. Lastly, A Goldsmith hath some Gold of 24 *Carats*, other of 21 *Carats*, and other some of 19 *Carats* fine; which he would so mix with *Alloy*, that 192 Ounces of the entire mixture might bear

17 *Carets fine*, now the *Question* is, how much of each sort; as also how much *Alloy* he must take to accomplish his desire? Before you

What a Caret fine, and what Alloy is.

can well understand this *Question*, it will be necessary to explain what a *Caret fine*, and what *Alloy* is: The Mint-Masters and Goldsmiths to distinguish the different *fineness* of Gold, esteem an entire Ounce to contain 24 *Carets*, and one Ounce of Gold that being tryed in the fire loseth nothing of the weight, is said to be 24 *Carets fine*: Again, the ounce that being tryed loseth one four and twentieth part of the weight, is said to be 23 *Carets fine*: In like manner that which loseth two four and twentieth parts of the Ounce, is esteemed to be 22 *Carets fine*, and so consequently of the rest: And as for the *Alloy*, it is Silver, Copper, or some other baser Metal, with which the Goldsmiths use to mix their Gold, to the intent they may moderate, or abate the *fineness* thereof. Here you may also observe, that as the *fineness* of Gold is measured by *Carets*, so is the *fineness* of Silver estimated by *Ounces*: In such sort, that a *Pound* of Silver which being tryed a certain time in the fire, loseth nothing of the weight, is said to be 12 *Ounces fine*: But a *Pound*, that being tryed loseth somewhat of the weight, is said to be the remainder of the weight *fine*. Example, A *Pound* of Silver, that loseth in the fire one Ounce 8 p. is estimated to be 10 *Ounces* 12 p. *fine*; and that which loseth 2 *Ounces*, 8 p. 10 Grains, is said to be 9 *Ounces*, 11 p. 14 Grains *fine*, &c. Now to rank the terms of the last mentioned *Question*, as also the differences of the terms in their due order, because the three given Branches (*viz.* 24

Carets

Cha

Carets
than
add
less t
mer
expre

XI.
found
them
again
a stra
right
So
pound
placed
write
the ne
Work,
order,
nexed.

Carects, 21 Carects, and 19 Carects) are all greater than 17 Carects the root or rate of the mixture. I add 0 as another branch, which I conceive to be less than the root, and then proceed as in the former operations; the whole frame of the Work is expressed here, as followeth.

$$\begin{array}{r|l}
 \begin{array}{l} 24 \\ 21 \\ 19 \\ 0 \end{array} & \begin{array}{l} 17 \\ 17 \\ 17 \\ 7. 4. 2. \end{array}
 \end{array}$$

XI. When in one and the same line there are found more differences than one, add them together, and write the sum just against the same differences before a straight line drawn towards the right hand of the Work.

How to add the differences.

So the first Example of the last Rule being propounded, the sum of 16 and 4 (the differences placed just against the first branch) being 20, I write it over against the same differences, before the new line drawn upon the right hand of the Work, and so consequently the rest in their due order, as appears by the Example hereunto annexed.

$$\begin{array}{r|l|l}
 \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} & \begin{array}{l} 16. 4. \\ 16. 4. \\ 20. 8. \\ 20. 8. \end{array} & \begin{array}{l} 20. \\ 20. \\ 28. \\ 28. \end{array}
 \end{array}$$

In like manner the last Example of the last Rule being offered, the whole Fabrick of the Work will stand as followeth:

17	{	24	17	17
		21	17	17
		19	17	17
		0	7. 4. 2	13

XII. Alligation Alternate is, either Partial or Total.

XIII. Alternation Partial is, when having the several rates of divers Simples, and the quantity of one of them given, we discover the several quantities of the rest, in such sort that a mixture of those Simples being made according to the quantity given, and the quantities so found, that mixture may bear a certain rate propounded: Of this kind is the Example of the sixth Rule, as also all the Examples of the tenth Rule except the last.

XIV. In Questions of Alternation Partial, the proportion is as followeth.

As the difference annexed to the first branch is to the several differences of the rest:

So is the quantity propounded to the several quantities required.

So the Example of the sixth and seventh Rules of this Chapter being again repeated, and the terms thereof, as also the differences of the terms being ordered after the first manner (shewed you in the ninth Rule aforegoing) it is evident that for

for every 16 Bushels
of Wheat that I take *The first*
in the mixture, I *Case.*
ought to take 4 Bu-
shels of Rye, 8 Bu-
shels of Barley, and 20 Bushels of Oats; and there-
fore I say,

28	{	48	D	16
		36		4
		24		8
		12		20

I. As 16 the difference annexed to the first branch
(being the rate of the Wheat) is to 4, the dif-
ference annexed to the next, being the rate of
the Rye; so is 10 the given quantity of the
Wheat to another number, which being found
by the Rule of *Three direct*, to be two Bushels
and an half (or two pecks) is the quantity of
Rye necessary in the *mixture*.

II. As 16 to 8, so is 10 to another Number,
which being likewise found by the Rule of
Three to be five Bushels, is the quantity of Bar-
ley necessary in the *mixture*.

III. As 16 to 20, so is 10 to another Number,
which being in like sort found by the Rule of
Three to be 12 Bushels, and half of a Bushel,
is the quantity of Oats requisite in the *mixture*.

So that at last I conclude, a heap of Corn being
composed of 10 Bushels of Wheat, 2 Bushels and a
half of Rye, 5 Bushels of Barley, and 12 Bushels and
an half of Oats (when those several Grains bear the
prices, *aforsaid*) may be afforded at 2 s. 4 d. the
Bushel.

The same Example being ordered after
the second manner (expressed likewise in *2 Case.*
the 9th Rule of this present Chapter) I say,

I 2

1. As

- I. As 4 the *difference* annexed to the *rate* of the Wheat, is to 16 the *difference* annexed to the *rate* of the Rye; so is 10 the *given* quantity of the Wheat, to 40 Bushels the *required* quantity of the Rye.
- II. As 4 to 20, so is 10 to 50 Bushels the *requisite* quantity of the Barley.
- III. As 4 to 8, so is 10 to 20 Bushels the *quantity* of the Oats *necessary* in the mixture.

$$\begin{array}{r|l}
 28 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right. & \begin{array}{l} 4 \\ 16 \\ 20 \\ 8 \end{array}
 \end{array}$$

So that I conclude *again*, a mass of Corn being compounded of 10 Bushels of Wheat, 40 Bushels of Rye, 50 Bushels of Barley, and 20 Bushels of Oats, (when those Grains bear the prices propounded in this Example) may be afforded at 2 s. 4 d. the Bushel as before.

3. That Example being disposed after the
3 *Case*. third manner (expressed in the tenth and eleventh Rules of this Chapter) I say,

- I. As 20 the *sum* of the differences annexed to the *rate* of the Wheat, is to 20 the *sum* of the differences annexed to the *rate* of the Rye; so is 10 the *given* quantity of the Wheat, to 10 Bushels the *required* quantity of the Rye.
- II. As 20 to 28, so is 10 to 14 Bushels the *requisite* quantity of the Barley.
- III. As 20 to 28, so is 10 to 14 Bushels, the *quantity* of Oats *demand*ed in the mixture.

28	{	48	D	16	4	20
		36		16	4	20
		24		20	8	28
		12		20	8	28

Whereupon this *third time* likewise I conclude, that (those Grains still retaining the given rates) 10 Bushels of Wheat, 10 Bushels of Rye, 14 Bushels of Barley, and 14 Bushels of Oats being all mixed together, will constitute a *mass* of Corn, that may be afforded at 28 *d.* or 2 *s.* 4 *d.* the Bushel.

By this Example thus *diversified* it plainly appears, that the quantities required may be altered as often as the Question given will admit divers *Alligations*, and yet the mixture produced will still hold the *rate* propounded; but when the *Question* propounded will admit but one only way of *Alligation*, the quantities required to make the *mixture*, cannot be varied; so the second *Example* of the tenth Rule of this Chapter, being again produced, and ordered according to the direction of the eleventh Rule foregoing, I say,

I. As 4 to 4, so 10 to 10 Bushels of Rye.

II. As 4 to 4, so 10 to 10 Bushels of Barley,

III. As 4 to 60, so 10 to 150 Bushels of Oats.

16	{	48	D	4		4
		36		4		4
		24		4		4
		12		32, 20, 8.		60

I 3

So

So that for this Question I conclude, to 10 Bushels of Wheat you ought to add 10 Bushels of Rye, 10 Bushels of Barley, and 150 of Oats, to the end that a mixture of Corn might be made, which may be sold at 16 *d.* the Bushel: And here the quantities found (*viz.* 10, 10, and 150) cannot be altered, because the terms of this Question will not admit any other variety of *Alligation*.

XV. In Alternation Partial, the Proof is likewise by comparing the total value of the several simples, with the value of the whole mixture: So in the second Example of the last Rule, the total value of the 10 Bushels of Wheat, 40 Bushels of Rye, 50 Bushels of Barley, and 20 Bushels of Oats, amounts to 14 *l.* which is also the value of the whole mixture at 2 *s.* 4 *d.* the Bushel, as appears by the Example of the fifth Rule of this present Chapter.

XVI. Alternation total is, when having the total quantity of all the simples, together with their several rates, we produce their several quantities, in such sort, that a mixture of them being made according to the quantities so found, that mixture may bear a certain rate propounded: Of this sort is the last Example of the tenth Rule aforegoing; as also *this*: A Goldsmith having divers sorts of Gold, *viz.* some of 24 Carats, other of 22 Carats, some of 18 Carats, and other some of 16 Carats *fine*, is desirous to melt of all these sorts so much together, as may make a mass containing 60 Ounces of 21 Carats *fine*: Now this Rule of *Alternation total* sheweth you how much you are to take of each sort, to the end the whole mass

may

may contain just 60 Ounces of 21 Carects, the fineness propounded.

XVII. In Questions of Alternation total, the proportion is as *The Proportions.* followeth:

As the sum of all the differences is to the total quantity of all the simples : So is the correspondent difference of each rate to the respective quantity of the same rate.

So the last Example of the last Rule being propounded, I say,

I. As 12 the sum of the differences is to 60 Ounces the *total* quantity of all the simples : So is 5 the correspondent *difference* of 24 Carects the first rate, to 25 Ounces, *viz.* the required quantity of the Gold of the same rate, which may be taken to make the mixture propounded.

II. As 12 to 60, so is 3 the correspondent *difference* of 22 Carects the second rate, to 15 Ounces, *viz.* the quantity of the Gold of 22 Carects, that ought to be used in the mixture.

III. As 12 to 60; so is 1 to 5 Ounces of the Gold of 18 Carects fine.

IV. As 12 to 60, so is 3 to 15 Ounces of the Gold of 16 Carects *fine*, which are requisite to be taken for the mixture propounded.

$$\begin{array}{r}
 21 \left\{ \begin{array}{l} 24 \\ 22 \\ 18 \\ 16 \end{array} \right. \begin{array}{l} 5 \\ 3 \\ 1 \\ 3 \end{array} \\
 \hline
 \end{array}$$

12

I 4

Where.

Whereupon I conclude, that 25 Ounces of 24 Carects fine, 15 Ounces of 22 Carects, 5 Ounces of 18 Carects, and 15 Ounces of 16 Carects fine, being all melted together, will produce a mass of Gold containing 60 Ounces of 21 Carects fine, which is the Resolution of the *Question* propounded.

Again, The last Example of the tenth Rule being here repeated, and ordered according to the direction of the eleventh Rule, I say,

I. As 64 to 192, so 17 to 51 Ounces of 24 Carects fine.

II. As 64 to 192, so is 17 to 51 Ounces of 21 Carects fine.

III. As 64 to 192, so is 17 to 51 Ounces of 19 Carects fine.

IV. As 64 to 192, so is 13 to 39 Ounces of Alloy.

$$\begin{array}{r|l}
 \begin{array}{l} 24 \\ 21 \\ 19 \\ 0 \end{array} \left. \begin{array}{l} 17 \\ 17 \\ 17 \\ 7.4.2. \end{array} \right\} \text{D} & \begin{array}{l} 17 \\ 17 \\ 17 \\ 13 \end{array} \\
 \hline & 64
 \end{array}$$

And therefore for conclusion I say, that 51 Ounces of Gold, 24 Carects fine, 51 Ounces of 21 Carects fine, 51 Ounces of 19 Carects fine, and 39 Ounces of Alloy being all mixed together, will produce a mass containing 192 Ounces of Gold, 17 Carects fine, which is the satisfaction of the *Question* premised.

And here observe (as before in the Exposition of the fourteenth Rule of this Chapter) that the operations of the first of these Examples may be varied according to the diversity of the *Alligations* which

which it will admit, whereas the last Example is not subject to any *variety*, the *Alligations* thereof remaining always the same.

XVIII. Here the operation is perfect, when the sum of the quantities found agrees with the total quantity propounded; So in the first Example of the last Rule, 25, 15, 5, and 15 (the quantities found) being all added together amount to 60, which is the total quantity propounded. *The Proof.*

C H A P. XV.

The Rule of False.

I. THE Rule of False is always performed by false and suppositional Numbers taken at pleasure after the Proposition is made, and the Question propounded; for things are said to be found out by the *Rule of False*, when by false terms *supposed*, we discover the true terms *required*.

II. The *Rule of False*, is either of single or double Position.

III. The Rule of single Position is *The Rule of single Position.* when at once, *viz.* by one false Position we have means to discover the true Resolution of the Question propounded.

For Example: *A*, *B*, and *C*, determining to buy together a certain quantity of Timber, that should cost them 36 *l.* agree among themselves that *B*. shall pay of that sum a *third* part more than *A*, and that *C* shall pay a *fourth* more than *B*. Now the Question is, What particular sum each of these Parties

parties ought to pay of the 36 *l.* To resolve this Question; first, put the case that *A.* ought to pay 6 *l.* of the 36 *l.* and then *B.* must pay 8 *l.* because he pays one third part more than *A.* And lastly *C.* ought to pay 10 *l.* because he is to lay out one fourth part more than *B.* This done although by addition of these three sums, *viz.* 6, 8, and 10, I find that I have made a wrong *Position* (their total amounting only to 24 *l.* which ought to have been 36 *l.*) nevertheless by those *suppositional* Numbers, I have means to discover the true sums which the several parties ought to pay: For I say by the Rule of *Three Direct*:

I. As 24 to 36, so is 6 to 9 *l.* the part that *A.* must pay.

II. As 24 to 36, so is 8 to 12 *l.* the part that *B.* ought to pay.

III. As 24 to 36, so is 10 to 15 *l.* the part of the 36 *l.* that *C.* must pay.

IV. Here for trial of this Rule the total of the sums found ought to accord with

The Proof. the sum given: So in the Example of the last Rule, 9, 12, and 15 being all added together, amount to 36, the sum propounded.

V. The Rule of double *Position* is, when two false *Positions* are supposed for the Resolution of the Question propounded. As in *this*, A Workman

The Rule of double Position.

having thresht out 40 quarters of Grain (part thereof being Wheat, and the rest Barley) received for his labour 28 *s.* being paid after the rate of 12 *d.* for every quarter of Wheat, and 6 *d.* for each quarter of Barley: Now here the Question is how many of those 40 quarters were Wheat, and how

how
at r
and
have
ney
the
Barle
26 Q
ters o
if my
fers f
perce
cover

differ
5 the
the err
the se
Wheat
second
the W
and fo
which
ceived
togethe
many o
the W
by the

VI. I

having
placed t
(viz. th
the upp
under h
same Cr

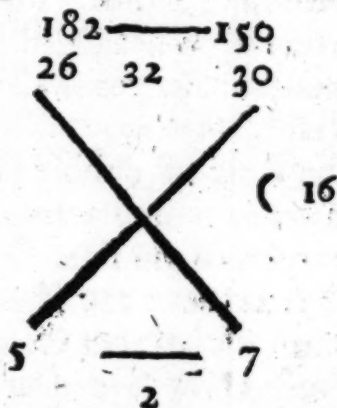
how many Barley? Here therefore I first suppose at random, that there were 26 quarters of Wheat, and 14 of Barley, and then to discover whether I have guessed right or wrong, I find how much Money is due to the Workman at the rate of 12 *d.* the Quarter of Wheat, and 6 *d.* the Quarter of Barley, which I find to be 33 *s.* (*viz.* 26 *s.* for the 26 Quarters of Wheat, and 7 *s.* for the 14 quarters of Barley,) which he ought to have received, if my *supposition* had been right; but because it differs from 28 *s.* the true sum that he received, I perceive I have mist the mark, and therefore discovering how much I have err'd by finding the difference betwixt 28 *s.* and 33 *s.* I keep in mind 5 their difference, which is called the *first error*, or the *error of the first Position*: Again, I propound for the *second Position*, that there was 30 quarters of Wheat, and 10 quarters of Barley; and then the *second error* I find to be 7; for there is then due to the Workman for the 30 quarters of Wheat 30 *s.* and for the 10 quarters of Barley 5 *s.* in all 35 *s.* which differs from 28 *s.* the true sum that he received, by 7 *s.* and here by these two *false Positions*, together with their *errors*, you may discover how many quarters of Wheat, and how many of Barley the Workman thresh'd, as shall be further explained by the *Rules following*.

VI. In the Rule of double *Position*,
 having drawn two lines across, and *The Operation.*
 placed the terms of the *false Position*,
 (*viz.* those that have the same Denomination) at
 the uppermost end of that Cross, as also each *error*
 under his respective *Position* at the lower end of the
 same Cross, multiply each *error* by the contrary
Position

Position; that is, the second *error* by the first *Position*, and the first *error* by the second *Position*; this done, when both the *errors* are of one and the same kind (*viz.* both excesses or both defects) subtract the less Product out of the greater, and then the remainder is your Dividend; but if the *errors* be of different kinds, (*viz.* one of them an excess, and the other a defect) add those Products together, and then the sum will be your Dividend, which if you divide by the difference of the *errors*, (when they are of one and the same kind) or by their sum (when they are of different kinds) the *Quotient* will give you a number you look for, having the same Denomination with the false *Positions* placed at the upper end of the Cross.

Example 1. The Question of the last Rule being again propounded, I place these terms, *viz.* 26 (having the Denomination of the Quarters of Wheat in the *first Position*) and 30 (having the same Denomination of the *second Position* at the upper end of the Cross :) As also 5 and 7 the two *errors* respectively under them at the lower end of the same Cross, as you may see it exemplified by the Pattern following.

Note, That this Character ——— signifies that the lesser of the two Numbers, betwixt which it is found, ought to be subtracted from the greater,

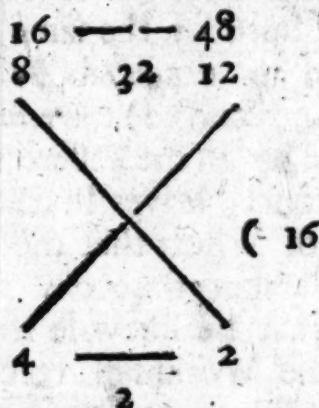


This

This done, having multiplied 26 by 7, the *Product* is 182, and likewise 30 by 5, the *Product* is 150, which being deducted out of 182 (because the *errors* here are both of the same kind, that is, are each of them an excess above 28 s. the sum that the Workman received) the *remainder* is 32, which being divided by 2 (the difference betwixt 5 and 7 the two errors) leaves in the *Quotient* 16, for the quarters of Wheat that the Workman threshed, whose complement to 40 *viz.* 24, are the quarters of Barley, that he likewise threshed; so at last I conclude, the Workman received 28 s. for his wages in threshing out 40 quarters of Grain (being part Wheat, part Barley) at 12 d. the quarter of Wheat, and 6 d. the quarter of Barley, threshed in all 16 quarters of Wheat, and 24 quarters of Barley.

Example 2. The same Question being again propounded, I suppose for my *first Position* that there are 8 quarters of Wheat, and 32 quarters of Barley, and then the first error will be 4 s. for 8 s. being accounted for the 8 quarters of Wheat, and 16 s. for the 32 quarters of Barley, make in all 24 s. which wants 4 s. of 28 s. the sum received: Again, *supposing* that there are 12 quarters of Wheat, and 28 quarters of Barley, the second error will be 2 s. for 12 s. being allowed for the 12 quarters of Wheat, and 14 s. for the 28 quarters of Barley, the sum is 26 s. which comes 2 s. short of 28 s. the right sum: Now then 8 being multiplied by 2, the *Product* is 16; likewise 12 by 4 produceth 48, out of which if you deduct 16 (because the errors in this case happen to be both defects under 28 s. the sum received) the remainder is 32, which being

being divided by 2 (the difference of the errors) gives you in the Quotient 16, viz. the quarters of Wheat, as before,



Example 3. The same demand being the third time produced, I take for my *first Position* 10 quarters of Wheat, and 30 quarters of Barley, and then proceeding as before, the first error will prove 3 s. which upon that *Position* I want of 28 s. the right sum: Again, here for the *second Position* I take 26 quarters of Wheat, and 14 quarters of Barley, and then the second error will be 5 s. which upon that *Position* I have exceeded 28 s. the true sum: Now then multiplying 10 by 5, the Product is 50, and 26 by 3, the Product is 78: And here (because the errors are of different kinds, one of them being a defect, and the other an excess of 28 s. the true sum) you are to add 50 and 78 the two Products together, whose sum is 128, which being divided by 8, the sum of 3 and 5 the two errors, gives you in the Quotient 16 for the quarters of Wheat, as before in the former Resolutions: So that what *Positions* soever you take in this *Question*, you shall always find, that the Workman threshed 16 quarters

Cha
ters
the

VI
is use
ample
found
of W
Barley
ed b
Exa
was th
his elc
his s
his thi
young
the Ag
the Ag
eldest
the Q
12, the
or you
Questi
half th

ters of Wheat, and 24 quarters of Barley, which is the Resolution of the Question propounded.

$$\begin{array}{r} 50 \\ 10 \\ \hline 128 \end{array} \quad \begin{array}{r} + \\ 78 \\ 26 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline 8 \\ \hline 5 \end{array} \quad (16$$

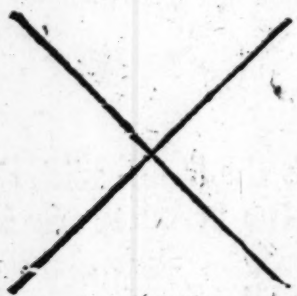
Note, That this Character & intimates that the Numbers, betwixt which it is found, ought to be added together.

VII. Here the trial is the same with that which is used in finding out the errors: So in the Example premised 16 and 24 being the Numbers found, and 16s. being allowed for the 16 quarters of Wheat, likewise 12s. for the 24 quarters of Barley, their sum is 28s. which was the sum received by the Workman.

Example 4. A certain Man being demanded what was the Age of each of his 4 Sons? Answered, that his eldest Son was 4 Years elder than the second; his second was 4 Years elder than the third; his third Son was 4 Years elder than the fourth or youngest; and his fourth or youngest was half the Age of the eldest, the Question is, what was the Age of each Son? Here I guess the Age of the eldest Son to be 16, then it may be inferr'd from the Question, that the Age of the second Son was 12, the Age of the third 8, and the Age of the fourth or youngest 4, this 4 should be half 16 (for the Question saith, that the Age of the youngest was half the Age of the eldest) but it wants 4 of what it ought

ought to be ; wherefore I make a second Position, and take 20 for the Age of the eldest, then the Age of the second must necessarily be 16, the Age of the third 12, and the Age of the fourth 8, which should be half 20, but it wants 2 : Now (according to the Rule) multiplying 16 (the first Position) by 2 (the second error) the Product is 32 : Also multiplying 20 (the

$$\begin{array}{r} 32 \text{ --- } 80 \\ 16 \quad 48 \quad 20 \end{array}$$



$$\begin{array}{r} 4 \text{ --- } 2 \\ 2 \quad 48 \quad (24 \end{array}$$

second Position) by 4 (the first error) the Product is 80, and because the errors are both of one kind, to wit, both defective ; I subtract the lesser Product from the greater, so the remainder is 48

or a Dividend, also subtracting the lesser error from the greater, the remainder is 2 for a Divisor : Lastly, dividing 48 by 2, the Quotient is 24, and such was the Age of the eldest Son, therefore the Age of the second was 20 ; the Age of the third 16 ; and the Age of the fourth 12 ; which is half the Age of the eldest, as was declared by the Question.

The Doctrine of Vulgar Fractions

CHAP. XVI.
Notation of Vulgar Fractions.

Ihus far of *Arithmetick* in whole numbers, only the Doctrine of *Fractions* ensueth, which depends upon this supposition, that Unity, or at least one whole thing, whatsoever it be, may in mind be conceived divisible into any Number of equal parts: Some will not allow 1 or unity to be a Number, when it is considered in the abstract, and separated from matter, but forasmuch as that Prince of Arithmeticians, *Diophantus* of *Alexandria*, in divers of his subtil Problems doth mention unity as a Number, and propounds it to be divided into Numbers; I shall take the like liberty to esteem 1 or unity as a Number, and likewise suppose it divisible into any Number of equal parts.

II. A broken number, otherwise called a *Fraction*, is only part of an integer or whole thing, as if you would express in Figures the length of a piece of cloth, that contains three fourths, or (which is all one) three quarters of a yard, you are to write it thus $\frac{3}{4}$, that is, an entire yard being supposed to be divided into four equal parts, the length of the piece propounded

pounded is three of those four parts: In like manner (a Foot being divided into 12 Inches) you must write six Inches thus $\frac{6}{12}$, that is, six twelve parts of a Foot; or if the Foot be divided into one hundred equal parts, to express five and twenty of those parts, set them down thus $\frac{25}{100}$ that is five and twenty hundredth parts of a Foot.

III. A Fraction consists of two parts, the Numerator and the Denominator, which are placed one above the other, and separated by a little line.

IV. The Numerator is the number placed above the line, and the Denominator is the number placed underneath: so in the aforementioned Fraction $\frac{3}{4}$, the number 3 placed above the line is the Numerator, and the number 4 placed underneath is the Denominator. Also in this Fraction $\frac{16}{12}$, the Numerator is 16, and the Denominator is 12. The Denominator is so called, because it Denominates or declares into how many equal parts the Integer or whole thing is supposed to be divided, and the Numerator is so called, because it numbereth or expresseth how many of those equal parts of the Integer are signified by the Fraction.

V. A Fraction is either proper or improper.

VI. A proper Fraction is that whose Numerator is less than the Denominator, such are the Fractions before-mentioned $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{100}$, and the like.

VII. A proper Fraction is either single or compound.

VIII. A single Fraction is that which consists of one Numerator, and one

one

12

sion

nish

as th

Divi

to th

part

expre

teger

mina

so if

mong

in the

rema

which

tion

the Fr

be 3

pound

A

lesser

ter, fo

Numer

minato

doth a

of an

Divide

vided

is, the

pound

In like

quotier

ways a

Fraction

one Denominator; such are $\frac{1}{4}$, $\frac{1}{12}$, $\frac{1}{24}$, and the like.

IX. A single Fraction doth often arise in Division of whole Numbers, for when Division is finished, if any number remain, it is to be esteemed as the Numerator of a Fraction, which hath the Divisor for a Denominator, and is to be annexed to the Integer or Integers in the Quotient as part of the Quotient; which Fraction doth always express certain parts (or at least a part) of an integer or entire unit, which hath the same Denomination with one of the Integers in the Quotient; so if 17 Pounds be given to be divided equally amongst 5 Persons, there will arise 3 entire pounds in the Quotient, and there will be a remainder or surplussage of 2 Pounds, $5 \overline{) 17} (3 \frac{2}{5}$ which 2 is to be placed, as the Numerator of a Fraction, over the Divisor 5 as a Denominator; so will the Fraction be $\frac{2}{5}$, and the compleat Quotient will be $3 \frac{2}{5}$, that is, 3 pounds, and 2 fifth parts of a pound for each Persons share.

A single Fraction doth likewise arise, when a lesser whole number is given to be divided by a greater, for in such case the Dividend is to be made the Numerator of a Fraction, and the Divisor the Denominator; which Fraction is the true quotient, and doth always express a certain part (or at least parts) of an Integer, which hath the same name with the Dividend; So if 3 Pounds sterling be given to be divided equally amongst 4 Persons, the share of each, that is, the quotient will be $\frac{3}{4}$, to wit, 3 fourth parts of a pound.

In like manner, if 5 be given to be divided by 8 the quotient is $\frac{5}{8}$, so that the Numerator of a Fraction is always a Dividend, the Denominator is a Divisor, and the Fraction it self is the quotient.

A Compound Fraction. X. A Compound Fraction (otherwise called a Fraction of a Fraction) is that which hath more Numerators and Denominators than one, and may be discovered by the word (of) which is interpos'd between the parts of such compound Fraction: So $\frac{3}{4}$ of $\frac{1}{2}$ is a Fraction of a Fraction, or compound Fraction, and expresseth two thirds of three fourths of an Integer, viz. a pound sterling being supposed the Integer, and first divided into four parts, three of those four parts are equal to 15 s. Again, if the said 15 s. be divided into three parts, two of those three parts are equal to 10 s. therefore the compound Fraction $\frac{3}{4}$ of $\frac{1}{2}$ of a pound sterling doth express 10 s. In like manner the compound Fraction $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of a pound sterling, that is, one fourth of three fourths of four fifths of a pound sterling doth express 3 s. as will be farther manifested by the sixteenth and ninth Rules of the seventeenth Chapter.

XI. An improper Fraction is that, whose Numerator is either greater, or at least equal unto the Denominator: So this Fraction $\frac{16}{4}$, that is 16 fourths, is called an Improper Fraction, and so is this $\frac{4}{3}$; for indeed a Fraction of this kind may well be surnamed Improper, because it will not admit the Definition of a true Fraction, since it is always greater than an entire Unity, or at least equal unto it; so sixteen Farthings, or $\frac{16}{4}$ of a Penny are equal to 4 entire Pence; and 4 Farthings, or $\frac{4}{4}$ of a Penny are equal to 1 Penny; therefore when the Numerator is greater than the Denominator, such improper Fraction signifieth more than 1 or an Integer, but when the Numerator is equal to the Denominator (be

Chap. XVII. *Reduction of, &c.*

(be it what number soever) such *improper Fraction* is always equal to unity, or 1 Integer.

XII. A mixt Number consists of entire unities (or Integers) or at least of unity (or 1 Integer) and a Fraction annexed: *A mixt Number.*

So $5\frac{1}{2}$, $1\frac{1}{4}$, and such like, are called mixt Numbers; So that if a piece of Timber be five Feet and eleven Inches in length, you are to write that length thus, $5\frac{11}{12}$; in like manner, one Mile and three quarters or fourths of a Mile are to be written thus, $1\frac{3}{4}$.

C H A P. XVII.

Reduction of Vulgar Fractions.

I. **T**HE same parts of *Numeration*, as have been wrought in *whole Numbers* in the preceeding Chapter, are likewise to be performed in *Fractions*, but first of all *Reduction of Fractions* in divers kinds must be known, which being the principal skill in the Doctrine of Fractions, must be diligently observed by the Learner.

II. A Number is said to be a common Measure or Divisor unto two or more Numbers given, when it will measure or divide every one of the Numbers given, and leave no remainder; so 4 is a common measure unto the Numbers 12 and 20; for if 12 be divided by 4, the *Quotient* will be exactly 3, without any remainder or surplusage; also if 20 be divided by the same Divisor 4, the *Quotient* will be

precisely 5 without any remainder; in like manner 5 is a *common Divisor* unto these three numbers 10, 25, and 40.

III. Two numbers being given, their greatest common Divisor, that is, the greatest number which will measure or divide each of the numbers given without leaving any remainder, may be found out in this manner; *viz.* Divide the greater number by the less, then divide the Divisor by the remainder (if there be any) and so continue dividing the last Divisors by the remainders, until there be no remainder (neglecting the Quotients;) so is the last Divisor the greatest common Divisor unto the numbers given.

Thus, If the greatest *common Divisor* unto the numbers 91 and 117 be sought, divide the greater number 117 by 91, the remainder is 26, by which dividing 91, the remainder is 13; by which dividing 26 the remainder is 0; so is 13 the greatest *common Divisor* unto the numbers 117 and 91, as is manifest in dividing each of them by 13; for 13 is found in 91 precisely 7 times, and in 117 precisely 9 times. In like manner, 29 will be found a *common Divisor* unto 116 and 145; and 51 a *common Divisor* unto 561 and 612.

To reduce a Fraction into the least terms, viz.
1. By a general Rule.

IV. A single Fraction may be reduced into the least terms by dividing the Numerator and Denominator

$$91 \overline{) 117} \quad (1$$

$$91$$

$$26 \overline{) 91} \quad (3$$

$$78$$

$$13 \overline{) 26} \quad (2$$

$$26$$

$$0$$

Ch
min
for
Den
and
S
to th
Div
will
Quo
vidin
Den
the
ner
But
men
be
the
term
foun
like
the
tion
tica
dy
V
nomi
may
Ther
the R
Num
the D
if
line
tor

minator by their greatest common measure (or divider;) for the Quotients will be the Numerator and Denominator of a Fraction equal to the former; and in the least terms.

So if the Fraction $\frac{91}{117}$ be given to be reduced into the least terms, search out the greatest common Divisor unto 91 and 117 by the last Rule, which will be found 13, and then dividing 91 by 13, the Quotient will be 7 for a new Numerator; also dividing 117 by 13, the Quotient will be 9 for a new Denominator: So the Fraction $\frac{91}{117}$ is reduced into the least terms, viz. into the Fraction $\frac{7}{9}$. In like manner $\frac{14}{21}$ will be reduced into $\frac{2}{3}$; And $\frac{6}{12}$ unto $\frac{1}{2}$: But here you are to observe, that if the greatest common Divisor unto the Numerator and Denominator be 1, such Fraction is in its least terms already: So the Fraction $\frac{1}{2}$ cannot be reduced into lower terms, because the greatest common Divisor will be found 1, (by the third Rule of this Chapter;) the like may happen of infinite others: And although the last be a general Rule for the Reduction of Fractions into their least terms, yet there are other Practical Rules, which in some cases will be more ready (especially unto Beginners) viz.

V. When the Numerator and Denominator are even Numbers, they may be measured or divided by 2. 2. By particular Rules.

Therefore in such case you may (as is taught in the Rules of the 6th Chapter) take the half of the Numerator for a new Numerator, also the half of the Denominator for a new Denominator. So if $\frac{16}{64}$ be given, draw at length the

16	8	4	2	1
64	32	16	8	4

line which separates the Numerator from the Denominator, and

precisely 5, without any remainder; in like manner 5 is a *common Divisor* unto these three numbers 10, 25, and 40.

To find the greatest common measure unto any two Numbers.

III. Two numbers being given, their greatest common Divisor, that is, the greatest number which will measure or divide each of the numbers given without leaving any remainder, may be found out in this manner; viz. Divide the greater number by the less, then divide the Divisor by the remainder (if there be any) and so continue dividing the last Divisors by the remainders, until there be no remainder (neglecting the Quotients;) so is the last Divisor the greatest common Divisor unto the numbers given.

Thus, If the greatest *common Divisor* unto the numbers 91 and 117 be sought, divide the greater number 117 by 91, the remainder is 26, by which dividing 91, the remainder is 13; by which dividing 26 the remainder is 0; so is 13 the greatest *common Divisor* unto the numbers 117 and 91, as is manifest in dividing each of them by 13; for 13 is found in 91 precisely 7 times, and in 117 precisely 9 times. In like manner, 29 will be found a

91)117 (1

91

26)91 (3

78

13)26 (2

26

0

common Divisor unto 116 and 145; and 51 a common Divisor unto 561 and 612.

To reduce a Fraction into the least terms, viz.

I. By a general Rule.

IV. A single Fraction may be reduced into the least terms by dividing the Numerator and Denominator

minator by their greatest common measure (or divisor;) for the Quotients will be the Numerator and Denominator of a Fraction equal to the former; and in the least terms.

So if the Fraction $\frac{91}{117}$ be given to be reduced into the least terms, search out the greatest common Divisor unto 91 and 117 by the last Rule, which will be found 13, and then dividing 91 by 13, the Quotient will be 7 for a new Numerator; also dividing 117 by 13, the Quotient will be 9 for a new Denominator: So the Fraction $\frac{91}{117}$ is reduced into the least terms, viz. into the Fraction $\frac{7}{9}$. In like manner $\frac{4}{12}$ will be reduced into $\frac{1}{3}$; And $\frac{6}{12}$ unto $\frac{1}{2}$: But here you are to observe, that if the greatest common Divisor unto the Numerator and Denominator be 1, such Fraction is in its least terms already: So the Fraction $\frac{1}{2}$ cannot be reduced into lower terms, because the greatest common Divisor will be found 1, (by the third Rule of this Chapter;) the like may happen of infinite others: And although the last be a general Rule for the Reduction of Fractions into their least terms, yet there are other Practical Rules, which in some cases will be more ready (especially unto Beginners) viz.

V. When the Numerator and Denominator are even Numbers, they may be measured or divided by 2. 2. By particular Rules.

Therefore in such case you may (as is taught in the Rules of the 6th Chapter) take the half of the Numerator for a new Numerator, also the half of the Denominator for a new Denominator. So if $\frac{16}{64}$ be given, draw at length the

16	8	4	2	1
64	32	16	8	4

line which separates the Numerator from the Denominator, and

cross'd the same with a downright stroke near the Fraction, as you may see in the *Margin*; then take the half of 16, which is 8, for a new Numerator, and the half of 64, which is 32, for a new Denominator; Again, the half of 8 is 4, for a new Numerator, also the half of 32 is 16, for a new Denominator, and proceeding in the like manner, there will be found $\frac{1}{4}$ equivalent unto $\frac{1}{16}$.

VI. When the Numerator and Denominator do each of them end with 5, or one of them ending with 5, and the other with a Cypher, they may be both measured or divided by 5. So $\frac{225}{475}$ will be reduced into $\frac{45}{95}$, and $\frac{45}{95}$ into $\frac{9}{19}$, as by the operation in the *Margin* is manifest.

VII. Whensoever you can espy any other Number, (which will exactly divide the Numerator and Denominator (although it be not the greatest common Divisor) you may divide the Numerator and Denominator by such Number as before: So $\frac{28}{84}$ may be first reduced into $\frac{7}{21}$ by 4, and $\frac{7}{21}$ may be reduced into $\frac{1}{3}$ by 7, as by the operation is manifest.

VIII. When the Numerator and Denominator do each of them end with a Cypher or Cyphers, cut off equal Cyphers in both, and the Fraction will be reduced into lesser terms: So $\frac{400}{9000}$ is reduced in 3, and $\frac{4}{90}$ into $\frac{2}{45}$.

IX. The

IX.
in the
may
multi
ction
know
which
duct b
lue of
and if
tient, y
ferious
ceed t
So t
sterling
viz. 1
9 by
lings
pound
180, 1
the De
tient is
manner
ling w
multip
12 (the
shilling
which
nomine
3 pene
of a d
found
be four
weight

IX. The value of a single Fraction in the known parts of the Integer, may be found out in this manner, viz. multiply the Numerator of the Fraction propounded by the number of known parts of the next inferiour Denomination which are equal to the Integer, and divide that product by the Denominator, so is the Quotient the value of the Fraction in that inferiour Denomination; and if there happen to be any Fraction in the Quotient, you may find the value thereof in the next inferiour Denomination, by the same Rule, and so proceed till you come to the least known parts.

So the value of $\frac{9}{16}$ of a pound sterling will be found 11s. 3d. viz. multiply the Numerator 9 by 20 (the number of shillings which are equal to 1 pound sterling) the Product is 180, which being divided by the Denominator 16, the Quotient is 11 $\frac{4}{16}$ shillings. In like manner, the value of $\frac{4}{16}$ of a shilling will be found 3 pence, for multiplying the Numerator 4 by 12 (the number of pence in a shilling) the Product is 48, which being divided by the Denominator 16, the Quotient is 3 pence. Also the value of $\frac{2}{3}$ of a pound sterling, will be found 10s. 9 $\frac{1}{3}$ d. And $\frac{3}{8}$ of a pound Troy will be found equivalent unto 3 Ounces 17 penny weight and 12 grains.

To find the value of a single Fraction in the known parts of the Integer.

$$\begin{array}{r} 16 \overline{) 180} \\ 16 \\ \hline 20 \\ 16 \\ \hline 4 \\ 12 \\ \hline 16 \overline{) 48} \\ 48 \\ \hline 0 \end{array}$$

To reduce a mixt
number into an im-
proper Fraction.

X. A mixt number may be reduced into an improper Fraction equivalent unto the mixt number, in this manner, *viz.* Multiply the Integer or Integers in the mixt number by the Denominator of the Fraction annexed to the Integer or Integers, and unto the Product add the Numerator of the said Fraction; so is the sum the Numerator of an improper Fraction, whose Denominator is the same with that of the said Fraction annexed.

So $4\frac{1}{2}$ will be reduced into the improper Fraction $\frac{9}{2}$ for 4 being multiplied by 2, the Product is 8, unto which adding the Numerator 1, the sum is 9 for a new Numerator, which being placed over the Denominator 2, gives the improper Fraction $\frac{9}{2}$; which is equivalent unto $4\frac{1}{2}$ (as will appear by the 13th Rule of this Chapter.) In like manner $7\frac{1}{2}$ will be reduced into $\frac{15}{2}$.

To reduce a whole
number into an im-
proper Fraction.

XI. A whole number is reduced into an improper Fraction, by placing the whole number given as a Numerator, and 1 as a Denominator.

So 14 Integers will be reduced into the improper Fraction $\frac{14}{1}$, and one Integer into the improper Fraction $\frac{1}{1}$.

XII. A whole number is reduced into an improper Fraction which shall have any Denominator assigned, in multiplying the whole number given by the Denominator assigned, and placing the Product as a Numerator over the said Denominator.

As if 13 be given to be reduced into an improper Fraction, whose Denominator shall be 4, multiply 13 by

by 4.
4, giv
(as w
13 m

be red
numbe
ner, v
the D
will gi
numbe
be rec
be divi
improp
whole

nomina
ctions
have e
Rule a
When
Denom
to two
shall h
the Num
of them
the Pro
dent un
also mu
Fraction
duct is a
Numera
ply the

by 4, the Product is 52, which being placed over 4, gives the *improper Fraction* $13\frac{1}{4}$ equivalent unto 13 (as will appear by the next *Rule*.) In like manner 13 may be reduced into $9\frac{1}{7}$.

XIII. An improper Fraction may be reduced into its equivalent whole number or mixt number in this manner, *viz.* divide the Numerator by the Denominator, and the quotient will give the whole number or mixt number sought; So the improper Fraction $\frac{59}{12}$ will be reduced into this *mixt number* $4\frac{7}{12}$, for if 59 be divided by 12, the Quotient is $4\frac{7}{12}$. Also this improper Fraction $13\frac{1}{4}$ will be reduced into the whole number 13.

To reduce an improper Fraction into its equivalent whole or mixt number.

XIV. Fractions having unequal Denominators, may be reduced into fractions of the same value, which shall have equal Denominators, by this Rule and the next following, *viz.* When two Fractions having unequal Denominators, are propounded to be reduced into two other Fractions of the same value, which shall have a common Denominator, multiply the Numerator of the first Fraction (that is either of them) by the Denominator of the second, and the Product shall be a new Numerator (correspondent unto the Numerator of that first fraction;) also multiplying the Numerator of the second Fraction by the Denominator of the first, the Product is a new Numerator (correspondent unto the Numerator of the second Fraction;) lastly, multiply the Denominators one by the other, and the Product

To reduce Fractions to a common Denominator, viz. 1. When two Fractions are propounded.

Product is a common Denominator to both the new Numerators.

Thus, If the Fractions $\frac{2}{3}$ and $\frac{4}{5}$ be propounded, multiply 2 by 5, the product 10 is a new

2 $\times$ 4 Numerator correspondent unto 2: also multiply 4 by 3, the product 12 is a new

3 5 Numerator correspondent unto 4: Lastly, multiply 3 by 5, and the Product 15

10 12 shall be a common Denominator unto the new Numerators, so the Fractions $\frac{10}{15}$ and $\frac{12}{15}$

$\frac{2}{3}$ are found out, which have equal Denominators, and each of these new Fractions is equal unto its correspondent Fraction first given, viz. $\frac{10}{15}$ is equal unto $\frac{2}{3}$ and $\frac{12}{15}$ is equal unto $\frac{4}{5}$, as will be manifest by the 4th Rule of this Chapter.

XV. When three or more Fractions having unequal Denominators, are given to be reduced into other Fractions of the same value with those given, but such as shall have one common Denominator; multiply continually (according to the Thirteenth Rule of the Fifth Chapter) the Numerator of the First Fraction into all the Denominators, except its own, i. e. the Denominator of that first Fraction; and reserve the last Product for a new Numerator instead of that first Numerator: In like manner, multiply continually the Numerator of the second Fraction into all the Denominators, except the Denominator of the second Fraction, and reserve the last Product for a new Numerator, instead of the second Numerator; proceed in like manner to find out new Numerators for the rest of the given Fractions: Lastly, multiply continually all

2 When three or more Fractions are to be reduced into others that shall have a Common Denominator.

$$\frac{1}{8} \frac{1}{2} \frac{1}{2} \frac{1}{2}$$

multi

rator

I mult

to the

ing, tw

ceth r

second

tinually

second

makes

for a r

rator 5

all the

saying,

produc

three n

found

and $\frac{2}{3}$

Denom

value u

viz. $\frac{2}{3}$

3; and

all the Denominators one into another, and the last Product shall be a common Denominator to all the new Numerators.

As for Example, If these three Fractions, $\frac{1}{8}$, $\frac{2}{5}$, $\frac{3}{7}$, having unequal (or different) Denominators, be given to be reduced into three other Fractions of the same value, which shall have equal Denominators (or one common Denominator.) First,

I multiply continually the first Numerator 3 into the second and third Denominators 5 and 7, say-

$$\begin{array}{r} \frac{1}{8} \quad \frac{2}{5} \quad \frac{3}{7} \\ \hline \frac{1 \times 2}{8 \times 5} \quad \frac{1 \times 3}{8 \times 7} \quad \frac{2 \times 3}{5 \times 7} \\ \frac{2}{40} \quad \frac{3}{56} \quad \frac{6}{35} \end{array}$$

ing 3 times 5 makes 15, which multiplied by 7 produceth 105, for a new Numerator instead of the first Numerator 3; Secondly,

I multiply continually the second Numerator 2 into the first and third Denominators 8 and 7, say-

$$\begin{array}{r} \frac{1}{8} \quad \frac{2}{5} \quad \frac{3}{7} \\ \hline \frac{2 \times 1}{8 \times 5} \quad \frac{2 \times 2}{5 \times 7} \quad \frac{2 \times 3}{5 \times 7} \\ \frac{2}{40} \quad \frac{4}{35} \quad \frac{6}{35} \end{array}$$

ing, twice 8 is 16, which multiplied by 7 produceth 112, for a new Numerator instead of the second Numerator 2; Thirdly, I multiply continually the third Numerator 5 into the first and second Denominators 8 and 5, saying 8 times 5 makes 40, which multiplied by 5 produceth 200, for a new Numerator instead of the third Numerator 5; Fourthly and lastly, I multiply continually

all the Denominators 8, 5 and 7 one into another, saying, 8 times 5 makes 40, which multiplied by 7 produceth 280 for a Denominator to each of the three new Numerators 105, 112 and 200 before found out; And so these three Fractions $\frac{105}{280}$, $\frac{112}{280}$, and $\frac{200}{280}$, are discovered, which have one common Denominator 280, and each of them is equal in value unto its correspondent Fraction first given,

viz. $\frac{105}{280}$ is equal unto $\frac{1}{8}$; Also $\frac{112}{280}$ is equal unto $\frac{2}{5}$; and $\frac{200}{280}$ is equal unto $\frac{3}{7}$; as may easily be proved

ved

ved by the fourth Rule of this Chapter.

After the same manner these four Fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$ are reducible into these, $\frac{2 \cdot 4 \cdot 5}{3 \cdot 6 \cdot 0}$, $\frac{3 \cdot 2 \cdot 5}{3 \cdot 6 \cdot 0}$, $\frac{2 \cdot 3 \cdot 8}{3 \cdot 6 \cdot 0}$ and $\frac{3 \cdot 0 \cdot 0}{3 \cdot 6 \cdot 0}$, which have 360 for a common Denominator, and are equal in value respectively to the four Fractions given to be reduced.

Note, Although by the foregoing fourteenth and fifteenth Rules, any multitude of Fractions may be reduced to a common Denominator; yet because Fractions in their least Terms are fittest for use, I shall shew how lesser Denominators than those that will be discovered by the said Rules, may oftentimes be found out, *viz.*

I. When the unequal Denominators of two Fractions have a common Divisor greater than 1, divide the Denominators severally by their greatest common Divisor (found out by the foregoing third Rule of this Chapter;) and then multiply cross-wise in this manner, *viz.* the Numerator of the first Fraction by the latter Quotient, and the Numerator of the latter Fraction by the first Quotient, and reserve the Products for new Numerators; Lastly, multiply the Denominator of the first Fraction by the latter Quotient (or the Denominator of the latter Fraction by the first Quotient,) so shall the Product be a common Denominator to the said new Numerators: As for Example, If $\frac{1}{12}$ and $\frac{2}{18}$ be proposed to be reduced to a common Denominator, I divide each of the Denominators, 12 and 18, by their greatest common Divisor 6, and the

the Quotients are 2 and 3; then I multiply 5 the Numerator of the first Fraction by 3 the latter Quo-
 tient; also 7 the Numerator of the latter Fraction by 2 the first Quo-
 tient; and the Products 15 and 14 I reserve for new Numerators instead
 of 5 and 7; Lastly, I multiply 12 the Denominator of the first Fraction by 3 the lat-
 ter Quotient (or 18 the Denominator of the latter Fraction by 2 the first Quotient,) and the Product
 36 is a Denominator to each of the new Numerators 15 and 14: So $\frac{15}{36}$ and $\frac{14}{36}$ are found out, which
 have the least common Denominator unto which the given Fractions $\frac{5}{12}$ and $\frac{7}{18}$ can be reduced;
 Also $\frac{15}{36}$ is equal to $\frac{5}{12}$, and $\frac{14}{36}$ to $\frac{7}{18}$.

$$\begin{array}{r} 6 \overline{) 12} \times 7 \quad 7 \\ \underline{42} \quad 18 \\ 36 \end{array}$$

II. Whenever the Denominator of a Fraction can be divided by the Denominator of a second Fraction, without any Remainder; then if by the Quotient you multiply severally the Numerator and Denominator of such second Fraction, a third will arise, having the same value with the second, and the same Denominator with the first Fraction: By this Rule Three or more Fractions may oftentimes be reduced to a lesser common Denominator, than that which will be discovered by the foregoing Rule XV. As for Example, Let these six following Fractions be given to be reduced to a common Denominator, viz.

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}$

Because 36 the Denominator of the first Fraction, being divided by the five other Denominators severally,

rally will give these Quotients 2, 3, 4, 6 and 12 without any Remainder. I multiply the Numerator and Denominator of each of the five latter Fractions, by its correspondent Quotient; viz. 11 and 18 by 2 the first Quotient; Also 7 and 10 by 3 the second Quotient, and in like manner the rest; so instead of those five latter Fractions, five others (hereunder placed after the first of those six) are produced, viz.

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}$

All which Fractions last express'd have a common Denominator 36, and are equal in value respectively to those given to be reduced.

XV. A compound Fraction (otherwise called a Fraction of a Fraction) may be reduced into a single Fraction in this manner, viz.

Multiply all the Numerators continually, and take the Product for a new Numerator; also multiply all the Denominators continually, and the Product shall be a new Denominator.

Thus, If the compound Fraction $\frac{1}{2}$ of $\frac{1}{4}$ be given to be reduced into a single Fraction, multiply the Numerators 1 and 3, one by the other, so is the Product 6 a new Numerator. Also multiplying

the Denominators 3 and 4 one by the other, the product 12 is a new Denominator, so $\frac{6}{12}$ (or $\frac{1}{2}$ is the single Fraction sought,) bring equivalent unto $\frac{1}{2}$ of $\frac{1}{4}$, the compound Fraction given to be reduced.

In

In lik

of $\frac{1}{2}$

merat

the r

tors 3

Denom

the fo

unto

the m

Integ

$\frac{1}{2}$ of

$\frac{1}{3}$ of

$\frac{1}{4}$ of

where

By

lesser n

ter na

duced

ling, 1

$\frac{1}{2}$ or

sterlin

aforefa

42 $\frac{1}{2}$

for $\frac{1}{2}$

in its le

Here

pound

Questio

Fraction

XVI.

ctions l

be wh

which

reason

In like manner, this compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ will be reduced unto $\frac{2}{5}$, or $\frac{2}{5}$; for the Numerator 2, 3, 4, being multiplied *continually*, produce the new Numerator 24, and the Denominators 3, 4, 5 multiplied *continually*, produce the new Denominator 60: Lastly, the new Fraction $\frac{24}{60}$ (by the fourth Rule of this Chapter) will be reduced unto $\frac{2}{5}$, which is equal to $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$: But to make the meaning hereof more evident, suppose the Integer to be one Pound of English Money; then

$\frac{2}{3}$ of 1 *l.* (*viz.* of 20 *s.*) ——— 16 *s.*
 $\frac{3}{4}$ of those $\frac{2}{3}$ (*viz.* of 16 *s.*) is ——— 12 *s.*
 $\frac{4}{5}$ of those $\frac{3}{4}$ (*viz.* of 12 *s.*) is ——— 8 *s.* or $\frac{2}{3}$ *l.*

whereby 'tis manifest that $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ *l.* is equal to $\frac{2}{5}$ *l.*
 By this Rule a Fraction or mixt number of a lesser name may be reduced to a Fraction of a greater name. As if 3 $\frac{1}{2}$ pence be propounded to be reduced into an improper Fraction of a Pound sterling, the operation will be in this manner, *viz.* 3 $\frac{1}{2}$ or $\frac{7}{2}$ of a penny is $\frac{7}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a Pound sterling, which compound Fraction will (by the aforesaid Rule be reduced, to $\frac{7}{40}$ *l.* In like manner 42 $\frac{1}{2}$ minutes of an hour are equal to $\frac{85}{96}$ of an hour, for $\frac{85}{96}$ (that is 42 $\frac{1}{2}$) of $\frac{1}{60}$ are equal to $\frac{85}{96}$ (or in its least terms) $\frac{85}{96}$.

Here you may also observe, that when a compound Fraction is one of the given terms in any Question, it is first of all to be reduced to a single Fraction by the aforesaid sixteenth Rule.

XVII. Two or more Fractions being given, there may be whole numbers found, which shall have the same reason or proportion as the

To find whole Numbers, which shall have the same reason as any Fractions or mixt numbers given.

Fractions given, *viz.* When the Fractions given have unequal Denominators, reduce them into equivalent Fractions which shall have a common Denominator (by the 14th or 15th Rule of this Chapter;) then rejecting the common Denominator, the Numerators shall have the same reason or proportion as the Fractions first given.

So $\frac{2}{3}$ and $\frac{5}{8}$ being given, will first of all be reduced into their equivalent Fractions $\frac{24}{48}$ and $\frac{25}{48}$; then rejecting the common Denominator 48, the Numerators 24 and 25 have the same reason with $\frac{2}{3}$ and $\frac{5}{8}$ *viz.* As $\frac{2}{3}$ is to $\frac{5}{8}$ so is 24 to 25. Also if the Fractions $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ were given, there will be found 8, 16 and 32, which are in the same proportion one to the other as the Fractions given: In like manner if mixt Numbers be given, there may be whole numbers found which shall have the same reason or Proportion, as the mixt numbers; so $5\frac{2}{3}$ and $3\frac{1}{2}$ being given, will be first reduced into the improper Fractions $\frac{17}{3}$ and $\frac{22}{5}$ (by the tenth Rule of this Chapter;) also the said $\frac{17}{3}$ and $\frac{22}{5}$ will be reduced into $\frac{136}{15}$ and $\frac{87}{15}$; then rejecting the common Denominator 15, the Numerators 136 and 87 will have the same reason as $5\frac{2}{3}$ and $3\frac{1}{2}$, *viz.* As 136 is to 87, so is $5\frac{2}{3}$ to $3\frac{1}{2}$. Also $16\frac{1}{2}$ and 18 being given, there will be found 33 and 36, which being divided by their common Divisor 3 (found out by the third Rule of this Chapter) will give 11 and 12 which have the same reason as $16\frac{1}{2}$ and 18.

W
nators
ther, f
a Frac
the sa
nator,
of the
So
will be
and 2
Denom
these F
(by the
found
of the
II.
have u
first to
the sam
commo
recenth
ter;) a
of this
So it
will be

C H A P. XVIII.

*Addition of Vulgar Fractions and
mixt Numbers.*

WHEN the Numbers given to be added are single Fractions, and have equal Denominators, add all the Numerators together, so is the sum the Numerator of a Fraction, whose Denominator is the same with the common Denominator, which new Fraction is the sum of the Fractions given to be added.

*To add single
Fractions, viz.
1. When they
have equal De-
nominators.*

So $\frac{1}{3}$ and $\frac{2}{3}$ being given to be added, their sum will be found $\frac{3}{3}$, viz. the sum of the Numerators, 3 and 2 is 5, which being placed over the common Denominator 9, gives $\frac{5}{9}$: Inlike manner the sum of these Fractions $\frac{7}{8}$, $\frac{1}{8}$, $\frac{3}{8}$, and $\frac{2}{8}$, will be found $\frac{13}{8}$, which (by the 13 Rule of the seventeenth Chapter) will be found equivalent unto $2 \frac{1}{8}$; so that $2 \frac{1}{8}$ is the sum of the Fractions given to be added.

II. When the Fractions given to be added have unequal Denominators, they are first to be reduced into Fractions of the same value, which shall have a common Denominator (by the fourteenth or fifteenth Rule of the seventeenth Chapter;) and then they may be added by the first Rule of this Chapter.

*2. When they
have unequal
Denominators.*

So if $\frac{1}{2}$ and $\frac{1}{3}$ were given to be added, their sum will be found $1 \frac{1}{6}$; for (by the fourteenth Rule of

the seventeenth Chapter) $\frac{2}{3}$ and $\frac{1}{3}$ will be reduced into their equivalent Fractions $\frac{2}{3}$ and $\frac{1}{3}$ which having equal Denominators may be added according to the first Rule of this Chapter, and so the sum will be found $1\frac{1}{3}$: In like manner the sum of these Fractions $\frac{1}{2}$ that is $1\frac{1}{3}$ ions $\frac{1}{2}$ and $\frac{1}{4}$ will be found $1\frac{1}{8}$. Also the sum of these six Fractions $\frac{1}{16}$ $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, after they are reduced to a common Denominator (according to the latter Example in the Note at the end of the fifteenth Rule of the seventeenth Chapter) will be found $3\frac{1}{2}$, that is, $3\frac{1}{2}$.

III. When any of the Fractions given to be added is a compound Fraction, such as $1\frac{1}{2}$ of $\frac{1}{3}$ the Addition of compound Fractions is first of all to be reduced into a single Fraction (by the sixteenth Rule of the seventeenth Chapter) and then you may proceed as before.

So $\frac{2}{3}$ and $\frac{1}{3}$ of $\frac{1}{4}$ being given to be added, their sum will be found $\frac{2}{3}$ for the compound Fraction $\frac{2}{3}$ of $\frac{1}{4}$ will (by the 16th Rule of the 17th Chapter) be reduced to $\frac{1}{2}$ (or in its least terms) $\frac{1}{2}$, which added to the single Fraction $\frac{1}{3}$ (according to the second Rule of this Chapter) gives $\frac{5}{6}$. Here you may observe, that the Fractions given to be added in all the former cases, are supposed to be Fractions of Integers, which have one and the same particular Denomination, viz. If one of the Fractions, given to be added, be a Fraction of a pound sterling, all the rest ought to be Fractions of a pound

By Denomination is meant the name of any Integer or thing.

pound *sterling*, and the like is to be understood of other denominations.

IV. When Fractions of Integers, of different Denominations are given to be added, they are first of all to be reduced into Fractions of Integers, which shall have one and the same particular Denomination (by the sixteenth Rule of the seventeenth Chapter;) and then they may be added by the first or second Rule of this Chapter.

To add Fractions of Integers which have different Denominations.

So if $\frac{2}{9}$ of a pound *sterling*, $\frac{1}{3}$ of a shilling, and $\frac{1}{8}$ of a penny were given to be added, reduce the two latter into Fractions of a pound *sterling* (by the sixteenth Rule of the seventeenth Chapter) *viz.* $\frac{1}{3}$ of a shilling is $\frac{1}{6}$ of $\frac{1}{20}$ of a pound *sterling*, which compound Fraction being reduced into a single Fraction gives $\frac{1}{120}$ *li.* Likewise $\frac{1}{8}$ of a penny, is $\frac{1}{96}$ of $\frac{1}{20}$ of a pound *sterling*, which compound Fraction being reduced, gives $\frac{1}{1920}$ *li.* Lastly, $\frac{2}{9}$ *li.* $\frac{1}{10}$ *li.* and $\frac{1}{384}$ *li.* being added according to the second Rule of this Chapter, their sum will be found $\frac{2}{34} \frac{0}{6} \frac{0}{0} \frac{6}{0} \frac{8}{0}$, or in its last terms $\frac{2}{3} \frac{1}{8} \frac{1}{8} \frac{1}{0} \frac{2}{0}$ *li.*

V. When mixt-Numbers are given to be added, find first of all the sum of the Fractions (by the first and the second Rule of this Chapter;) then add the Integer or Integers (if there be any found) in the sum of the fractions, unto the whole numbers, and collect the sum of them as you were taught by the Rules of the third Chapter.

To add mixt Numbers.

So if $3\frac{1}{2}$, $4\frac{1}{3}$ and $16\frac{1}{8}$ were given to be added, their sum will be found $24\frac{1}{24}$ *viz.* the sum of the Fractions $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{8}$, will be found (by the second Rule of this Chapter) to be $\frac{1}{24}$, and the sum of the

whole numbers, 3, 4, and 16, is 23, unto which adding 1 (the Integer found in the sum of the Fractions) the sum is 24; so that $24\frac{1}{4}$ is the sum of the mixt numbers given to be added.

C H A P. XIX.

Subtraction of Vulgar Fractions and mixt Numbers.

I. **W**hen the Numbers given are both single Fractions and have equal Denominators, subtract the lesser Numerator from the greater, and place the remainder over the common Denominator, so is such new Fraction the difference between the Fractions given:

The Subtraction of single Fractions, viz.

1. When they have a common Denominator.

Thus the difference between the Fraction $1\frac{2}{11}$ and $1\frac{7}{11}$ is $1\frac{2}{11}$, which is found by subtracting the lesser Numerator 7 from the greater Numerator 9, and placing the remainder 2 over the common Denominator 11: Also the difference between the Fractions $2\frac{1}{11}$ and $2\frac{2}{11}$ is $2\frac{6}{11}$, that is, the Fraction $2\frac{7}{11}$ exceeds $2\frac{1}{11}$ by $2\frac{6}{11}$.

II. When the Numbers given are both single Fractions and have not a common Denominator, reduce them into Fractions of the same value which shall have a common Denominator (by the fourteenth or fifteenth Rule of the seventeenth Chapter) and then find their differences by the last Rule.

2. When they have unequal Denominators.

So

So the difference between the Fractions $\frac{1}{2}$ and $\frac{1}{8}$ will be found $\frac{3}{8}$, viz. reducing the Fractions given into their equivalent Fractions $\frac{4}{8}$ and $\frac{1}{8}$, which have a common Denominator, the difference sought will be found $\frac{3}{8}$, by the first Rule of this Chapter. Likewise $\frac{1}{2}$ being subtracted from $\frac{3}{4}$ there will remain $\frac{1}{4}$.

III. When one of the Numbers given is a whole Number or a mixt Number, also when both of them are mixt Numbers, reduce such whole, or mixt Numbers into an improper Fraction or Fractions, by the 10th or 11th Rule of the seventeenth Chapter, and then the operation will be according to the first or second Rule of this Chapter.

The subtraction of mixt numbers, viz. I By a general Rule.

So $7\frac{3}{5}$ being given to be subtracted from 12, the remainder will be found $4\frac{2}{5}$; viz. First $7\frac{3}{5}$ will be reduced into the improper Fraction $\frac{38}{5}$, also 12 will be reduced to $\frac{60}{5}$, then these two improper Fractions $\frac{38}{5}$ and $\frac{60}{5}$ will be reduced into their equivalent Fractions $\frac{38}{5}$ and $\frac{60}{5}$ (which have a common Denominator. Lastly, the difference between $\frac{38}{5}$ and $\frac{60}{5}$ is $\frac{22}{5}$ or $4\frac{2}{5}$. In like manner $9\frac{1}{2}$ being given to be subtracted from $12\frac{1}{2}$, the remainder will be found $2\frac{1}{2}$; as by the subsequent operation is manifest.

$$\begin{array}{r} 12 \\ \frac{12}{1} \\ \hline 60 \\ 38 \\ \hline 22 \\ \frac{22}{5} \text{ that is } 4\frac{2}{5} \end{array}$$

$$\begin{array}{r} 12\frac{1}{2} \\ \frac{12\frac{1}{2}}{2} \\ \hline 122 \\ 95 \\ \hline 27 \\ \frac{27}{10} \text{ that is } 2\frac{7}{10} \end{array}$$

Although the three last Rules be sufficient for all cases in *Subtraction of Fractions*, mixt numbers, or whole and mixt; nevertheless the following Rules will be more expeditious in the subtraction of mixt Numbers; or whole and mixt, especially when the Integers consist of many places, as will be manifest by the operation, viz.

IV. When a whole Number is given to be subtracted from a mixt Number, subtract the said whole Number from the Integer or Integers of the mixt Number (as is taught by the Rules of the fourth Chapter) and unto the remainder annex the Fractional part of the mixt Number given, so is the mixt Number thus found, the remainder or difference sought.

As if 7 be given to be subtracted from $24 \frac{3}{8}$, the remainder will be $17 \frac{3}{8}$ as by the operation is manifest.

$$\begin{array}{r} 24 \frac{3}{8} \\ 7 \\ \hline 17 \frac{3}{8} \end{array}$$

V. When a Fraction is given to be subtracted from an Integer, subtract the Numerator from the Denominator, and place that which remains over the Denominator, which new Fraction thus found, is the remainder or difference sought,

So $\frac{2}{3}$ being subtracted from an Integer, or 1, the remainder is $\frac{1}{3}$: Also $\frac{1}{9}$ being subtracted from 1, the remainder is $\frac{8}{9}$.

VI. When a Fraction is given to be subtracted from a whole Number greater than 1, subtract the said Fraction from one of the Integers (given by the last Rule;) so the remaining

3. A Fraction from a whole Number greater than 1.

remaining Fraction being annexed to the number of Integers lessened by unity or 1, gives the remainder or difference sought.

Thus $\frac{2}{3}$ being subtracted from 17, the remainder is $16\frac{2}{3}$; also $1\frac{2}{3}$ being subtracted from 39, the remainder is $38\frac{1}{3}$.

VII. When a mixt Number is given to be subtracted from a whole Number, subtract first of all (by the fifth Rule of this Chapter) the Fractional part of the mixt Number from an Integer borrowed from the whole Number given, and set down the remaining Fraction, then adding the Integer borrowed unto the Integer or Integers of the mixt Number, subtract the said sum from the whole Number, given (as is taught in subtraction of whole Numbers;) so that which remains, together with the remaining Fraction before found, is the remainder or difference sought.

4. A mixt number from a whole number.

So if $9\frac{2}{3}$ be subtracted from 50, the remainder is $40\frac{1}{3}$, as by the operation is manifest.

50
9 $1\frac{2}{3}$
40 $1\frac{1}{3}$

VIII. When a Fraction is given to be subtracted from a mixt Number, and the said Fraction is less than the Fractional part of the mixt Number, subtract the lesser Fraction from the greater by the first or second Rule of this Chapter, then the remaining Fraction being annexed to the Integer or Integers of the mixt Number, gives the remainder or difference sought.

5. A Fraction from a mixt Number by this and the next Rule.

So

So $\frac{2}{9}$ being subtracted from $12 \frac{7}{8}$ the remainder is $12 \frac{23}{24}$, as by the operation is manifest.

$$\begin{array}{r} 12 \frac{7}{8} \\ 0 \frac{2}{9} \\ \hline 12 \frac{23}{24} \end{array}$$
 IX. When a Fraction is given to be subtracted from a mixt Number, and the said Fraction is greater than the Fractional part of the mixt number, subtract the said greater Fraction from an Integer borrowed from the mixt Number (by the fifth Rule of this Chapter) and add the remaining Fraction unto the Fractional part of the mixt Number (by the first or second Rule of the eighteenth Chapter) so the Fraction found by that addition, being annexed to the Integers of the mixt Number lessened by an Integer, or 1, gives the remainder or difference sought.

Thus $\frac{2}{9}$ being subtracted from $13 \frac{3}{8}$, the remainder $12 \frac{12}{8}$ viz. subtracting $\frac{2}{9}$ from 1, the remainder is $\frac{7}{9}$, which added to $\frac{3}{8}$ gives $\frac{12}{72}$, which being annexed to 12 (the number of Integers in the mixt number lessened by 1 or unity) gives $12 \frac{12}{72}$ the remainder sought.

X. When a mixt Number is given to be subtracted from a mixt Number, and the Fractional part of the mixt Number to be subtracted, is less than the Fractional part of the mixt Number from which you are to subtract, subtract the said lesser Fraction from the greater (by the first or second Rule of this Chapter) and set down the remaining Fraction: Also subtract the Integers of the lesser mixt Number from the Integers of the greater (as in subtraction of whole Numbers,) so is the mixt Number thus found, the remainder or difference sought.

6. A mixt number from a mixt number by this and the next Rule.

So

So
from
 $3 \frac{1}{2}$,
mainc
20, th

.XII.

ated fr
of the
than t
from
said gr
from
Rule o
Frac
mixt N
18th C
the Fr
then a
ger or
subtrac
ter mi
Number
the Fra
or diffe

Thus i
the rem
subtrac
remainc
 $\frac{29}{40}$, then
unto 20
from 35
differen

So if $17 \frac{1}{8}$ be given to be subtracted from $20 \frac{2}{7}$, the remainder will be found $20 \frac{2}{7}$
 $3 \frac{1}{3} \frac{2}{6}$, viz. subtracting $\frac{1}{8}$ from $\frac{2}{7}$, the remainder is $\frac{1}{56}$, also subtracting 17 from 20 , the remainder is 3. $17 \frac{1}{8}$
 $3 \frac{1}{3} \frac{2}{6}$.

XI. When a mixt Number is given to be subtracted from a mixt Number, and the Fractional part of the mixt Number to be subtracted is greater than the Fractional part of the mixt Number from which you are to subtract, subtract the said greater Fraction from an Integer borrowed from the greater mixt Number (by the fifth Rule of this Chapter) and add the remaining Fraction unto the Fractional part of the greater mixt Number (by the first or second Rule of the 18th Chapter;) so is the sum to be reserved as the Fractional part of the remainder sought; then add the Integer borrowed unto the Integer or Integers of the lesser mixt Number, and subtract the sum from the Integers of the greater mixt Number (as in subtraction of whole Numbers;) so that which remains, together with the Fraction before reserved, is the remainder or difference sought.

Thus if $20 \frac{2}{8}$ be given to be subtracted from $35 \frac{2}{5}$, the remainder will be found $14 \frac{2}{40}$, viz. subtracting $\frac{2}{8}$ from an Integer or 1, the remainder is $\frac{1}{8}$, which added to $\frac{2}{5}$ gives $\frac{29}{40}$, then adding the Integer borrowed unto 20, it will be 21, which subtracted from 35, the remainder is 14, so the remainder or difference sought is $14 \frac{2}{40}$.

When

When you cannot clearly discern which is the greater of two Fractions, having unequal Denominators, reduce them into Fractions of the same value which shall have a common De-

To discern the greater of two Fractions.

ominator (by the fourteenth Rule of the seventeenth Chapter) and then it will be apparent which of the two Fractions is the greater. As if it be desired to know which of these two Fractions $\frac{4}{7}$ and $\frac{1}{3}$ is the greater, after they are reduced to $\frac{2}{3}$ and $\frac{1}{3}$, it is evident that the former exceeds the latter by $\frac{1}{3}$.

CHAP. XX.

Multiplication of Vulgar Fractions, and mixt Numbers.

I. **W**Hen the Numbers given to be multiplied are both single Fractions, multiply the Numerators one by the other, and take the Product for a new Numerator; also multiply the Denominators one by the other, and the Product is a new Denominator, which new Fraction is the Product sought.

To multiply single Fractions. So $\frac{1}{2}$ and $\frac{1}{8}$ being given to be multiplied, the Product will be found $\frac{1}{16}$, for 7 multiplied by 5 produceth 35 for a new Numerator, and 12 multiplied by 8 produceth 96 for a new Denominator: also $\frac{1}{2}$ and $\frac{1}{7}$ being multiplied one by the other, the Product will be found $\frac{1}{14}$. Here you may observe that in the multiplication of proper Fractions, the Product is always less than either of the terms given; for in multiplication such proportion

as unity or 1 hath to either of the terms given, the same proportion hath the other term to the Product.

II. When one of the numbers given is a whole number or a mixt number; also when both of them are mixt numbers, reduce such whole number, mixt number or numbers into an improper Fraction or Fractions by the tenth or eleventh Rules of the seventeenth Chapter, and then the operation will be the same as in the last Rule.

To multiply mixt Numbers.

So $8\frac{2}{3}$ being given to be multiplied by 5, the Product will be found $43\frac{2}{3}$; viz. $8\frac{2}{3}$ being reduced into the improper Fraction $\frac{26}{3}$: Also 5 unto $\frac{2}{3}$ multiply 26 by 5, the Product is 130 for a new Numerator: Also multiplying 3 by 1, the Product is 3 for a new Denominator, which new Fraction $\frac{130}{3}$ being reduced (according to the thirteenth Rule of the seventeenth Chapter) will be $43\frac{2}{3}$ the Product sought. In like manner $7\frac{1}{2}$ being multiplied by $5\frac{1}{3}$, the Product will be found 41. Here observe, that when either of the terms given is a compound Fraction, it is first of all to be reduced into a single Fraction, and then the operation is as before.

Note, 1. Sometimes the Work of Multiplication in Fractions may be very usefully contracted by this following Rule, viz.

When two Fractions propos'd to be multiplied (whether they be proper or improper) are such, that the Numerator of the one, and the Denominator of the other, may be severally divided by some common Divisor without a remainder; you may take

take the Quotients instead of the said Numerator and Denominator, and then multiply as before in the first Rule of this Chapter: As for Example, If $\frac{6}{7}$ be to be multiplied by $1\frac{1}{2}$ because 6 the Numerator of the first, and 12 the Denominator of the latter Fraction, being severally divided by their common Divisor 6, give the Quotients 1 and 2, I set these (or imagin them to be set) in the places of 6 and 12; by which exchange there arise $\frac{1}{2}$ and $\frac{1}{2}$, these multiplied one by the other (according to the first Rule of this Chapter) produce $\frac{1}{4}$ the desired Product of $\frac{6}{7}$ into $1\frac{1}{2}$, in the smallest terms.

Again, to multiply $1\frac{1}{4}$ by $1\frac{3}{8}$; because the Numerator of the first Fraction and the Denominator of the latter, being each divided by 16 give the Quotients 1 and 1, I set 1 and 1 in the places of 16 and 16; likewise because 48 the Denominator of the first, and 3 the Numerator of the latter Fraction, being each divided by their common Divisor 3, give 16 and 1, I take 16 and 1 instead of 48 and 3, so by those exchanges there arise $\frac{1}{16}$ and $\frac{1}{16}$, which multiplied one by the other produce $\frac{1}{256}$, which is the Product in the smallest terms made by the multiplication of $1\frac{1}{4}$ into (or by) $1\frac{3}{8}$.

2. To take any part or parts of a Number propounded, is nothing else but to multiply the said Number by the Fraction which declareth what part is to be taken: So if you desire to know what is $\frac{1}{3}$ of 320, multiply $1\frac{2}{3}$ by $\frac{1}{3}$, or $\frac{2}{3}$ by $\frac{1}{3}$, and the Product will be 200. In like manner $\frac{2}{3}$ of 45 $\frac{3}{4}$ is 30 $\frac{1}{4}$: Also $\frac{1}{4}$ of 120 is 30.

3. Sometimes the work of multiplication in mixt
B
Numbers

Numbers may be compendiously performed after the manner of these following Examples, viz. If it be required to multiply $120 \frac{1}{4}$ by $48 \frac{1}{2}$, first multiply the whole Numbers mutually, to wit, 120 by 48, and place the particular Products orderly one under the other as in Multiplication of whole Numbers; then multiply the said whole Numbers first given by the Fractions alternately, viz. take $\frac{1}{4}$ of 48 which is 12, also take $\frac{1}{2}$ of 120 which is 60, and place the said 12 and 60 orderly to be added to the former particular Products: Lastly, add all together, and to the sum annex the Product of the two Fractions, to wit in this Example, the Product of the Multiplication of $\frac{1}{4}$ by $\frac{1}{2}$, which is $\frac{1}{8}$; to the total Product required will be $5832 \frac{1}{8}$, as you see by the Example in the Margin. In like manner, if $18 \frac{1}{2}$ be multiplied by $40 \frac{1}{3}$, the Product will be $746 \frac{1}{6}$; and if $29 \frac{1}{2}$ be multiplied by 50, the Product will be 1475, as you see by the Examples following.

$$\begin{array}{r} 120 \frac{1}{4} \\ 48 \frac{1}{2} \\ \hline 960 \\ 480 \\ 12 \\ 60 \\ \hline 5832 \frac{1}{8} \end{array}$$

$18 \frac{1}{2}$	$29 \frac{1}{2}$
$40 \frac{1}{3}$	50
720	1450
20	25
746 $\frac{1}{6}$	1475

41 When a Fraction is to be multiplied by a Number which happens to be the same with the Denominator, take the Numerator for the Product; so if this Fraction $\frac{1}{4}$ be propounded to be multiplied by the Denominator 4, the Product will be

be $1\frac{1}{4}$ that is 3, which is the same with the Numerator 3. In like manner if $\frac{1}{8}$ be multiplied by the Denominator 8, the Product is equal to 5 the Numerator of the said $\frac{5}{8}$.

C H A P. XXI.

Division of Vulgar Fractions and mixt Numbers.

I. **W**hen the Numbers given are both single Fractions, multiply the Denominator of the Divisor by the Numerator of the Dividend; and take the Product for a new Numerator. Also multiply the Numerator of the Divisor by the Denominator of the Dividend, and the Product is a new Denominator; which new Fraction is the Quotient sought.

So if $\frac{4}{5}$ be given to be divided by $\frac{3}{4}$, the Quotient will be found $2\frac{2}{3}$; viz multiplying 5 by 4 the Product is 20 for a new Numerator, $\frac{4}{5}$ also multiplying 3 by 9, the Product is 27 for a new Denominator, so is $2\frac{2}{3}$ the Quotient sought; in like manner if $\frac{1}{8}$ be given to be divided by $\frac{3}{7}$, the Quotient will be found $2\frac{3}{8}$, that is, $2\frac{3}{8}$, as you see in the Example: Here $\frac{3}{7}$ you may observe, that in Division by proper Fractions, the Quotient is always greater than either of the Fractions given; for in Division, as the Divisor is in proportion to 1 or unity, so is the Dividend to the Quotient.

II. When

II. When one of the Numbers given is a whole number or a mixt number; also when both are mixt numbers, reduce such whole number or mixt number or numbers into an improper Fraction or Fractions, by the tenth or eleventh Rule of the seventeenth Chapter, and then the Operation will be the same as in the last Rule.

So if 42 be divided by $7\frac{1}{2}$, the Quotient will be found $5\frac{2}{3}$, for $7\frac{1}{2}$ and 42 will be reduced into these improper Fractions $1\frac{1}{2}$ and $4\frac{2}{2}$, then multiplying 42 by 2, the Product is 84 for a new Numerator, also multiplying 15 by 1, the product is 15 for a new Denominator, so is $\frac{84}{15}$ the Quotient sought, which is equal to $5\frac{2}{3}$ (as is evident by the thirteenth Rule of the seventeenth Chapter.) In like manner, if $6\frac{1}{2}$ be divided by $3\frac{2}{3}$, the Quotient will be $1\frac{3}{4}$. Also if $5\frac{1}{3}$ be divided by $12\frac{1}{2}$ the Quotient will be $\frac{2}{7}$.

Note, Sometimes the Work of Division in Fractions may be very usefully contracted by this following Rule, viz. When either the two Numerators, or the two Denominators of the Fractions proposed can be divided severally by some common Divisor without a remainder, you may take the Quotients instead of the said Numerators or Denominators, and then divide by the first Rule of this Chapter: As for Example, If $\frac{12}{8}$ be to be divided by $\frac{3}{2}$, because the Numerators 12 and 8 being each divided by their common Divisor 4, will give the Quotients 3 and 2; I take these instead of 12 and 8, by which exchange there arise $\frac{3}{2}$ and $\frac{2}{2}$, the former of which being divided by the latter, (accord-

ing to the first Rule of this Chapter) given $\frac{15}{34}$, which is the Quotient in the least terms that ariseth by dividing $\frac{15}{17}$ by $\frac{2}{5}$.

Again, To divide $\frac{25}{8}$ by $\frac{15}{8}$; because the Numerators 25 and 15 being severally divided by their common Divisor 5, give the Quotients 5 and 3, likewise because the Denominators 8 and 8 being each divided by 8 give the Quotients 1 and 1, I set 5 and 3 in the places of the Numerators 25 and 15, also 1 and 1 in the Places of the Denominators 8 and 8, whence arise $\frac{5}{1}$ and $\frac{3}{1}$. Lastly, dividing $\frac{5}{1}$ by $\frac{3}{1}$, that is 5 by 3, there ariseth $\frac{5}{3}$, that is $1\frac{2}{3}$, which is the desired Quotient of $\frac{25}{8}$ divided by $\frac{15}{8}$.

Questions to exercise the Rules of Vulgar Fractions before delivered.

Quest. 1. The difference of two numbers is $1\frac{3}{4}$, the lesser number is $2\frac{1}{8}$, what is the greater? *Ans.* $3\frac{2}{3}$, (found by *Addition*.)

Q. 2. What number is that, which if added to $3\frac{1}{8}$ gives the sum $8\frac{2}{8}$? *Ans.* $4\frac{7}{8}$ (found by *Subtraction*.)

Quest. 3. There is in three Bags the sum of $121\frac{2}{10}$ l. viz. in the first Bag $50\frac{2}{8}$ l. in the second $40\frac{4}{1}$ l. what is in the third Bag? *Answer* $30\frac{1}{3}$ l. (found by *Addition* and *Subtraction*.)

Quest. 4. Two Merchants *A* and *B*, having certain shares in a Ship, the share of *A* is $\frac{2}{10}$ of the Ship, that of *B* $\frac{2}{3}$, what is the difference between their parts? *Ans.* The share of *A* exceeds the share of *B* by $\frac{1}{30}$, (found by *Subtraction*.)

Quest.

Quest. 5. What is $\frac{2}{3}$ of $130 \frac{2}{3}$? *Ans.* $81 \frac{2}{3}$
(found by *Multiplication.*)

Quest. 6. What Number is that, which being multiplied by $\frac{2}{3}$ produceth $25 \frac{2}{3}$? *Ans.* $42 \frac{1}{3}$ (found by *Division.*)

Now followeth the *Doctrine of Decimal Fractions.*

The Doctrine of Decimal Fractions.

C H A P. XXII.

Notation of Decimal Fractions.

IT is hard to determine, who was the first that brought *Decimal Arithmetick* to light, though it be a late Invention; but without doubt it hath received much Improvement within the compass of a few Years, by the Industry of *Artists*, and now seems to be arrived at Perfection. The excellency thereof is best known to such as can apply it to the practical Part of the *Mathematicks*, and to the Construction of *Tables*, which depend upon standing or constant Proportions, such are *Trigonometrical Canons*, *Tables* for computing of compound Interest, &c. in which cases decimal Operations do afford so great help, (that in my Opinion) many Ages have not produced a more useful Invention. But it may be objected, that *Decimal Arithmetick* for the most part gives an imperfect Solution to

*The proper use of
Decimal Arithmetick.*

a Question. This I grant, yet the answer so given may be as useful as that which is exactly true; for in common Affairs, the Loss of $\frac{1}{1000}$ part of a Grain, or of an Inch, &c. to wit, any quantity which cannot be seen, is inconsiderable: but I would not be mistaken, for in extolling *Decimals* I do not cry

Decimal Fractions sometimes abused.

down *Vulgar Fractions*, since experience sheweth that *Decimal Fractions* are commonly abused, by being applyed to all manner of Questions about *Money, Weight, &c.* when indeed many Questions may be resolved with much more facility by *Vulgar Arithmetick*, as may partly appear by this Example, viz. at 9 l. — 6 s. — 8 d. the hundred weight of Tobacco, what will 987 hundred weight cost? *Ans.* 9212 l. which by the common Rule of *Practice* by *Aliquot Parts* is found out in a quarter of the time, that will necessarily be required to work it by *Decimals*, which at last will give an imperfect Answer; I might instance the like inconvenience divers ways, were it not for loss of time; so that the right use of *Decimals* depends upon the Discretion of the Artist.

II. When a single Fraction hath for its Denominator a Number consisting of 1 or Unity in the extream Place towards the Left hand, and nothing but a Cypher or Cyphers towards the Right, it is more particularly called a *Decimal*: Of this kind are these that follow, $\frac{1}{10}$, that is, five tenths; $\frac{1}{100}$, five hundredth parts; likewise these are decimal Fractions, $\frac{34}{1000}$, $\frac{201}{10000}$, $\frac{1023}{100000}$, &c.

The Definition of a Decimal Fraction.

III. A Decimal Fraction may be express'd without the Denominator, by prefixing a Point or Comma before (to wit, on the Left hand of) the Numerator, so $\frac{2}{10}$ may be written thus .5, or thus .5, $\frac{25}{100}$ thus, .25, or thus, .25.

IV. In Decimals when the Numerator consists not of so many Places as the Denominator hath Cyphers, fill up the void Places in the Numerator with Cyphers prefixed on the Left hand: so $\frac{5}{100}$ is written thus, .05; likewise $\frac{50}{1000}$ thus .050; and $\frac{205}{10000}$ thus, .0205, likewise $\frac{6}{10000}$ thus .0006.

V. In Decimals thus express'd, the Denominator is discoverable by the places of the Numerator: For if the Numerator consists of one place, the Denominator consists of 1 or Unity with one Cypher; if of two places, the Denominator consists of 1 with two Cyphers annexed; if of three, the Denominator consists of 1 or Unity with three Cyphers annexed: So the Denominator of .25 is 100; the Denominator of .050 is 1000, and the Denominator of .096 is 1000.

VI. Cyphers at the end of a Decimal do neither augment nor diminish the Value thereof: So .2, .20, .200, .2000 are *Decimals*, which have one and the same Value, for $\frac{200}{1000}$ being abbreviated by the eighth Rule of the seventeenth Chapter, will be made $\frac{2}{10}$ and so will $\frac{2000}{10000}$ or $\frac{20000}{100000}$.

VII. Wherefore Decimal Fractions are easily reduced to a common Denominator (which is a troublesome Work in *Vulgar Fractions*;) for if all the Numerators of as many decimal Fractions as are given, be made to consist of the same Number of places, by annexing a Cypher or Cyphers at the

end (that is on the Right hand) of such Numerators as are defective, they will all be reduced to a common Denominator, to these *Decimals*, .2, .03, .027, (which signifie $\frac{2}{10}$, $\frac{3}{100}$, $\frac{27}{1000}$) may be reduced into these, .200, .030, .027, which have a 1000 for a common Denominator.

VIII. The order of Places in any Decimal proceedeth from the Left hand to the Right, contrary to the order of places in Integers, which is from the Right hand to the left: So in this *Decimal*, .247 the Figure 2 standeth in the first place (being the outermost towards the Left hand, and next to the Point,) the Figure 4 standeth in the second place, and 7 in the third. Also in this *Decimal* .0245, a Cypher stands in the first place, 2 in the second, 4 in the third, and 5 in the fourth.

IX. Every place in the Numerator of a Decimal Fraction hath a peculiar Denominator, or proper value, viz. the Denominator of the first place is 10, of the second, 100; of the third, 1000, &c. so that the first place of a Decimal signifies tenth parts of an Unite or Integer; the second place, hundredth parts of an Integer; the third place, thousandth parts of an Integer, &c. Hence it is manifest, that this *Decimal* .3254 (every place thereof being considered apart by it self) consists of .3, .02, .005, .0004, (viz. $\frac{3}{10}$, $\frac{2}{100}$, $\frac{5}{1000}$, $\frac{4}{10000}$), which being reduced to a common Denominator (by the seventh Rule of this Chapter) will give these, $\frac{3000}{10000}$, $\frac{200}{10000}$, $\frac{50}{10000}$, $\frac{4}{10000}$, (to wit, $\frac{3000}{10000}$, $\frac{200}{10000}$, $\frac{50}{10000}$, $\frac{4}{10000}$) all which collectively make .3254 (or $\frac{3254}{10000}$)

X. In whole Numbers, the first place above (that is on the Left hand of) the place of Units, signifies

fies Tens of Units ; but the first place beneath, (that is on the right hand of) the place of Units, signifies tenth parts of 1 or Unity, and is called the first place of Decimal parts, or place of Primes ; likewise the second place above the place of Units signifies hundreds of Units, but the second place beneath the place of Units, signifieth hundredth parts of 1 or Unity, and is called the *second place of Decimals*, or place of seconds ; so that as the values of the places in Integers, do ascend in a decuple proportion from the place of Units towards the left hand, so the values of the places of Decimals do descend in a subdecuple proportion beneath the place of Units towards the right hand : *viz.* Among the places of Integers, every following place towards the left hand, is ten times the value of the next preceding place : But among the places of decimal Parts, every following place towards the right hand is one tenth part of the value of the next preceding place : All which will be evident by the following *Table*.

TABLE for the Notation of Integers and Decimals.

Integers.					Decimal parts.				
of Units.					of 1 or Unity				
&c.	Ten Thousands	Thousands	Hundreds	Tens	Units (under 10)	Ten parts	Hundredth parts	Thousandth parts	Ten Thousandth parts
	7	2	2	8	5	8	2	3	7
&c.	Fifth place	Fourth place	Third place	Second place	First place	First place	Second place	Third place	Fourth place

In the foregoing Table you may observe, that the places of Integers, or whole numbers are separated from the places of *Decimal Parts* of 1 (or unity) by a point ; so the number on the left hand of the

Chap

the p
the r
seth
fed
man
eight
285
parts

Conc

I. I

a pou
Coin
piece
man
taug
now
form
ceed
taini
are i

II
of t
to b
duce

Chap. XXIII. *Reduction of Vul. Fract. &c.* 173

the point expresseth 72285 Integers or Units, but the number on the right hand of the point expresseth only 8237 parts of 1 (or an Integer) supposed to be divided into 10000 equal parts. In like manner this number 5.8 signifies 5 Integers and eighth tenth parts of an Integer, and this number 285.82 signifies 285 Integers (or Units) and $\frac{82}{100}$ parts of an Integer.

C H A P. XXIII.

Concerning the Reduction of Vulgar Fractions to Decimal Fractions.

I. IF the greatest Integer of money, as also of weight, measure, &c. were subdivided decimally, to wit, a pound of English Money into ten equal pieces of Coin, and every one of these into ten other equal pieces, &c. and weights, measures, &c. after the same manner: The Doctrine of Arithmetick would be taught with much more ease and expedition than now it is; but it being improbable that such a reformation will ever be brought to pass, I shall proceed in directing a course to the studious for obtaining the frugal use of such Decimal Fractions as are in his power.

II. Forasmuch as in *Arithmetical Questions*, some of the given numbers do for the most part happen to be Fractions, a way must be shew'd how to reduce a *Vulgar Fraction* to a *Decimal Fraction*; yet in some

174 Reduction of Vulgar Fractions. Book I

Some cases there is no need of this *Reduction*; for Example, A *foot* in length is vulgarly subdivided into 12 inches, an inch into 4 quarters, and each quarter into two half quarters; but a *foot* may as easily, and a great deal more commodiously be divided, first into ten equal parts, and then each of those into ten other equal parts, and each of these into ten other equal parts; (or at least such division may be supposed or imagined when it cannot actually be made.) This *foot* in length so divided being applied to the sides of *superficial figures*, or of *solids*, will at first sight give the quantities of lines in feet and decimal parts of a foot (as readily as a foot vulgarly divided will shew you how many feet, inches, quarters, and half quarters are contained in any line) from whence the *superficial or solid content* may be found in feet by *multiplication* only; and how much this excels the *vulgar way*, I shall partly manifest in the fifth Rule of the 26th Chapter. The like subdivision I would have to be made of a *Yard*, *Perch*, &c.

III. A single Fraction, which is no decimal Fraction, may be reduced into a decimal of the same value, or infinitely near (for all vulgar Fractions cannot be exactly reduced to Decimals) by the Rule of Three Direct; for as the Denominator of any single fraction whatsoever, is to the Numerator thereof, so is any other Denominator to his correspondent Numerator: Example, Let it be required to reduce $\frac{5}{8}$ into a Decimal, whose Denominator is assigned to be 1000, say by the Rule of Three, If the Denominator 8 hath 5 for a Numerator, what will the Denominator 1000 require for

How to reduce a vulgar Fraction to a decimal Fraction.

CH
a M
Th
tion
four
equ
quin
Den
Thre
rator
for a
but
Num
there
almo
reduc
penin
mals
must
in the

IV.
accoust
&c. m
to kno
equal
pound
to thar
now i
10000
minator
found
05000
are of
the 22

Chap. XXIII. *to Decimal Fractions.* 175

a Numerator? Multiply and divide as the Rule of *Three direct* doth require, so will the fourth *proportional* be found to be 625, which is the Numerator sought; therefore $\frac{625}{10000}$, or .625, is a *decimal Fraction* equal in value to $\frac{5}{8}$. Another *Example*: Let it be required to reduce $\frac{7}{240}$ into a *decimal Fraction*, whose Denominator shall be 100000, say by the Rule of *Three*, If 240 the Denominator give 7 for a Numerator, what will the Denominator 100000 require for a Numerator? *Ans.* 2916 and somewhat more; but that which the said 2916 wants of being a true Numerator is less than $\frac{1}{10000}$ part of an Integer, therefore the *decimal Fraction* $\frac{2916}{100000}$ or .02916 is almost equal to $\frac{7}{240}$, which $\frac{7}{240}$ cannot be exactly reduced into a *decimal Fraction*. The like will happen in the reduction of most *vulgar Fractions* to *Decimals*, in which case, the Denominator of the *Decimal* must be assigned to be so great, that what is wanting in the Numerator may be an inconsiderable value.

IV. Upon the aforesaid Ground, the known or accustomed Parts of *Money, Weight, Measure, Time, &c.* may be reduced to *Decimals*: for if you desire to know what *decimal Fraction* of a Pound sterling is equal in value to one shilling, consider first that a pound is the Integer, and that 20 shillings are equal to that Integer, therefore 1 shilling is $\frac{1}{20}$ of a pound, now if we conceive one pound to be divided into 100000 parts, viz. if we assign 100000 for the Denominator of a *decimal Fraction*, the Numerator will be found by the last Rule to be 5000, so that $\frac{5000}{100000}$ or .05000 or .05 (for Cyphers at the end of a *decimal* are of no use, as hath been shewn in the 6th Rule of the 22 Chapter) is a *decimal Fraction* of a pound, and is exactly

176 *Reduction of Vulgar Fractions.* Book I.

exactly equal to 1 s. or $\frac{1}{20}$ part of a pound sterling.

In like manner forasmuch as 240 pence are equal to a pound of English Money, 7 pence are $\frac{7}{240}$ parts of a pound, which fraction will be reduced into this decimal .02916 l. which is very near equal to $\frac{7}{240}$, for it wants not $\frac{1}{100000}$ part of a pound. Moreover, since 960 farthings are equal to a pound English, one farthing is $\frac{1}{960}$ part of a pound, which will be reduced into this decimal .00104 l. very near; but if you please to proceed near to the truth, you will find this decimal .0014166, &c. to answer a farthing, and so by augmenting the Denominator with Cyphers, you may proceed infinitely near, when you cannot attain unto the truth it self. After the same Method may the vulgar Sexagenary Fractions used in Astronomy be reduced to Decimals; for since a degree is usually subdivided into sixty Parts called minutes or primes; a prime or minute into sixty parts called seconds; a second into sixty thirds; a third into sixty fourths, &c. and consequently a degree is equal unto 60 minutes (or Primes) or unto 3600 seconds, or 216000 thirds, or 1296000 fourths, &c. It is evident that 7 minutes (or Primes) are $\frac{7}{60}$ parts of a degree, which by the third Rule of this CHAPTER may be reduced into the Decimal .1166, &c. Also 29 thirds are $\frac{29}{216000}$ parts of a degree, which may be reduced into the Decimal .000134, &c.

Moreover, 58 : 33 : 14 : 12, that is, 58 primes, 33 seconds, 14 thirds, and 12 fourths may be reduced to a decimal in this manner, viz. reduce them all into fourths, (according to the sixth Rule of the seventh Chapter) so will you find 12647652 fourths, which are

Chap. XXIII. to Decimal Fractions. 177

are $\frac{12647632}{12960000}$ parts of a degree, which vulgar fraction may be reduced into this decimal of a degree, to wit, .975899, &c. (by the Third Rule of this Chapter.)

This to the Ingenious will be a sufficient light for the finding of the *Decimals* congruent to the *Shillings, Pence, and Farthings*, which are under a *Pound Sterling*; also the *Decimals* of the known parts of *Weight, Measure, Time, &c.* as they are express'd in the following *Table*, wherein you may observe, that most of the *Decimals* consist of 7 or 8 Figures, yet in ordinary Practice, you shall have Occasion to use only the first Five, and sometimes fewer.

The

The TABLE OF REDUCTION.

TABLET I. Of English Money, the Integer being a Pound.		Pence with Farthings.	Decimals of a Pound.
Shillings.	Decimals of a Pound.		
19	95		.0489583
18	.9		.0479166
17	.85		.046875
16	.8	11	.0458333
15	.75		.0447916
14	.7		.04375
13	.65		.0427082
12	.6	10	.0416666
11	.55		.040625
10	.5		.0395833
9	.45		.0385416
8	.4	9	.0375
7	.35		.0364583
6	.3		.0354166
5	.25		.034375
4	.2	8	.0333333
3	.15		.0322916
2	.1		.03125
1	.05	7	.0302083
			.0291666
			.028125
			.0270833
			.0260416
		6	.025

	.0239583	14.7	
	.0229166	13.65	
	.021875	12.6	
5	.0208333	11.55	
	.0197916	10.5	
	.01875	9.45	
	.0177708	8.4	
4	.0166666	7.35	
	.015625	6.3	
	.0145833	5.25	
	.0135416	4.2	
3	.0125	3.15	
	.0114583	2.1	
	.0104166	1.05	
	.009375		Decimals
2	.0083333		Grains. of an Ounce.
	.0072916	23	.0479166
	.00625	22	.0458333
1 Pen. & 1 Far.	.0052083	21	.04375
Penny	.0041666	20	.0416666
3. Farth.	.003125	19	.0395833
2. Farth.	.0020833	18	.0375
1. Farth.	.0010416	17	.0354166
		16	.0333333
		15	.03125
		14	.0291666
		13	.0270833
		12	.025
		11	.0229166
		10	.0208333
		9	.01875
		8	.0166666
		7	.0145833
		6	.0125

TABLET II.

Of Troy weight, the Integer being an Ounce.

Penny Decimals of weight: an Ounce.

19.95

18.9

17.85

16.8

15.75

5	0104166	11	0982142
4	0083333	10	0892857
3	00625	9	0803571
2	0041666	8	0714285
1	0020833	7	0625
TABLET III.			
<i>Of Averdupois great weight, the Integer being an hundred weight, to wit, 112 pounds.</i>			
<i>quarters of</i>		<i>decimals of</i>	
<i>1 hundred.</i>		<i>1 hundred.</i>	
3	75	<i>decimals of</i>	
2	5	<i>Ounces. 1 hundred.</i>	
1	25	15	0083705
<i>decimals of</i>		14	0078125
<i>Pounds. 1 hundred.</i>		13	0072544
27	2410714	12	0066964
26	2321428	11	0061383
25	2232142	10	0055803
24	2142857	9	0050223
23	2053571	8	0044642
22	1964285	7	0039062
21	1875	6	0033482
20	1785714	5	0027901
19	1696428	4	0022321
18	1607142	3	0016741
17	1517857	2	0011160
16	1428571	1	0005580
15	1339285	<i>quarters of</i>	
14	125	<i>1 Ounce. 1 hundred.</i>	
13	1160714	3	0004185
12	1071428	2	0002790
		1	0001395

TABLET

TABLET IV.		6.0234375	
Of Avoirdupois little		5.01953125	
weight, the Integer being		4.915625	
a Pound.		3.01171875	
Ounces.	Decimals of	2.0078125	
	a Pound.	1.00390625	
15	9375	Quarters of	Decimals of
14	.875	a Dram	a Pound.
13	8125		3.0029296
12	75		2.0019531
11	6875		1.0009765
10	.625	TABLET V.	
9	.5625	Of Liquid Measures, the	
8	.5	Integer being a Gallon.	
7	.4375	Pints	Decimals of
6	.375		a Gallon.
5	.3125		7.875
4	.25		6.75
3	.1875		5.625
2	.125		4.5
1	.0625		3.375
			2.25
			1.125
	Decimals of a	Quarters of	Decimals of
Drams.	Pounds	a Pint.	a Gallon.
15	.05859375		3.09375
14	.0546875		2.0625
13	.05078115		1.03125
12	.046875		
11	.04296875		
10	.0390625		
9	.03515625		
8	.03125		
7	.02734375		

N

TABLET

TABLE VI.

Of dry Measures, the Integer being a Quarter.

Bushels.	Decimals of a quarter.
7	.375
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125
Pecks.	Decimals of a quarter.
3	.09375
2	.0625
1	.03125
Quarters of a Peck.	Decimals of a quarter.
3	.0234375
2	.015625
1	.0078125
Pints.	Decimals of a quarter
3	.005859
2	.003906
1	.001953

TABLE VII.

Of Long Measures, one Yard or one Ell being the Integer.

Quarters of 1 yard or 1 Ell.	Decimals of 1 yard or 1 Ell.
3	.75
2	.5
1	.25
Nails. 1 yard.	Decimals of 1 yard.
3	.1875
2	.125
1	.0625
Quarters of nails.	Decimals of 1 yard.
3	.046875
2	.03125
1	.015625

TABLE VIII.

Of the Reduction of Inches, &c. to Decimals, the Integer being a Foot in length.

Inches.	Decimals of a Foot.
11	.9166666
10	.8333333
9	.75

Parts of a Dozen.	Decimals of a Gross.
12	0.076388
10	0.069944
9	0.0625
8	0.055555
7	0.048611
6	0.041666
5	0.034722
4	0.027777
3	0.020833
2	0.013888
1	0.006944

TABLET X.
Of Time, a day being the Integer.

TABLET IX.
Of Dozens, the Integer being a Gross.

Decimals of a Gross.	Hours a day.
23	0.9583333
22	0.9166666
21	0.875
20	0.8333333
19	0.7916666
18	0.75
17	0.7083333
16	0.6666666
15	0.625
14	0.5833333
13	0.5416666
12	0.5
11	0.4583333
10	0.4166666

9.375	38.0263888
8.3333333	37.0256944
7.2916666	36.0249999
6.25	35.0243055
5.2803333	34.0236111
4.1666666	33.0229166
3.125	32.0222222
2.0833333	31.0215277
1.0416666	30.0208333
	29.0201388
	28.0194444
	27.01875
	26.0180555
	25.0173611
	24.0166666
	23.0159722
	22.0152777
	21.0145833
	20.0138888
	19.0131944
	18.0125
	17.0118055
	16.0111111
	15.0104166
	14.0097222
	13.0090277
	12.0083333
	11.0076388
	10.0069444
	9.00625
	8.0055555
	7.0048611
	6.0041666
	5.0034722
	4.0027777
	3.0020833
	2.0013888
	1.0006944
	0.0000000

Decimals of
Minutes. a day.

59.0409722
58.0402777
57.0395833
56.0388888
55.0381944
54.0375
53.0368055
52.0361111
51.0354166
50.0347222
49.0340277
48.0333333
47.0326388
46.0319444
45.0312500
44.0305555
43.0298611
42.0291666
41.0284722
40.0277777
39.0270833

Ta
En
Co
&ic
Far
lum
&iv
Dec
.00
of
is th
V
tain
part
weig
and
cima
Penn
.025
VI
great
lumn
Quar
Quar
acco
their
Quar

4	0027777
3	0020833
2	0013888
1	0006944

V. This Table aforegoing consists of Ten several Tablets, of which the first (intituled *English Money*) contains in the first Column thereof the particular Fractions (*viz.* the *Shillings*, *Pence*, and *Farthings*) of a pound *sterling*; and in the other Column the *Decimals*, unto which they may be respectively reduced; So in the same Tablet .65 is the *Decimal* answerable to 13 s. 0208333 to 5 d. and .003125 to 3 f. Likewise, .0489583 is the *Decimal* of 1 d. together with 3 *Farthings*: Also .03125 is the *Decimal* of 7 *Pence* half-penny.

VI. The next Tablet (intituled *Troy Weight*) contains in the first Column thereof the particular Fractions (*viz.* the *Penny weights*, and *Grains*) of an Ounce *Troy* and in the other their respective *Decimals*: So .6 is the correspondent *Decimal* of 12 *Penny weight*, and .0020833 of 1 *Grain*. Likewise .025 is the *Decimal* of 12 *Grains*.

VII. The third Tablet (intituled *Averdupois great weight*) contains in the first Column thereof the Fractions (*viz.* the *Quarters*, *Pounds*, *Ounces*, and the *Quarters of an Ounce of an Hundred* according to *Averdupois weight*), and in the other their proper *Decimals*: So .5 is the *Decimal* of two *Quarters* or half a hundred, .1517857 of 17 pounds

205 *Reduction of Vulgar Fractions.* Book I.

.0033482 of 6 Ounces, and .0004185 the Decimal of 3 quarters of an Ounce.

VIII. The fourth (intituled *Averdupois little weight*) sheweth you the Fractions (*viz.* the Ounces, Drams, and quarters of a dram) of a pound *Averdupois*, together with their respective Decimals: So the Decimal of 3 Ounces is .1875, the Decimal of 9 Drams is .03515625, and the Decimal of one quarter of a Dram is .0009765.

IX. The fifth (intituled *Liquid Measures*) bath the Fractions (*viz.* the Pints and Quarters of a pint) of a Gallon, and likewise their several Decimals: So the Decimal of 5 Pints is .625, and the Decimal of two quarts or half a pint is .10625.

X. The sixth (intituled *Dry Measures*) gives you the Fractions (*viz.* the Bushels, Pecks, quarters of Pecks and Pints) of a quarter, together with their peculiar Decimals: So .375 is the Decimal of three Bushels, .03125, of one Peck, .0334375 of $\frac{1}{4}$ of a Peck, and .003906 of two Pints.

XI. The seventh (intituled *Yards and Ells*, offers you the Fractions) *viz.* the Quarters of Yards and Ells, Nails, and quarters of Nails of Yards) and their respective Decimals: So .25 is the Decimal of one quarter of a Yard or Ell, .125 of two Nails, and .046875 of three quarters of a Nail.

XII. The eighth (intituled *Reduction of Inches, &c. to Decimals of a Foot*) presents unto you the Fractions (to wit, the Inches, Quarters and Half quarters of an Inch) of a Foot, together, with

with their correspondent Decimals: So $.24766666$ is the *Decimal* of $\frac{5}{16}$ Inches, $.0625$ of $\frac{1}{4}$ of an Inch, and $.0104166$ of $\frac{1}{8}$ or half a quarter of an Inch.

XIII. The ninth Tablet (intituled *Dozens*) yields you the Fractions (*viz.* the *Dozens* and particulars) of a *Gross*, as also their respective *Decimals*: So $.25$ is the *Decimal* of 3 *Dozen*, and $.048611$ of 7 particulars.

XIV. The tenth and last Tablet (intituled *Time*) gives you the Fractions (*viz.* the *Hours* and *Minutes*) of a *Day*: So $.625$ is the *Decimal* of 15 hours, $.0375$ of 54 *Minutes*, and $.0006944$ of one *Minute*.

XV. When a single Fraction of any of the premised Tablets is propounded to be reduced to a *Decimal*, find it in the first Column of the Tablet, unto which it belongs; this done, just against that Fraction so found, you shall have the *Decimal* required: So $13 s.$ being propounded, taking the first premised Tablet, I find $13 s.$ in the first Column of the Tablet of *Money*, and just against the same thirteen shillings, I observe $.65$, before which having prefixed a point, and by that means signed it for a *Decimal* (according to the third Rule of the 22 Chapter of this Book) I conclude the same $.65$ so ordered, to be the correspondent *Decimal* of thirteen shillings the Fraction propounded. In like manner $.0229166$ is the *Decimal* of 31 grains in the Tablet of *Troy* weight, and $.0357142$ the *Decimal* of 4 *lb.* in the Tablet of *Averdupois* great weight, &c.

The use of the same Table for the Reduction, 1. Of single Fractions to Decimals.

XVI. When two or more Fractions are propounded, and it is required to find a Decimal equivalent unto the sum of them, find the Decimal of each of the Fractions given according to the last Rule; then adding together the Decimals so found, that intire sum is the Decimal sought: So $13 \text{ s. } 5 \text{ d.}$ being produced to a Decimal, is $.670833$; for the Decimal of 13 s. is $.65$, and the Decimal of 5 d. $.020833$, which being added together (by the second Rule of the 24th Chapter of this Book) amount to $.670833$, viz. the Decimal which represents $13 \text{ s. } 5 \text{ d.}$ the Fraction propounded: In like manner the Decimal of $9 \text{ penny weight, and } 13 \text{ Grains,}$ is $.4770833$, and the Decimal of $\frac{1}{2} \text{ C. } 19 \text{ lb. } 7 \text{ Ounces,}$ is $.67354$, &c.

13 s.	$.65$
5 d.	$.020833$
	$.670833$
9 p. w.	$.45$
13 gr.	$.027083$
	$.477083$
$\frac{1}{2} \text{ C.}$	$.5$
19 lb.	$.16964$
7 ounce.	$.00390$
	$.67354$

And here as you see meer Fractions reduced, so likewise may the Fractions of mixt Numbers be reduced to Decimals; for Example, These Numbers 97 lb.

lb. 7 ounces, $13 \frac{1}{4}$ Drums. *Item* of 67 Gallons $5 \frac{1}{2}$ Pints. *Item* 28 Quarters, 0 Bushels and $2 \frac{1}{3}$ Pecks, after Reduction are 97.4891, 67.7187, and 28.0781.

97.4375	67.625	28.0625
.0507	.0937	.0156
.0009		
97.4891	67.7187	28.0781

Again 22 $\frac{1}{2}$ yards, $3 \frac{1}{4}$ Nails; *Item* 36 Gross, 3 Dozen and 5 particulars, being reduced, are 22.7031, 36.2847.

22.5	36.25
.1875	.0347
.0156	
22.7031	36.2847

XVII. When a Decimal is propounded to know what Fraction it represents, search the same Decimal in the second Column of the Tablet, unto which it belongs, where if you find it expressly, the Number just against it in the first Column is the Fraction you look for: So .65 (representing the Fraction of a Pound sterling) being given, I find it in the second Column of the Tablet of Money, and over against it in the first Column I find 13s. which is the Fraction represented by .65, the Decimal propounded. In like manner 3.025 (representing 3 Ounces and .025 of an Ounce *Troy*) being propounded, the Number represented by it, is 3 Ounces, 0 p. w. 12 Grains.

XVIII. When in the second Column of the Tablet,

210 Reduction of Vulgar Fractions, &c. Book I.

Tablet, unto which you are directed, you cannot precisely find the Decimal propounded, search that which being less, comes nearest unto it, and take the Number that answers unto it in the first Column for the greatest Fraction of the Number required: Then deducting the Decimal so found out of the Decimal given, find likewise the remainder as another Decimal, and take his correspondent Number for the next Fraction of the Number required; and so proceed in that order, till you have discovered the intire Number represented by the Decimal propounded.

Example, .6739 being propounded, I demand the Fraction of a pound sterling represented by it; the Decimal in the Tablet of Money, which being less, comes nearest to .6739 is .65, whose correspondent Number in that Tablet is 13, which are the shillings of the Number required; then subtracting (by the 1 Rule of the 25th Chapter of this Book) .65 out of .6739, the remainder is .0239, and the nearest Decimal in the same Tablet to .0239, is .0208, whose correspondent Number is 5, which are the pence of the Number required. Last of all, deducting .0208 out of .0239, the remainder is .0031, which gives you in the first Column 3, being the farthings of the Number required: So that I conclude the intire Fraction represented by the Decimal .6739, is 13 s. 5 d. 3 f.

.6739 l. sterling.
Subtract 13 s. ——— .65

————— .0239
Subtract 5 d. ——— .0208
3 f. ——— .0031

In

Chap. XXIV. Addit. of Decim. Fractions. 211

In like manner 7.359 C. being reduced by the Table of *Averdupois* great weight, is 7½ C. 12 lb. 4 ounce: And 94. 58 lb. reduced by the Table of *Averdupois* little weight is 94 lb. 3 Ounces and 6 drams.

$$\begin{array}{r} 7.359 \text{ C.} \\ \text{Subtract 1 quarter } .25 \\ \hline \end{array}$$

$$\begin{array}{r} 109 \\ \text{Subtract 12 lb } .109 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \text{ oz. } .002 \\ \hline \end{array}$$

$$\begin{array}{r} 94.58 \text{ lb.} \\ \text{Subtract 9 oz. } .56 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \text{ Drams } .02 \\ \hline \end{array}$$

CHAP. XXIV.

Addition of Decimal Fractions.

I. TO such as well understand the Notation of Decimal Fractions, all the varieties of their Numeration, to wit, Addition, Subtraction, &c. will be as easie as the operations by whole numbers; therefore he that would be a good Proficient in Decimal Arithmetick, must thoroughly understand the 22 and 23 Chapters aforegoing.

II. When divers Decimal Fractions are given to be added together, they must first of all be orderly placed one under another according to the Doctrine of their Notation. So if these Decimal Fractions, to wit, .125, .39 and .7 were given to be added, they must be written down thus;

$$\begin{array}{r} .125 \\ .39 \\ .7 \\ \hline \end{array}$$

or

or if you will have the same Number of places to be in all the *Decimals* given, without altering their values, they may be writteng thus,

$$\begin{array}{r} .125 \\ .390 \\ .700 \\ \text{Not thus,} \\ .125 \\ .39 \\ .7 \end{array}$$

For the Figures or Cyphers, which are of like degrees or places must be subscribed directly one under another, *viz.* *Tenth parts* of or *Primes* must be written down directly underneath *Tenths*; also *hundredths Parts* or *Seconds* must be placed under *hundredths parts*. As you see in the first Example, where .3 or three tenthparts in the second Decimal stands directly under .1 or one tenth Part in the first Decimal; likewise .7 or seven tenths in the third Decimal stands directly under the tenths in the former, and so of the rest.

In like manner, when mixt numbers, which consist of Integers, and decimal Parts are given to be added, due respect must be had of their subscription one under another: So if these mixt Numbers, to wit, 32.056, 7.07, and 1.9 were given to be added, they must be written down thus,

$$\begin{array}{r} 32.056 \\ 7.07 \\ 1.9 \end{array}$$

III. Having placed the Decimals and drawn a Line underneath in manner aforesaid, add them together,

gether, beginning with the outermost rank towards the right hand (as hath been taught in Addition of whole Numbers of one Denomination in the third Chapter :) so if the Decimals in the first Example of the second *Rule* of this *Chapter* were given to be added, I first subscribe 5, which is all that stands in the first rank towards the Right hand ; then proceeding to the second rank, I say 9 and 2 makes 11, wherefore I write down 1, which is the excess of 11 above 10, and for the 10 I carry 1 in mind to the next rank, saying 1 in mind added to 7 makes 8 which added to 3 and 1 make 12, wherefore I write 2, which is the excess of 12 above 10, under the line, reserving 1 in mind for the 10 ; then I prefix a Point before 2, which stands in the first place of Decimals ; and on the left hand of the Point, to wit in the place of Units or first place of Integers, I write down 1 (being the 1 in mind) which done, I find that the sum of the Decimals given is 1.215, that is, one Integer (whether it be a Perch, Yard, Foot, &c) and $\frac{215}{1000}$ parts of an Integer, as you see in the Example. In like manner these mixt Numbers 32.056 ; 7.07 and 1.9 being given to be added, their sum will be found to be 41.026, that is, 41 Integers, and $\frac{26}{1000}$ parts of an Integer, as you see in the Margin ; more *Examples* for the Learners Exercise are these :

.125

.39

.7

1.215

32.056

7.07

1.9

41.026

.65

24.7

503.75

.025

0.35

0.32

.03

5.27

0.12

705

30.37

504.19

CHAP.

C H A P. XXV.

Subtraction of Decimal Fractions.

HAVING first written down the greater of the two Numbers given (whether it be a whole Number, mixt Number, or Decimal) and the lesser underneath the greater, according to the directions in the second Rule of the 24th Chapter, proceed as you are taught in Subtraction of whole Numbers (by the Rules of the 4th Chapter:) So if this Decimal Fraction $.784$ were given to be subtracted from this Decimal $.837$, the remainder will be $.053$, that is $\frac{53}{1000}$ parts of an Integer; in like manner if this mixt Number 295.094 were given to be subtracted from 78.919 , the remainder will be 216.175 , that is $216\frac{175}{1000}$. In each of which Examples you may observe that 10 is borrowed as often as need requires, according to the Rules of Subtraction of whole Numbers of one Denomination: Note also, when the Decimals in both the Numbers given consist not of the same Number of places, that Decimal which is defective in places towards the right hand, must have the void places filled up with Cyphers, or at least Cyphers must be supposed to be annexed: So if this Decimal $.04338$ be given to be subtracted from this $.65$, the remainder will be found to be $.60662$, and the Work will stand as in the Margin, where you see the three void places are supplied with Cyphers, and then the Operation is as in whole Numbers, by borrowing 10 as often as the lower Figure

II.
Multi
given a
always

Figure cannot be subtracted from the upper. More Examples of Subtraction of Decimals are these following:

$$\begin{array}{r} 2494338 \\ .65 \\ \hline 23.29338 \end{array} \quad \begin{array}{r} 37 \\ 0.104 \\ \hline 36.896 \end{array} \quad \begin{array}{r} .394 \\ .35 \\ \hline .044 \end{array}$$

CHAP. XXVI.

Multiplication of Decimal Fractions.

WHen two Numbers are given to be multiplied, and are both mixt Numbers, or both Decimal Fractions, or one of them a whole Number, and the other a Decimal or mixt Number (which are all the cases that can happen) there is no necessity of writing them down precisely one under the other as in Addition and Subtraction, for the Product or Number sought in Multiplication depends not upon any regular placing of the two Numbers given: So if this mixt Number 56.3 were given to be added to this mixt Number

1. 30526, they ought to be written one under the other, as you see (according to the second Rule of the 24th Chapter;) but if they are to be multiplied one by the other, they may be written thus,

$$\begin{array}{r} 1.30526 \\ 56.3 \\ \hline \end{array}$$

II. In any of the Cases which may happen in Multiplication of Decimals, multiply the Numbers given as if they were whole Numbers, then cut off always from the Product by a Point, Comma, or

Line

Line, so many places towards the Right hand, as there are places of Decimal parts in both the Numbers given to be multiplied; that done, the Figure or Figures (if any happen to be) on the Left hand of the said Point or Line of separation doth declare the Integer or Integers in the Product, and those on the Right-hand of the Point are decimal Parts of an Integer: So if this mixt Number 56.3 (that is, 56 Integers and $\frac{3}{10}$ of an Integer) be given to be multiplied by this next Number 1.30526 , the Product will be found 73.486138 , that is, 73 Integers and $\frac{486138}{1000000}$ Parts of an Integer; for having chosen that to be the Multiplier, which will cause least Work, and subscribed it under the Multiplicand (to wit, 56.3 underneath 1.30526) I proceed according to the Rules of Multiplication of whole Numbers, viz. having drawn a Line underneath the Numbers given, I multiply all the Multiplicand, to wit, 1.30526 , as if it were a whole Number, by 3 the first multiplying Figure, and subscribe

1.30526	the Product thereof, which is 391578
563	underneath the Line, and proceeding
391578	in like manner with the other multi-
783156	plying Figures 6 and 5, at last I find
652620	the total of the particular Products
73486138	to be 73486138; and because there

are six places of decimal Parts in both the Numbers given (to wit, 5 places of parts in the multiplicand, and a place in the multiplier) I cut off 6 places to the Right-hand from the total before produced, so will it stand thus 73.486138 : Wherefore I conclude that the true Product is 73.486138 , that is, 73 Integers and almost $\frac{1}{2}$ of an Integer.

In like manner, if this mixt number 246.25 (that is $246 \frac{1}{4}$) were given to be multiplied by 35 Integers the true product will be found 8618.75 , that is $8618 \frac{3}{4}$ Integer, and $\frac{3}{4}$ parts of an Integer, as you see by the operation in the Margin, where you may observe that two places are cut off from the total number produced of the multiplication, towards the right hand, because there are two places of decimals in the multiplicand (the multiplier consisting of Integers only;) but if there had been decimal parts also in the multiplier, so many more places should have been cut off, as was shewed in the first Example.

$$\begin{array}{r} 246.25 \\ \times 35 \\ \hline 123125 \\ 77250 \\ \hline 8618.75 \end{array}$$

Again, If these two decimals $.87$ and $.9$ (to wit $\frac{87}{100}$ and $\frac{9}{10}$) were given to be multiplied one by the other, the true product will be found to be $.783$, that is $\frac{783}{1000}$ parts of an Integer, as you see in the Example, where you may observe that the product is a fraction only; for after 3 places (being the number of places of decimals in both the numbers given to be multiplied) are cut off to the right hand, there remains no Integer on the left hand.

$$\begin{array}{r} .87 \\ \times .9 \\ \hline .783 \end{array}$$

III. When the multiplication is finisht, if there arise not so many places in all as ought to be cut off by the second Rule of this Chapter (which may often happen when the product is a fraction;) in such case, as many places as are wanting, so many cyphers must be prefixed in the product on the left hand thereof, and then a point must be prefixe

to sign the product so increased for a decimal: So these decimals .0375 and .05 being given to be multiplied one by the other, I multiply 375 by 5, and there ariseth 1875: Now according to the second Rule of this Chapter, I should cut off 6

places to the right hand, and here are but 4 in all; wherefore I prefix two Cyphers, to wit, as many as there are places wanting, and then prefixing a point, the true product will be .001875 or 1875. In like manner if this

mixt number 5.525 be multiplied by this decimal .0026, the true product will be found to be .0143650 (or 143650) as you may see by the operation in the Margin, where one Cypher is prefixed to the Numbers arising from the total Multiplication to discover the true product.

IV. Decimal parts of an Integer may be reduced to the known or accustomed

parts of such Integer by Multiplication only, for if the decimal Fraction given be multiplied by that number which declareth how

many known parts are equal to the Integer, the Product gives the number of known parts required: So this decimal Fraction of a Pound sterling, to wit, .86871. being propounded, I multiply it first by 20 (the number of Shillings contained in a Pound) and the product gives 17 Shillings and

.3740 parts of a Shilling; which decimal .3740 being multiplied by 12 (the number of Pence in a Shilling) produceth 4 pence, and .488 parts of a Penny; Lastly, multiplying .488 by 4 (the number of Farthings, which make a Penny) the product gives one Farthing, and .9520 parts of a Farthing, which are very near in value to another Farthing, so it appears that .8687 parts of a Pound sterling are 17 s. 4 d. 2 f. very near. After the same manner, a decimal fraction of any Integer whatsoever may be reduced into the known or accustomed parts of such Integer.

$$\begin{array}{r}
 .8687 \text{ l.} \\
 \times 12 \\
 \hline
 17 \text{ s. } 10 \text{ d.} \\
 \times 4 \\
 \hline
 7480 \\
 \times 12 \\
 \hline
 89760 \\
 \times 4 \\
 \hline
 359040 \\
 \times 12 \\
 \hline
 4308480 \\
 \times 4 \\
 \hline
 17233920
 \end{array}$$

A briefer way to value any decimal part of a Pound of *English Money*, without loss of a Farthing, may be this, viz. the figure (if any happen) in the first place of the decimal being doubled, gives Shillings; also if there be 5, or a figure greater than 5 in the second place, one shilling more is to be added to the former; lastly, when 5 is taken from the figure in the second place, if every Unit in the remainder be accounted as Ten, and the figure in the third place as Units, these Tens and Units taken as one number and lessened by 1, give the number of Farthings, which with the Shillings before found declare the value of the decimal propounded; likewise if the figure in the second place

A brief way to find the value of any decimal Fraction of a Pound of English Money.

(when any happens) be less than 5; every unit in such figure is to be accounted ten as before, so in the decimal before mentioned, to wit, 8687 $\frac{1}{2}$ the figure 8 in the first place being doubled gives 16 shillings, also because 5 is contained in 6 which stands in the second place, one shilling more is to be added to the aforesaid 16 shillings, which will now be made 17s. that done, the remainder of the said 6 after 5 is subtracted, to wit, 1 being esteem'd as 10, and added to 8 (which stands in the third place, and to be esteem'd as units) gives 18, from which abating 1, the remainder is 17 farthings or 4 pence and a farthing; so that the value of the said decimal 8687 $\frac{1}{2}$ l. is found as before to be 17 shillings 4 pence 1 farthing. After the same manner this decimal of a pound of *English Money*, to wit, .319 will be reduced to 6 shillings and 18 farthings, or 6 shillings 4 pence 2 farthings, which wants less than a farthing of the exact value of the decimal .319 l.

V. Having explain'd all the cases in *Multiplication of Decimals*, I shall here give the learner a taste of their excellent use by some familiar questions whereby it will be evident, that what is oftentimes performed by many tedious *Multiplications and Divisions* in the vulgar way, is effected for the most part by one or two *Multiplications in Decimals*.

The first *Example* may be this: Suppose there is a certain piece of *Wainscot*, in form of a *Rectangled Parallelogram*, commonly called a *long square*, whose breadth is 3 yards, $\frac{3}{4}$ of a yard, 1 nail and $\frac{1}{4}$ of a nail, and the length of 6 yards, and $\frac{1}{2}$ of a yard, the question is to know

know how many *square yards* are contained in that piece of *Wainscot*; here because it is desired that the *superficial content* may be given in *yards*, the parts of a *yard* as well in *breadth* as in the *length* of the *Wainscot* which are before express'd by the accustomed parts of *quarters, nails, &c.* must be reduced into decimal parts of a *yard*, which are as easy to be found by a *yard* subdivided decimally, as the common parts of *quarters* and *nails* are found by a *yard* vulgarly subdivided: but for want of a *yard* subdivided decimally, this *Reduction* may be performed by the seventh *Table* of the precedent *Table* Of *Reduction*, viz. looking into the said *Table*, right against $\frac{3}{4}$ of a *yard*, I find this

decimal .75

Also the decimal correspondent to 1 nail is 0.0625

And the decimal of $\frac{1}{4}$ of a nail } .015625

The sum of those three decimals is .828125

Wherefore the breadth of the
Waincoat in yards and decimal
parts is 3.828125

Again the decimal of half a yard
is .5, wherefore the length of the
Wainscot is 6.5

The length and breadth being multiplied one by the other produce the *superficial content*, therefore the number of square yards required is 24.8828125

Wherefore I conclude that 24 square yards and
somewhat more are contained in that piece of
O 3 Wainscot,

Wainfcot; and it is evident by the First place of the decimal, that what is above 24 yards is more than $\frac{1}{10}$ but less than $\frac{2}{10}$ of a square yard; or more strictly, it is more than $\frac{1}{100}$, but less than $\frac{2}{100}$ of a square yard: but by taking all the places in the decimal you have the exact answer to this question, because the common parts of *quarters, nails, and quarters of nails*, may be always exactly reduced into *decimals*, but that seldom happens in other things; nevertheless, albeit by decimal operations you cannot always hit the mark, yet you may come as near it as is possibly to be imagined, and that with much more ease than by vulgar computations in questions of this nature, as will appear by comparing the precedent operation with the common way of working here in your view, viz.

y. q. n. q.n.
3---3---1---1.

4
35
4
61
4

245 quarters of nails. of the seventh Chapter; so there will be found

245 quarters of Nails, as you see by the operation.

Again the 6 yards and half which express the length aforesaid, must likewise be reduced into quarters of Nails by the aforesaid Rule; so there will be found 416 quarters of nails of a yard, as you see by the operation.

I now I leave it to the Reader to judge, which of these two ways is the more expeditious, and which liketh him best.

Example 2. There is a square piece of Timber terminated at both ends with equal squares, the breadth of the piece of Timber is 1 foot 2 inches 3 quarters of an inch, and 1 half quarter of an inch; the depth or thickness is 1 foot 1 inch 4 quarters of an inch, and 1 half a quarter of an inch, and the length of the piece is 11 feet 11 inches, and 3 quarters; the quarters of Timber are contained in that piece of Timber 11 feet 11 inches, and 3 quarters.

Then Multiplying the breadth and length one by the other, to wit, 24 by 416, the product will give 101920 for the superficial content of the piece of Wainscot in square quarters of nails of a yard; now these square quarters of nails of a yard must be reduced to square yards, and the readiest way to perform that, is to find first of all how many quarters of nails of a yard are contained in one yard in length, viz. since there are 16 nails in a yard, there are consequently 4 times 16 quarters of nails, to wit, 64 quarters of nails in a yard in length; therefore 64 multiplied by 64 produceth 4096 square quarters of nails in a yard square; lastly, I say by the Rule of three, if 4096 square quarters of nails of a yard give 1 yard square, how many yards square will 101920 square quarters of nails give? So will the answer be found 24 $\frac{16}{4096}$ yards, which is the same with 24.8828125 before found by the decimal operation (for $\frac{16}{4096}$ is equal to the decimal .8828125, as will appear by reducing them to a common denominator by the four-

teenth Rule of the seventeenth Chapter.) Now I leave it to the Reader to judge, which of these two ways is the more expeditious, and so let him take which liketh him best.

Example 2. There is a squared piece of Timber terminated at both ends with equal long squares, viz. the breadth of the piece of Timber is 1 foot 5 inches 3 quarters of an Inch, and 1 half quarter of an Inch; the depth or thickness is 1 foot 3 inches 1 quarter of an Inch, and $\frac{1}{8}$ or half a quarter of an Inch; and the length of the piece is 11 feet 10 inches, and 3 quarters; the question is how many solid or cubical feet are contained in that piece of Timber? The Answer may be found by decimal Multiplication in manner following, viz. Forasmuch as it is desired that the solid content may be given in feet, the parts of a foot as well in the breadth, depth, and length, which are before express'd by the accustomed parts of inches, quarters, and half quarters, must be reduced into the decimal parts of a foot, which are as easie to be found by a foot subdivided decimally, as the other common parts by a foot vulgarly subdivided; but for want of a foot subdivided decimally, this Reduction may be performed by the eighth Tablet of the precedent Table of Reduction, viz.

The decimal correspondent to 5 In- }
ches is ————— .416

The decimal of $\frac{3}{4}$ of an Inch is ————— .062

The decimal of half a quarter of an }
Inch is ————— .01

The sum of those 3 decimals is ————— .488

Wherefore the breadth of the piece }
of Timber is ————— 1.488

In

In like manner the common parts of Inches, &c. in the depth or thickness of the piece of Timber, will be reduced by the said *Tablet*, into these decimals, viz.

The decimal correspondent to 3 Inches is ————.25

The decimal of $\frac{1}{4}$ of an Inch is ————.03

The decimal of half a quarter of an Inch is ————.01

The sum of these 3 decimals is ————.28

Wherefore the depth or thickness is ————.28

Again the accustomed parts of Inches, &c. in the length of the piece of Timber will be reduced to these decimals, viz.

The decimal of 10 Inches is ————.833

The decimal of $\frac{1}{4}$ of an Inch is ————.062

The sum of those 2 decimals is ————.895

Wherefore the length of the piece is ————.895

Now if the breadth, depth and length be multiplied continually, the last product is the solid content required, viz. 1.488 multiplied by 1.28 produceth 1.90464, which multiplied by 11.895 produceth 22.65, &c. Wherefore I conclude that 22 solid feet, half a foot, and somewhat more than half a quarter of a foot are contained in that piece of Timber.

Example 3. How many *Equinoctial Degrees* are correspondent unto 136 days, 21 hours, and 40 minutes? The Answer is found by multiplying the time given by 360, for as 1 day is to 360 degrees, so 136 days, 21 hours, and 40 minutes, to the *Equinoctial degrees* required. But first the 21 hours and 40 minutes must be reduced to decimal parts of a day by the tenth *Tablet*, thus.

The

The decimal of 21 hours is $\frac{21}{24} = .875$

The decimal of 40 minutes is $\frac{40}{24 \times 60} = .02777$

The sum of these 2 decimals is $.875 + .02777 = .90277$

Therefore the time propounded is 136.90277

Which being multiplied by 360 }
 produceth 49284.99 , &

Wherefore I conclude, that 49284.99 or very near 49285 Equinoctial Degrees are correspondent unto 136 days, 21 hours, and 40 minutes, which was required by the question.

CHAP. XXVII.

Division by Decimal Fractions.

I. IN any of the Cases which may happen in Division, if the Dividend be greater than the Divisor, the quotient will be either a whole number or else a mixt number: But when the Dividend is less than the Divisor, the quotient must necessarily be a fraction; for a lesser number is contained in a greater once, at the least, but a greater is not contained once in a lesser.

II. Sometimes the Dividend, whether it be a whole number, mixt number, or decimal fraction, is to be prepar'd by annexing a competent number of Cyphers thereunto, to make room for the Divisor: So if 32.5 were given to be divided by 17.325 the Dividend 32.5 must be increased with cyphers at pleasure after this manner 32.50000 &c. Likewise if 1 were given to be divided by 360 , the Division

vision cannot be made till the Dividend be increased with cyphers, which being annexed the Dividend will stand thus 1.000000, &c. Here note that the cyphers annexed in manner aforesaid do supply places of decimal parts, and will be useful in discovering the quality of the quotient according to the fourth *Rule* of this *Chapter*.

III. After the Dividend is prepared by annexing cyphers, when occasion requires (as in the last *Rule*,) all the places thereof must be esteemed as one whole number (to wit consisting of units or Integers;) and so is the Divisor to be esteemed whether it be a decimal fraction or mixt number; for in all cases the Division must be performed in every respect according to the Rules of Division of whole numbers in the sixth *Chapter*. So if this mixt number 326.25 were given to be divided by this mixt number 12.3, you must divide in the same manner, as when you divide 32625 integers by 123 integers. Also if this decimal .8356 were given to be divided by this decimal .05, you are to divide in the same manner, as when you divide 8356 Integers by 5 integers; and after the quotient is found the degree or place of the first figure which ariseth in the quotient must be enquired after; viz. you must know how far such first figure is distant from the place of units, to the end that the point or line which is used to separate between the place of units (or first place of integers) and the first place of decimals may be duly placed: This is the only knot in decimal Division, and may be resolved by the following *Rule*, viz.

IV. In any of the *Cases* which may happen in Division of decimals, the first figure which ariseth in the Quotient, will be always of the same place or degree with that figure or cypher of the Dividend, which at the first question standeth over, or at least belongeth unto the place of *units* in the Divisor. To illustrate this Rule I shall give examples in all the principal cases; and first let a mixt number be given to be divided by a mixt number, viz. Let it be required to divide 172.5 by 3.746 here (according to the second Rule of this Chapter) the Dividend must be encreased with cyphers at pleasure, so will it stand thus 172.500000, &c. then Division being made according to the Rules of Division of whole Numbers in Chapter 6, the Quotient arising will be 46.049, &c.

To illustrate this Rule I shall give examples in all the principal cases; and first let a mixt number be given to be divided by a mixt number, viz. Let it be required to divide 172.5 by 3.746 here (according to the second Rule of this Chapter) the Dividend must be encreased with cyphers at pleasure, so will it stand thus 172.500000, &c. then Division being made according to the Rules of Division of whole Numbers in Chapter 6, the Quotient arising will be 46.049, &c.

Now it remaineth to separate the *Integers* in this quotient from the *decimal parts*; to perform which, I subscribe the Divisor 3.746 orderly underneath.

3.746) 172.500000 (46, 049, &c.

3.746.

the first Dividual 172.50 (being that part of the Dividend whereof the first question must be asked) or at least I imagine the Divisor to be so subscribed, and so I find that the figure 3 which stands in the place of *Units* in the Divisor will be placed under

under 7, which is the place of tens (or second place of Integers) in the Dividend; wherefore by the fourth Rule before given, I conclude that the first figure arising in the quotient must likewise stand in the place of tens (or second place of integers) and consequently the next place on the right hand must be the place of *Units*; so it is evident that the separating point or line must be placed between the figure 6 and 0 in the quotient, that done, the true quotient is found to be 46.049. &c. to wit, 46 integers and $\frac{49}{1000}$ of an Integer, and somewhat more: for $46\frac{49}{1000}$ is less than the true quotient, but $46\frac{50}{1000}$ is greater than it, and therefore albeit, after the aforesaid Division of 172.500000 by 3.746 is ended, there will be a remainder, to wit, 446 which seems to be greater, yet here it is less in value than $\frac{1}{1000}$ part of an unit or Integer, and if to that remainder you annex another cypher and continue the division, you will proceed nearer the truth and not miss $\frac{1}{10000}$ part of an unit of the true quotient, and in that order you may proceed infinitely near when you cannot obtain the quotient exactly by Division of Decimals.

Example 2. Suppose this mixt number 2.34 be given to be divided by this mixt number 52.125 (where you may observe that the Dividend is less than the Divisor;) first (as before) annex cyphers at pleasure to the Dividend, to make room for the Divisor, then the division being prosecuted as in whole numbers, at length these figures will arise in

52.125) 2 3400000 (0448. &c.

...
52, 125

the

the quotient, to wit, 448: and to the end the degree or quality of the first figure 4 may be discovered, I subscribe the divisor 52.125 under the first dividial 2.34000 (for so far the first question did extend in the Division) and thereby I find that the figure 2 which stands in the place of units in the divisor will be seated under 4, which is in the second place of decimals, wherefore I conclude that the first figure arising in the quotient must also stand in the second place of decimals, and consequently the first place of *decimals* (which is next on the left hand to the second) must be supplied with a cypher; so that if a cypher be prefixed on the left hand of 4, and then a point placed before that cypher, the quotient will at length be discovered to be .0448, &c. or $\frac{448}{10000}$, and somewhat more that is to say, $\frac{448}{10000}$ is less than the true quotient but $\frac{442}{10000}$ is greater than it; and if you will proceed nearer the truth, you may continue the division, as is directed in the first Example of this Rule

Example 3. Where a whole number is divided by a decimal fraction, viz. suppose 82 Integers were given to be divided by this decimal .056; After cyphers are annexed to the dividend at pleasure, and

.056) 82.00000 (146428, &c.

the

Chap. XXVII. *Decimal Fractions.* 231

the division prosecuted as in whole numbers (to wit, 8200000 being divided by 56) these figures 146428 will arise in the quotient: now to the end the degree or seat of 1, the first figure in the quotient may be known, I subscribe the Divisor .056 under the first dividual 82 (for so far did the first question in the division extend;) and because the divisor is less than unity, I supply the place of units by a cypher or 0 prefixed on the left hand of the point of separation in the divisor; also I pre-

.056) 0082.00000 (1464.28, &c.
0.05.6

fix cyphers before (to wit on the left hand of) the integers in the dividend to represent a succession of places of integers (for the order of places in integers is from the right hand towards the left;) then I find that the cypher or 0 which represents the place of units in the divisor, doth stand under that cypher which represents the fourth place of integers in the dividend (as you see by the Example;) wherefore I conclude that the first figure arising in the quotient must also be seated in the fourth place of integers, and consequently the 4 first places in the quotient will be integers, and the rest a decimal, so that the true quotient is 1464 integers, and $\frac{28}{100}$ parts of an Integer, and somewhat more, viz, 1464.28 is less than the true quotient, but 1464.29 is greater than it.

Example 4. Suppose this decimal .0125 be given to be divided by this decimal .5; after division is finished according to the Rules of division of

whole

whole numbers (to wit after 125 is divided by 5) these figures 25 will arise in the quotient; now to discover the degree or seat of 2 the first figure in the quotient, I subscribe the divisor 5 under the first dividend 012. and having .5).0125(.025 (as in the last Example) prefixed a cypher on the left hand of the point of separation in the divisor, to denote or represent the place of units, I find that such cypher or place of units, do stand under the figure 1, which is seated in the second place of decimals in the dividend, wherefore I conclude by the Rule, that the first figure which ariseth in the quotient must also be in the second place of decimals, and therefore prefixing a cypher to supply the first place of decimals, and putting a point before that cypher the quotient is at length discover'd to be .025 or $\frac{1}{40}$.

Example 5. Suppose this decimal .8564 be given to be divided by this .008, first I annex cyphers to the dividend at pleasure, then prosecuting the division as in whole numbers, to wit, dividing .856400 by 8, the quotient arising is 107.050, now to discover the degree or place of 1, the first figure in the quotient, I subscribe the divisor .008 under the first dividend .8, then I prefix .008)000.85640(.107.05 a cypher to set forth, or supply the place of units in the divisor; also I prefix cyphers to represent places of Integers in the dividend; that done, I find that the cypher or 0 which supplieth

Chap. XXXII. *Decimal Fractions* .233

plie the place of units in the divisor, doth stand under the Cypher which is seated in the third place of Integers in the dividend; wherefore I conclude by the Rule, that the first figure arising in the Quotient must be also in the third place of Integers, and consequently the three first places in the Quotient will be Integers, and the rest a decimal; so that the true Quotient is 107.03 or 107. $\frac{3}{100}$.

Example. 8. Let it be required to divide this decimal fraction .73952 by this .32; first dividing 73952 by 32 as if they were whole numbers, the figures arising in the quotient will be 2311. Now to discover the quality or value of the said figures I subscribe the *Divisor* .32 under the first *dividual* .73, then prefixing a Cypher as well on the left (32)0.73952 (2. 311) hand of the *dividend*, as of the *divisor* so subscribed (or 0.32 imagined to be subscribed) as aforesaid, to represent the place of units in each of them, I find the cypher or 0, which supplieth the place of units in the *Divisor*, to stand under the 0 which represents the place of units in the *dividend*; wherefore I conclude by the preceeding fourth Rule, that the first figure arising in the *quotient* will stand in the place of units, and consequently the following places of the *quotient* will be a decimal fraction, so that the true *quotient* is 2.311 or 2 $\frac{311}{1000}$.

The reason of the foregoing fourth Rule will appear from the following Considerations.

P

I. If

I. If the product of the Multiplication of two numbers be divided by one of them, the quotient is the same with the other number; As, if 269.0625 , the product of 14.35 multiplied by 18.75 , be divided by 14.35 , the quotient will give 18.75 .

II. If the Divisor be multiplied by the first figure in the Quotient, the Product is the first number to be subtracted from the Dividend (being the same with the last particular Product in the Multiplication of the two Numbers that produced the Dividend;) and every particular place of that Product is of the same degree with that figure or cypher of the Dividend, which stands over such particular place when the subtraction is made; For a figure of one degree (or place) cannot be subtracted from a figure of a different degree: As in the last mentioned Example, the work whereof is here in view; the Divisor 14.35 being taken as in a whole number and multiplied by 1 , the first figure in the quotient, produceth 1435 , which must be conceived to consist of the same degrees as are in 269.0 in the Dividend, from which the said Product is to be subtracted and therefore the said Product 1435 is really but 143.5 , as you may see by the last particular Product, in the Multiplication of the mixt number 14.35 by 18.75 .

IV. Now among many Rules that might be given to discover the degree of the first figure in the quotient, and consequently the degrees of all the rest, the preceeding fourth Rule of this Chapter is sufficient, namely, The first figure which ariseth in the quotient, is always of the same place or degree with that figure or cypher in the Dividend, which at the first question stands over, or at least belongs unto the place of units in the Divisor: The reason is, because if a figure standing in the units place of the Divisor, be multiplied by (or doth multiply) a figure of the same degree with that degree in the Dividend, which at the first question belongs to the said units place of the Divisor, the first place in the product shall be of that degree also, whether it be of Integers or decimal Parts; and consequently the rest of the places in the said Product shall be of the same degrees with their correspondent degrees (or places) in the Dividend; as they ought to be, to the end that due subtraction may be made (according to Observ. 2.)

So in the Example before given, the first figure 1 in the quotient, shall be of the degree or place of Tens, because if the figure 4 standing in the units place of the Divisor 14.35, be multiplied by Ten, to wit, the degree which the figure 6 in the Dividend is of that belongs to the said 4 at the first question, it will produce four Tens, to be subtracted from the said six Tens: In like manner if a figure in the place of units be multiplied by units, the first place in the product shall be units; if by tenth parts of an unit, or Integer, the first place in the Product shall be Tenths, &c.

Having explained all necessary Rules in Division concerning

Chap. XXVII. *Decimal Fractions.* 237

concerning decimal fractions. I shall give a taste of their excellent use by the two following questions, and then conclude this Chapter.

Quest. 1. A Merchant bought of Gold Plate 356 ounces, 13 Penny weight, and 15 grains for 1160 Pounds sterling, the question is what he paid for an ounce? Answer, 3*l.* — 5*s.* — $\frac{1}{2}$ *l.* very near. The operation by decimals may be after this manner, viz.

By the second Tablet of Reduction }
the decimal of 13 Penny-weight is --- } .65

The decimal of 15 grains is --- .03125

The Sum of these 2 decimals is --- .68125

Wherefore the quantity of Plate }
in ounces and decimal parts of an ounce } 356.68125
is ---

Then by the Rule of Three, I say, if 356.68125 ounces cost 1160 pounds, what will 1 ounce? Here 'tis evident that if I divide 1160 by 356.68125, the quotient will give the value of an ounce, to wit, 3.252 pounds, or three pounds, 5 shillings and $\frac{1}{2}$ *l.* very near.

356.68125) 1160.000000 (3.252, &c.

Quest. 2. Suppose the length of the Tropical Year (or the space of time wherein the Sun running through the whole Ecliptick Circle consisting of 360 degrees, is returned to the same Equinoctial or Solstitial point from whence he departed) to consist of 365 days 5 hours, and 49 minutes, the question is to know the Sun's mean or equal motion for 1 day, to wit, what part of 360 degrees the Sun moveth in a whole day? The operation by decimals, thus,

By the tenth *Tablet of Reduction* }
 the decimal correspondent to 5 hours } 12083333
 is —————

The decimal of 49 minutes is ————— .0340277

The sum of those decimals is ————— .2423610

Wherefore the time given in }
 days and decimal parts of a day is } 365 .2423610

Then by the rule of three, if 365 .242361 days
 give 360 degrees (or a total circumference) what
 will 1 day give? Here if I divide 360 by 365
 .242361, the *quotient* will give the diurnal motion
 required; which will be found very near .98564,
 &c. or $\frac{28564}{289000}$ parts of a degree, which decimal being
 reduced into the common *Sexagenary* parts (by the

Fourth Rule of the 26 Chapter) will give 59 — 8,
 &c. and such is the Sun's diurnal motion very near,
 according to the aforesaid supposition of the length
 of the *Tropical Year*.

I shall here add the vulgar *Sexagenary* resolution
 of this question, that by comparing both ways
 together, the excellency of decimal *Arithmetick* in
 Calculations of this Nature may be the more per-
 spicuous.

The aforesaid question being stated according to
 the Rule of Three will stand thus,

days hours degrees day.

If 365 : 5 : 49 ————— 360 ————— 1

The first term in the Rule must be reduced into
 minutes (by the sixth Rule of the seventh Chapter;)
 so there will be found 525949 minutes.

C H A P. XXVII.

365 — 5 — 49

24

1465

730

8765

60

525949 *minutes.*

Likewise the third term 1 *day* must be reduced into *minutes*, which will be found to be 1440, as you see by the following operation.

1 *Day* or 24 *Hours.*
60

1440 *minutes.*

Then multiply the third term by the second, to wit, 1440 by 360, the product is 518400, which being divided by the first term 525949 (according to the note in the ninth *Rule* of the 16th *Chapter*) the quotient will give $\frac{518400}{525949}$ parts of a degree, which fraction being reduced into the accustomed *Sexagenary* parts (by the ninth *Rule* of the seven-teenth *Chapter*) will give as before 59' 8", &c. for the Sun's mean diurnal motion; now which of these two ways is the more expeditious, I leave to him who is vers'd in both to determine.

C H A P. XXVIII.

The Rule of Three Direct in Fractions.

1. **T**O repeat such things as have already been declared in reference to the definition of this Rule, as also to the due placing of the 3 given numbers, would be superfluous; and if respect be had to the Rules of *Multiplication* and *Division* in *Fractions* delivered in the 20, 21, 26 and 27 Chapters, the working of the *Rule of Three Direct in Fractions* as well *vulgar* as *decimal*, is the same with that in whole numbers, *viz.* multiply the second number by the third (or the third by the second,) and divide the product by the *first* number, so the quotient is the fourth number sought; to wit, the answer of the question.

Otherwise thus in Vulgar Fractions.

Multiply the Denominator of the first number by the Numerator of the second, also multiply that Product by the Numerator of the third number, and reserve this last Product for a new Numerator; again multiply the Numerator of the first number by the Denominator of the second, also multiply this Product by the Denominator of the third number, so shall this last product be a new Denominator; lastly, the new fraction (whose Numerator and Denominator is found as afore-said) is the fourth number sought, which, if it be a proper

proper fraction, may (if occasion require) be reduced into the known parts of the Integer (by the ninth Rule of the seventeenth Chapter;) if an improper fraction, it is to be reduced into its equivalent whole number or mixt number, by the thirteenth Rule of the seventeenth Chapter.

Example, If $\frac{3}{4}$ of a yard of Velvet be sold for $\frac{3}{4}$ of a pound sterling, what shall $\frac{5}{6}$ of a yard cost? *Answer* $14s. 9\frac{1}{4}d.$ For according to the Rule I multiply the Denominator 4 by the Numerator 2, and the Product is 8, this 8 I again multiply by the Numerator 5, and the Product $\frac{3}{4} \times \frac{5}{6} = \frac{15}{8}$ gives 40 for a new Numerator: Moreover, I multiply the Numerator 3 by the Denominator 3, and the product which is 9 I again multiply by the Denominator 6, so the last product is 54 for a new Denominator; wherefore I conclude that $\frac{40}{54}$ is the fourth number sought which if it be reduced (according to the ninth Rule of the seventeenth Chapter) gives $14s. 9\frac{1}{4}d.$ (or $9\frac{1}{4}d.$) for the Answer of the question.

II. When any of the three given numbers is a whole number, or mixt number, such number must first of all be reduced into an improper fraction (by the tenth or eleventh Rule of the seventeenth Chapter) to the end that all the three given numbers may be 3 fractions: Moreover, If after such Reduction, the first and third numbers be not fractions of Integers of the same particular denomination such of the said numbers which is of the lesser denomination, must be reduced to a fraction of the greater (by the sixteenth Rule of the seventeenth Chapter;) which preparations being performed, the rest

rest of the Work is to be prosecuted according to the first Rule of this Chapter. An Example of this second Rule here followeth. If a quantity of Ambergreese weighing $\frac{1}{2}$ lb. Troy, to be sold for 60 l. sterling, what are 19 $\frac{1}{8}$ grains worth at that rate? Answer, $\frac{61940}{352960}$ l. or 2 s. 4 $\frac{112}{192}$ d.

This question being stated according to the 7 Rule of the 8 Chapter will stand thus, —

lb.	l.	gr.
$\frac{1}{2}$	60	19 $\frac{1}{8}$

which 3 numbers will be reduced (by the tenth and eleventh Rules of the seventeenth Chapter) into these improper fractions. —

lb.	l.	gr.
$\frac{1}{2}$	60	117

But since the third number 117 grains Troy is not a fraction of an Integer of the same name with the first (which is a fraction of a pound Troy,) it must be reduced into a fraction of a pound Troy, thus, 117 gr. is $\frac{117}{8}$ of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of a pound Troy, which compound fraction will be reduced (by the 16 Rule of the 17 Chapter) into this single fraction, to wit, $\frac{117}{46080}$ lb. Troy and so the 3 numbers will at length stand thus in the Rule.

$$\frac{1}{2} \text{ lb.} \text{ — } 60 \text{ l.} \text{ — } \frac{117}{46080} \text{ lb.}$$

Then working as in the first Example of this Chapter, the Answer will be found $\frac{61940}{352960}$ l. which being reduced (according to the 3 and 4 Rules of the 17 Chapter) is found equal unto 2 s. 4 $\frac{112}{192}$ d.

Another Example. When the $\frac{2}{3}$ of $\frac{3}{4}$ of a Ship is valued at 147 l. — 11 s. — 3 d. how much is the whole Ship worth? Answer, 491 l. — 17 s. — 6 d.

Note,

Note, when in any question whatsoever a compound fraction, to wit, a fraction of a fraction, is one of the given numbers, such compound fraction must first of all be reduced to a single fraction (by the 16 Rule of the 17 Chapter; so here the compound fraction $\frac{3}{4}$ of $\frac{1}{2}$ being reduced into a single fraction gives $\frac{3}{8}$ or $\frac{3}{8}$; then say if $\frac{3}{8}$ be worth 147 l 11 s. 3 d. what is 1 or the whole Ship worth? Ship l. s. d. Ship
After due reduction $\frac{3}{8} = 147:11:3 = 3 = 1$
is made by converting the 147 l 11 s 3 d into pence, and that number of pence, as also the third number 1. into improper fractions, the 3 numbers will stand in the Rule thus,

$$\begin{array}{r} \text{Ship} \quad \text{pence} \quad \text{Ship} \\ \frac{3}{8} \quad \frac{35413}{1} \quad \frac{1}{2} \\ \hline \end{array}$$

Lastly, Proceeding as in the first Rule of this Chapter, the fourth number will be found to be $35413^{\circ}d.$ which being reduced first by the 13 Rule of the 17 Chapter, and then by the 7 Rule of the 7 Chapter, the Answer at length is 491 l. — 17 s. — 6 d.

An Example of the Rule of three direct in Decimals may be this that follows. If 19 ounces, 3 penny weight, and 5 grains of Gold, be worth 62 l. — 10 s. — 6 d. what is the value of $1\frac{1}{2}$ ounce? Answ. 4 l. — 17 s. — 10 $\frac{1}{2}$ d. very near.

By

By the 2 *Tablet* in the *Table of Reduction* in the 23 Chapter, the decimal fraction correspondent to 3 penny weight, is ———— } .15

Also, the decimal of 5 grains is ———— } .010416

The sum of those 2 decimals is ———— } .160416

Wherefore the first number in the Rule of Three is ———— } oz. 19.160416

Again, by the first *Tablet* of the a-
formentioned *Table* the decimal of 10 shillings is ———— } .5

Also the *Decimal* of 6 pence is ———— } .025

The sum of these two decimals is ———— } .525

Wherefore the second number in the Rule of Three is ———— } l. 62.525

Moreover by the said *Tablet* 2. the decimal of $\frac{1}{4}$ of an ounce or 10 penny weight is 5, wherefore the third number in the Rule of Three is ———— } oz. 1 5.

So that after the said *Reduction* is finisht the 3 given numbers will stand in the Rule thus:

oun. l. oun.
19.160416 ———— 62.525 ———— 1 5

Lastly, multiplying the second number by the third, and dividing the product by the first number, (according to the *Rules of Multiplication and Division of Decimals* deliverd in the 26 and 27 Chapters) the fourth number will be this, to wit, 4.8948, &c. that is four pound sterling, and $\frac{8948}{10000}$ parts of a pound, which decimal being reduced according to the fourth *Rule* of the 26 Chapter) gives 17s. ———— 10d. ———— 3 far.

The

The proof of the Rule of three direct in Fractions is the same as in whole numbers, respect being had to the Rules of Multiplication in Fractions.

CHAP. XXIX.

The Inverse Rule of Three in Fractions.

I. **A**fter a question belonging to this Rule is duly stated (according to the seventh rule of the eighth Chapter) and prepared if need require, according to the second Rule of the 28 Chapter; the operation will be the same as in the Rule of three Inverse in whole numbers, respect being had to the Rules of Multiplication and Division in Fractions, *viz.* multiply the first number by the second, and divide the Product by the third; the quotient is the fourth number sought, to wit, the answer of the question.

Or thus, in Vulgar Fractions.

Multiply the Denominator of the third fraction by the numerator of the second; also multiply that Product by the numerator of the first fraction and reserve the last product for a new Numerator; again multiply the numerator of the third fraction by the Denominator of the second; also multiply this Product by the denominator of the first fraction, so is the last Product a new Denominator; lastly, this new fraction is the fourth number sought, or answer of the question.

Example,

Example, if of cloth, which is, $1\frac{1}{2}$ yard in breadth $3\frac{1}{2}$ yards in length will make a Cloak, how much in length of stuff which is $\frac{1}{2}$ yards in breadth will make a Cloak of the same bigness with the former? *Answer* $9\frac{1}{2}$ yards

The 3 numbers being duly placed will stand thus — } *brea. leng. brea.*
 $1\frac{1}{2}y. \quad 3\frac{1}{2}y. \quad \frac{1}{2}y.$

Then (after the first and second numbers are reduced into improper fractions) the three Numbers will stand thus — } $\frac{7}{4} \quad \frac{7}{2} \quad \frac{1}{8}$

Lastly, 8, 7 and 7 being multiplied continually give 392 for a numerator; also 5, 2 and 4 being multiplied continually give 40 for a denominator, whereby this improper fraction $\frac{392}{40}$ ariseth which (by the thirteenth rule of the seventeenth Chapter) will be found to be $9\frac{3}{5}$, or (the fraction being reduced into its least terms) $9\frac{1}{2}$, which is the *Answer* of the question.

Ex. 2. Suppose when Wheat is at 2*l.*--0*os.*--6*d.* the Quarter, the penny white loaf ought to weigh 8 ounces and $1\frac{1}{2}$ penny weight of Troy weight; what ought it to weigh when wheat is at 36 shillings the the Quarter? *Answer*, 9 ounces and $1\frac{1}{2}$ penny weight.

The 3 given numbers being duly placed in the rule and reduced will stand thus — } *pence p. wt. pence*
 $48q. \quad 14q. \quad 43q.$

And if the operation be prosecuted according to the rule before given, the *Answer* will be found 181 $\frac{3226}{12728}$ penny weight, or 9 ounces, $1\frac{1}{2}$ penny weight.

C H A P. XXX.

The double Rule of Three in Fractions.

THE Double Rule of Three is so called, because it is composed of two single Rules, and may either be resolved at one Work by the Rule compound of 5 numbers, or else by two distinct single Rules of three; which latter way, to such as understand the Rule of three in fractions, is (as I conceive) less troublesom in the stating, and (in the method whereby I intend to prosecute it) the same in operation with the former. This I shall manifest first in whole numbers, then in fractions.

Example 1. If I pay 28 shillings for the carriage of 3 C. weight for 50 miles, how much ought I to pay for the carriage of 17 C. for 84 miles? *Answer*
 $13\text{ l.} \text{ --- } 6\text{ s.} \text{ --- } 6\text{ d. } \frac{18}{25}$

Of the 5 given numbers I make choice of three such which will make a single rule of three and say

C. shil. C
 If 3 ————— 28 ————— 17

Which Rule I find (by the third rule of the ninth Chapter) to be direct and therefore I multiply the third number 17 by the second 28, and the product which is 476 I place as a numerator over the divisor as denominator. Then with this fraction (whether it happen to be a proper or improper fraction) and the remaining two numbers in the question, which have not yet been used, I form a second rule of Three, and say,

miles

miles shill. miles.
 If $\frac{5}{2}$ ——— $\frac{476}{3}$ ——— $\frac{84}{1}$

Which being a *Rule of Three Direct*, I work as a *Rule of Three in Fractions*, according to the first rule of the 28 Chapter, and so find the fourth number to be $\frac{39984}{170}$ s. or 13 l. — 6 s. — $\frac{618}{370}$ d.

Or the first single Rule being varied, the operation will be thus,

1. By a *Rule Inverse*, miles C. miles C.
 50 — 3 — 84 — $(\frac{110}{170})$

2. By a *Rule Direct*, C. sh. C. sh.
 $\frac{110}{84}$: $\frac{3}{1}$: $\frac{17}{1}$: $(\frac{39984}{370})$

Otherwise thus,

1. By a *Rule Inverse*, C. m. C. m.
 3 — 50 — 17 — $(\frac{110}{170})$

2. By a *Rule Direct*, m. sh. m. sh.
 $\frac{110}{17}$: $\frac{3}{1}$: $\frac{84}{1}$: $(\frac{39984}{370})$

Thus you see the two single rules to be varied three manner of ways in resolving the question propounded, and each way produceth the same Answer; the like diversity may be found in all questions resolvable by the *double Rule of Three*, or rule compound of 5 numbers.

Example 2. If 4c $\frac{3}{4}$ l. in $\frac{1}{2}$ of a year, gain 2 $\frac{1}{2}$ l. what will 100 l. gain after that rate in 1 $\frac{1}{2}$ of a year?

Ans^r. $\frac{1500}{374}$ l. or 5 l. — 7 s. — 9 $\frac{1}{2}$ d.

By

By 2 Single Rules of Three, thus,

$$1. \text{ By a Rule Direct, } \frac{100}{1} : \frac{1}{2} :: \frac{100}{1} : (\frac{100}{400}).$$

$$2. \text{ By a Rule Direct, } \frac{1}{2} : \frac{100}{400} :: \frac{1}{12} : (\frac{100}{96}).$$

Or by these two single Rules.

$$1. \text{ By a Rule Direct, } \frac{1}{2} : \frac{1}{12} :: \frac{1}{12} : (\frac{100}{48}).$$

$$2. \text{ By a Rule Direct, } \frac{100}{48} : \frac{100}{96} :: \frac{100}{96} : (\frac{100}{192}).$$

Otherwise thus,

$$1. \text{ By a Rule Inverse, } \frac{1}{2} : \frac{1}{12} :: \frac{100}{1} : (\frac{400}{12}).$$

$$2. \text{ By a Rule Direct, } \frac{400}{12} : \frac{1}{2} :: \frac{1}{12} : (\frac{100}{96}).$$

Thus by 2 single Rules of Three, varied three several ways, you see the Answer of the question to be $\frac{100}{96}$ l. to wit, 5 l. — 7 s. — 9 d.

Q

CHAPTER

C H A P. XXXI.

The Rule of False in Fractions.

I. **W**Hen a *question* propounded cannot readily be applied to the *Rule of Three*, or any of the vulgar *Rules* in *Arithmetick*; the best refuge for such as are not acquainted with *Algebra* is the *Rule of two false Positions*, which, for that it hath already been handled in *whole Numbers*, I shall the more briefly touch upon in *Fractions*.

II. When a number is sought by a question, you are to feign or suppose some number taken by guess to the number sought, and to make trial whether that feigned number will answer the conditions in the question or not, by comparing the number resulting at the end of the Work, with the given number resulting from the true number sought; and if you find both these results to be the same, then is the *number* which you first took by guess, the true *number* or *answer* of the question; but if the *number* resulting from the suppositious *number* be either greater or less than the given result, with which it ought to be compared (to see whether you have *hit* the *mark* or not) such excess or defect must be noted for the *Error* of the first Position, to wit, an *Excess* must be signified by this note $\dagger$; and a Defect by this—

III. In like manner a second number must be feigned, and after trial is made *therewith*, to see whether it will perform the conditions prescribed in the *question*, by comparing the results as afore-said,

said, the error of this second position, if too much, is to be noted by $+$, if too little by $-$, as before.

IV. After the errors of both positions are discovered, the two numbers before supposed or feigned to be the number sought, must be multiplied by the altern errors, that is, the first Position by the second Error, and the second Position, by the first Error; then if the notes of the Errors be unlike, to wit, one of them $+$, and the other $-$, the sum of the said Products is to be taken for a dividend, and the sum of Errors for a Divisor: but if the notes of the Errors be both alike, to wit, both of them $+$, or both $-$, the difference of the said Products is to be taken for a Dividend, and the difference of the Errors for a Divisor; lastly, the Quotient arising from the Division (made by the said Dividend and Divisor) gives the true number sought, or answer of the question, if it be solvable by the *Rule of False*. These Rules, are the same in substance with those delivered in the 15 Chapter, and may be farther illustrated by the following Questions.

Quest. 1. A Gentleman hired a Servant for a Year for 6 pounds sterling, and a Livery Cloak valued at a certain rate, but it happened that $\frac{1}{2}$ of the Year being expired they fell at variance, and the Gentleman put away his Servant, giving him the Cloak together with 50 Shillings in Money, which was the Servant's full due for the time of his Service, the Question is to find what the Cloak was valued at? *Ans.* 2*l.* — 8*s.* — 0*d.*

I. I suppose the Cloak to be valued at 3 pounds, and then seek how much thereof was due to the

Servant, saying, if one
 y. — l. — y. Year give 3 l. how much
 1. — 3 — $\frac{7}{12}$ — ($\frac{7}{12}$ l. $\frac{7}{12}$ of a Year? *Ans.* $\frac{7}{12}$ l.

2. I likewise find what parts of the 6 Pounds
 was due to the Servant
 y. — l. — y. at the end of $\frac{7}{12}$ of the
 1 — 6 — $\frac{7}{12}$ — ($\frac{7}{12}$ l. Year, saying, if 1 Year
 give 6 Pounds, how much
 $\frac{7}{12}$ of the Year? *Answer,* $\frac{7}{12}$ l.

3. For as much as the Cloak together with the
 Money which the Servant received, ought to be
 equal to the part of the Cloak, together with the
 part of the 6 Pounds wages due to him at the end
 of $\frac{7}{12}$ of the Year, therefore 3 l. (the supposed value
 of the Cloak) together with 2 $\frac{1}{2}$ l. (the Money
 which the Servant received) should be equal to
 $\frac{7}{12}$ of a Pound (the value of part of the Cloak due
 to the Servant at the end of $\frac{7}{12}$ of the Year) toge-
 ther with $\frac{7}{12}$ l. (the wages due for the same time) that
 is to say, $\frac{11}{12}$ l. (the sum of 3 l. and 2 $\frac{1}{2}$ l.) should be
 equal to $\frac{7}{12}$ l. (the sum of $\frac{7}{12}$ l.) and $\frac{7}{12}$ l. but it is grea-
 ter by $\frac{1}{2}$, wherefore the first Position for the value
 of the Cloak being 3 Pounds, the Error is found to
 be $\frac{1}{2}$ too much.

4. I make a second Supposition guessing the value
 of the Cloak to be 2 Pounds, and Proceeding in
 every respect as with the first Supposition I find the
 Error to be $\frac{1}{2}$ too little, so that the two Positions with
 their Errors will be as you see:

Pos. — Er. —
 3 — $\frac{1}{2}$ too much
 2 — $\frac{1}{2}$ too little

Now

Now in regard the *errors* are *fractions*, I may take in their stead whole numbers in the same proportion, to wit, multiplying the *Numerator* of the first fraction (or first *error*) by the *Denominator* of the second, I take the Product which is 6 instead of the first *error* $\frac{1}{2}$, likewise multiplying the *Numerator* of the second fraction by the *Denominator* of the first, I take the Product which is 4 instead of the second *error* $\frac{1}{3}$, Or instead of the said 6 and 4 I may take 3 and 2 which are in the same proportion with 6 and 4, or with $\frac{1}{2}$ and $\frac{1}{3}$. Then multiplying the *positions* and new *errors* crosswise, and adding the Products together (because the signs are unlike) the sum is 12 for a *Dividend*, and the sum of *errors* 3 and 2 is 5 for a *Divisor*, so the Quotient will be found to be $2\frac{1}{5}$! so much therefore was the value of the Cloak, as will easily appear if trial be made with $2\frac{1}{5}$ l. in the same manner as with the first feigned number.

Pos.	Er.
3	$\frac{1}{2}$
2	$\frac{1}{3}$
6	
4	
12	
5	
2	$\frac{1}{5}$

5)12($2\frac{1}{5}$ poun.

Quest. 2. *Vitruvius* (in lib. 9. cap. 3.) reporteth that King *Hiero* having given commandment for the making of a *Crown* of pure *Gold*, was informed that the Workman had detained part of the *Gold*, and mixt the rest with as much *Silver* as he had stole of *Gold*; the King being much displeased at the deceit, recommended the examination of the Business to the famous *Archimedes* of *Syracuse*, who without defacing the *Crown* discovered the Cheat in this manner; viz. Experience telling him that a quantity of *Gold* would possess less room or space than the same quantity of *Sil-*

ver, and consequently that a mixt mass of Gold and Silver of the same quantity would take up some mean space between the two former, he made a mass of pure Gold of the same weight with the Crown, likewise another mass of Silver of the same weight, then having put the Crown, as also the other two Masses severally into a Vessel filled up to the brim with Water, he diligently reserved the Water flowing over into another Vessel, and from those 3 several quantities of Water so expell'd, he found out the quantity of Gold and of Silver in the Crown. But forasmuch as *Vitruvius* delivers not the practical operation, I shall here shew the same after the manner of *Cardanus*, *Gemma Frisius*, and other *Arithmeticians*.

Let us therefore suppose the weight of the Crown, as also of the two several Masses to have been 5l. Suppose also that by putting of the Mass of Gold into the Vessel, 3l of Water was expell'd; by putting in of the Crown, $3\frac{1}{2}$ l. and by putting in of the Mass of Silver, $4\frac{1}{2}$ l. The question therefore is to know how much Gold and how much Silver the Crown was composed of. This may be resolved after this manner. Suppose 3l. of Gold to

be in the Crown, then there remained 2l. of Silver, now say
 $5 - 3 = 2$ — $(1\frac{1}{2})$ by the Rule of 3, if 5l. of Gold expel 3l. of Water, how much 3l. of Gold? Answer, $1\frac{1}{2}$ l. Also if 5l. of Silver? expel $4\frac{1}{2}$ l. of Water, how much 2l. of Silver? Answer, $1\frac{1}{2}$ l. of Water; add therefore the Water of the Silver and of the Gold together to wit, $1\frac{1}{2}$ and $1\frac{1}{2}$, so there will arise $3\frac{1}{2}$ l. of Water: This ought to have been $3\frac{1}{2}$ (for so much overflowed

flowed by putting in of the *Crown*;) but it is too much by $\frac{7}{10}$, wherefore $\frac{7}{10}$ is to be noted with $\dagger$ for the error of the first Position 3l. Again, feign another quantity of *Gold* to have been in the *Crown*, to wit, 2l. therefore there remained 3 l. of *Silver*; then say if 5 l. of *Gold*

expel 3 l. of *Water*, how much 2l. of *Gold*? *Answ.* 5 — 3 — 2 — (1 $\frac{1}{2}$ 1 $\frac{1}{2}$ of *Water*; Also if

5 l. of *Silver* expel 4 $\frac{1}{2}$ l. of *Water*, how much 3l. of *Silver*? *Answ.* 2 $\frac{7}{10}$, then add 1 $\frac{1}{2}$ l. unto 2 $\frac{7}{10}$, the sum will be 3 $\frac{7}{10}$ of *Water*: This ought to have been 3 $\frac{1}{4}$ l. but it is too much by $\frac{13}{10}$. wherefore $\frac{13}{10}$ is to be noted with $\dagger$ for the

error of the second Position 2 l. Here because the errors are fractions

Pos.	Er.
3 $\dagger$	$\frac{7}{10}$ 7
2 $\dagger$	$\frac{13}{10}$ 13

having a common Denominator, I take their Numerators 7 and 13 in-

39
14

stead of the errors, then multiplying cross-wise,

6) 25 (4 $\frac{1}{6}$ lb. of *Gold*.

to wit, 3 by 13 the Product is 39, also 2 by 7 the Product is 14, which subtracted from the former Product 39 (because the errors are like) leaves 25 for a *Dividend*; also the difference between the errors 7 and 13 is 6 for a *Divisor*; Lastly, dividing 25 by 6, the Quotient is 4 $\frac{1}{6}$; so much *Gold* therefore was in the *Crown*, and consequently (because the weight of the *Crown* was 5 l.) there was $\frac{1}{2}$ l. of *Silver* which may be proved thus: Say, if 5 l. of *Gold*, expel 3l. of *Water*, how much 4 $\frac{1}{2}$ l. of *Gold*? *Answ.* 2 $\frac{1}{2}$ l. of *Water*: Again, if 5 l. of *Silver* ex-

ple 4 $\frac{1}{2}$ of Water, how much $\frac{1}{2}$ of Silver? *Answ.* $\frac{1}{2}$ of Water, which being added to $2\frac{1}{2}$ l. the sum is $3\frac{1}{2}$ l. of Water, to wit, as much as flowed over when the *Crown* was put into the Vessel.

Here note, that in making a trial of this nature, there is no necessity that the *Mass* of Gold or of Silver be of the same weight with the *Crown*, or whatsoever thing is to be examined, but of what notable part of weight you please.

Note also; that for the more easie discovering of the Dividend and Divisor by the notes of $\dagger$ and --- according to the fourth Rule of this Chapter, the following Verse may be a help, to wit,

Addito dissimiles, subtrahitoque pares.

Or thus,

*Notes being unlike, Addition make;
If like, lesser from greater take.*

The Reader may see more questions to exercise the *Rule of False* in the tenth Chapter of the *Appendix*, and the demonstration thereof in the ninth Chapter of the same.

CHAP. XXXII.

The Extraction of the Square
(or Quadrate) Root.

I. **T**HE Extraction of the Square-root is that, by which having a number given, we find out another number, which being multiplied by it self, produceth the number given.



II. In the Extraction of the Square-root, the number propounded is always conceived to be a square number, that is, a certain number of little squares comprehended within one intire great square, and the root or number *required* is the side of that great square, as will readily appear by this Diagram, where you see 25 little squares contained within one great square; now if the said content 25 be given, and the side or root of the square containing the said 25 little squares is required, the invention of such side or root is called the Extraction of the square-root; which root must be

be such, that if it be squared, that is, multiplied by it self, the Product must be equal to the square constant first given: So 5 is the square root of 25, for 5 times 5 is 25. Likewise this square number 49 being propounded, his root is 7.

III. Square numbers are either single or compound.

IV. A single square number is that, which being produced by the multiplication of one single figure by it self, is always less than 100: So 25 is a single square number produced by 5; likewise 4 is a square number produced by 2.

V. All the single square numbers together with their respective roots are expressed in the Table following.

Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

Here in the uppermost rank of the Table are placed the single square numbers of every particular figure, and in the other their respective roots; and therefore if it were demanded, What is the square-root of 36, the answer will be 6. So the square-root of 16 is 4; the square-root of 9 is 3, &c. And contrarily the square of the root 6 is 36: Also the square of 3 is 9.

VI. When a square number is given, that exceeds not 100, and yet is none of the square numbers mentioned in the Table, for his root you are to take the root of the square number that being less, yet comes nearest unto it: So 45 being given, the root that belongs unto it is 6, and 10 being given, his correspondent root is 3.

VII. A

VII. A compound square number is that which being produced by a number (that consists of more places than one multiplied by it self, is never less than 100: So 1024 is a compound square number produced by the multiplication of 32 multiplied by it self.

A compound square number.

VIII. To prepare any square number given for extraction, put a point over the first place thereof on the right hand (being the place of Units;) then proceeding towards the left hand, pass over the second place, and put another point over the third place, also passing over the fourth place put another point over the fifth, and so forward in such manner that between every two points which are next one to the other, one place will be intermitted: So if the square root of 1024 be required, the first point is to be placed over 4, and the second over 0 as you see, 1024 and so many points as are in that manner placed, of so many figures the root demanded will consist.

IX. Having thus prepared your number, you may see it distributed by the points into several squares: So in the last *Example*, 10 is the first square and 24 the second; likewise if this number 144 were propounded for extraction after points are duly placed according to the last Rule, you will see 1 to be the first square and 44 the second.

X. Having drawn a crooked line on the right hand of the number propounded for extraction (after the same manner as is usually done in Division to denote the place of the Quotient,) find the root

root of the first square, and place it in the Quotient; So I find, by the sixth Rule
 aforegoing, 3 to be the correspondent
 1024 (3 root of 10; wherefore I write 3 in
 the Quotient, and then the Work will
 stand as you see.

XI. Subscribe the square of the figure
 placed in the Quotient under the first
 1024 (3 square of the number given, as you see
 9 in the Margin.

XII. Having drawn a line under the square (of
 the figure placed in the Quotient, subscribed as
 afore said,) subtract the same out of the
 first square of the number propounded,
 1024 (3 and placing the *remainder* or *orderly un-*
 9 *derneath* the line; so the square of 3 which
 — is 3 being subtracted from 10, the re-
 1 mainder is 1, and the Work will stand
 as you see in the Margin.

XIII. To the said remainder bring down the
 next square of the number propounded, that is,
 write down the *figures* or *cyphers* stand-
 1024 (3 ing in the two following places of the
 9 number propounded on the right hand
 — of the said remainder, so the square 24
 being placed next to the remainder 1,
 there will be found this number 124
 which may be called the *Resolvend*.

XIV. Double the root being the
 number placed in the Quotient, and
 1204 (3 place the said double on the left hand
 9 of the Resolvend, like a Divisor: So
 — the double of 3 is 6, which being
 6) 124 placed before a crooked line on the
 left

left hand of the Resolvend, 124, the work will stand as you see.

XV. Let the whole Resolvend, Except the first place thereof on the right hand (being the place of units) be always esteemed as a Dividend, then demanding how often the Divisor before found, is contained in the said Dividend, and observing in that behalf the Rules before taught in Division, write the answer in the Quotient, and also on the right hand of the Divisor, to wit, between the Divisor and the crooked line: So if you ask how often the Divisor 6 is found in the Dividend 12, the answer is 2, wherefore I write 2 in the Quotient and also after the Divisor 6, as you see in the Margin.

XVI. Multiply all the number which standeth on the left hand of the Resolvend, to wit, before the crooked line) by the figure last placed in the Quotient, and write the Product orderly underneath the Resolvend (to wit, units under units, tens under tens, &c.) then having drawn a line under the said Product subtract it from the Resolvend, and subscribe the remainder under the line: So 62 being multiplied by 2, the Product is 124, which if I subtract out of the Resolvend 124 the remainder is 0; and thus the whole Work being finished, the square root of 1024 (the number propounded) is found to be 32.

$$\begin{array}{r} 1024(32 \\ 9 \end{array}$$

$$62)124$$

$$\begin{array}{r} 1024(32 \\ 9 \end{array}$$

$$\begin{array}{r} 62)124 \\ 124 \end{array}$$

0

Note,

Note 1. When the Product before mentioned exceeds the *Resolvend* placed above it, the work is erroneous, and then you are to reform it by placing a lesser figure in the Quotient.

Note 2. For every one of the particular squares (distinguished by the points) except the first on the left hand, a *Resolvend* is to be set apart, by bringing down to the remainder the congruent particular square, as is directed in the 13 *Rule*; and as often as a *Resolvend* is set apart, so often a new *Divisor* is to be found by doubling or multiplying by 2 all the root in the quotient (consisting of what number of places soever.)

Note 3. The work of the 10, 11, and 12 *Rules* for finding of the first figure in the root, is but once used in the extraction of the root of a number consisting of what number of places soever; but the Work of the 13, 14, 15, and 16 *Rules* is to be repeated for the finding of every place in the root, except the first.

The practice of these 3 *Notes* will be seen in the following *Examples*.

Example 1. Let it be required to extract the square root of 43623.

Having distributed the number propounded into several squares by points, as is directed in the eighth *Rule* of this Chapter, I demand the square root of 4 the first square, which I find by the 5 rule of this Chapter to be 2; wherefore placing 2 in the quotient, and the square thereof, which is 4, under the first square 4, I draw a line, and subtracting 4 from 4 the remainder is 0, which I subscribe underneath,

derneath the line. This is always the first Work, which is no more repeated in the whole *Extraction* (as was intimated in the third *Note* aforegoing.)

Then bringing down the next square, which is 36, and placing it next after the remainder 0, the *Resolvend* is 36; and doubling the root 2 in the quotient, the *Product* is 4 for a *Divisor*. (by the 13 and 14 *Rules*) and the *Dividend* will be 3 (by the 15 *Rule*;) wherefore I demand how often the *Divisor* 4 is contained in the *Dividend* 3, and not finding it once contained in it, I place 0 in the quotient, and also next after the *Divisor* 4; and because the *Product* of 40 multiplied by 0 (the last Character in the quotient) is 0, the *Resolvend* 36, from which the said *Product* ought to be deducted, remains the same without alteration; therefore I bring down 23 the next square, and place it after the remainder 56, so will 03623 be a new *Resolvend*, then doubling the whole root in the quotient, which is 20, the *Divisor* will be 40 (according to the second *Note* before mentioned,) and the *Dividend* will be 362 (to wit, all the *Resolvend* except the first place on the right hand by Rule 15.) wherefore I demand how often the *Divisor* 40 is contained in the *Dividend* 362, or how often 4 in 36, and though it be 9 times in it, yet (according to the first *Note* aforegoing) I can take but 8, (for if I should take 9, and proceed according to the 15

43623 (20

4

—————

40) 036

43623

4

—————

40) 03623

and

and 16 Rules, a number would arise greater than the *Resolvend*, from which such number arising ought to be subtracted;) wherefore I write 8 in the quotient, and also after the Divisor 40; this done, I

multiply 408 (the number on the left hand of the *Resolvend*) by 8 the figure last placed in the quotient, and the Product, to wit 3264, I subscribe under, and subtract from the *Resolvend* 3623, so there will remain 359, thus the work being finished I find 208 to be the number of

$$\begin{array}{r} 43623(208 \ 8 \\ 4 \text{ ---} \\ 408)03623 \\ \underline{3264} \\ 349 \end{array}$$

units contained in the root sought; and because after the extraction is ended there happens to be a remainder, to wit, 359, I conclude that the root sought is greater than the said 208. but less than 209, yet how much it is greater than 208, no Rules of Art hitherto known will exactly discover, altho we may proceed infinitely near, as in the next Rule will be manifest.

XVII. To find the fractional part of the root very near, a competent number of pairs of cyphers, to wit, 00, 0000, 000000, or 00000000, &c. are to be annexed to the number first propounded, then esteeming the number propounded with the cyphers annexed to be but one entire number, the extraction is to be made according to the precedent Rules, and look how many points were placed over the number first given so many places of Integers will be in the root, the rest of the root towards the right hand will be the Numerator of a decimal fraction, which Numerator consisteth of so many places as there were points over the cyphers

Chap. XXXII. *the Square root.* 265

cyphers annexed: So if 43623 were given as before, to find the root thereof (according to this rule) annex cyphers in this manner, and then if you extract it according to the *Rules* aforegoing, you

43623.000000 (208.861, &c.

will find the *root* arising in the quotient to be 208.861, that is 208 $\frac{861}{1000}$; and because after the extraction is *finisht* there happens to be a remainder, I conclude that 208 $\frac{861}{1000}$ is less than the *true* or *exact* root, but 208 $\frac{861}{1000}$ is greater than it; so that by annexing three pair of *Cyphers* to the number propounded, you will not miss $\frac{1}{1000}$ part of an *unit* of the *true* root; also by annexing 4 pair of *cyphers*, you will not miss $\frac{1}{10000}$ part of an *unit*, and in that order you may proceed infinitely near, when you cannot obtain the true root. The whole operation of the said *Example* here followeth.

43623.000000 (208.861, &c.

4 *The root*

408) 03623

3264

4168) 35900

33344

41766) 255600

250596

417721) 500400

417721

82679

R

Again,

Again, if 10 were propounded to be extracted, you must prepare it thus,

10. 00000000000000)

And then the root thereof } 3 ¹⁶²²⁷⁷⁶ 10000000, &c.
being extracted will be _____

Which (according to the } 3. 1622776, &c.
third Rule of the 22d Chapter }
may be written thus _____

See here part of the Work in the extraction of the Root of 10, which may give you a light and understanding of the rest.

10. 00000000000000 (3. 16227, &c.

9

61) 100

61

616) 3900

3756

6322) 14400

12644

63242) 175600

126484

632447) 4911600

4427129

84471

XVIII. The extraction of the square root is proved by multiplying the root by it self, for that done, the Product (in such case, when there is no remainder after the extraction is finished) will be equal to the number whose square root was enquired; so in the first *Example* of this *Chapter*, the root 32 being multiplied by it self, produceth 1024 the number propounded: But when after the extraction is finished, there happeneth to be a remainder, and that the root is found as near as you please, in a mixt number of Integers and Decimal parts (by annexing cyphers, as in the 17 Rule) then such mixt number being multiplied by it self must produce a mixt number less than the number first propounded for extraction, yet so near unto it, that if the figure standing in the last place of the Numerator of the Decimal fraction in the root be made greater by 1, and then the mixt number so increased be multiplied by it self, the Product must be greater than the number first propounded: So in the *Example* of the 17 Rule, if 208. 861 be multiplied by it self, it produceth 43622. 917, &c. which is less than the propounded number 43623, but if 208. 862 be multiplied by it self, the Product will be 43623. 939, &c. which is greater than 43623.

XIX. The square root of a Fraction is found in this manner, viz. *To extract the square root of a Fraction.* extract the root of the Numerator (by the precedent Rules of this Chapter) which root shall be a new Numerator: Also the root of the Denominator is to be taken for a new Denominator: So the new Fraction shall be the square root of the Fraction first propounded.

ed. Thus the square root of $\frac{9}{16}$ is $\frac{3}{4}$, viz. the root of 9 is 3 for a new numerator, also the root of 16 is 4 for a new denominator. In like manner the square root of $\frac{4}{9}$ is $\frac{2}{3}$. But here note diligently, that if the Fraction whose square root is required be not in its least terms, it must first of all be reduced by the 4 Rule of the 17 Chap. before any extraction be made; for oftentimes it happens that the Fraction first given hath not a perfect root, but when such Fraction is reduced into its least terms, the root thereof may be extracted: So in this Fraction $\frac{8}{18}$ each term is incommensurable to its square root, viz. neither 8 nor 18 hath a square root expressible by any true or rational number; but the said $\frac{8}{18}$ being reduced to its least terms $\frac{4}{9}$, the root of this may be extracted, for the root of 4 is 2 for a new Numerator; also the root of 9 is 3 for a new Denominator; so that $\frac{2}{3}$ is found to be the square root of $\frac{4}{9}$ (equivalent unto $\frac{8}{18}$.)

XX. When either the Numerator or Denominator of a Fraction hath not a perfect square root, such root is usually express'd by prefixing this Character, $\sqrt{\quad}$ or $\sqrt{q.}$ before the Fraction given: So the square root of $\frac{13}{16}$ is signified thus, $\sqrt{\frac{13}{16}}$, or thus $\sqrt{q. \frac{13}{16}}$, because the root of $\frac{13}{16}$ cannot be express'd by any true or rational number whatsoever, yet it may be found very near, as in the next Rule.

XXI. The square root of a Fraction which is incommensurable to its root, may be found near, in this manner viz. reduce the fraction proposed into a decimal by the third Rule of the 23 Chapter: The more places are in the decimal, the nearer will the root be found, but the decimal must consist of an even number

To extract the square root near, of a fraction incommensurable to its square root.

Chap. XXXII. *Of the Square Root.* 269

number of places, viz. either of two, four, six, eight, or ten, &c. places; then extract the square root of that decimal, as if it were a whole number, according to the Rules foregoing, which root found shall be a decimal expressing near the square root of the fraction proposed.

So if the square root of $\frac{1}{2}$ be required near, reduce the said $\frac{1}{2}$ into a decimal (by the 3d Rule of the 23d Chapter) which will be found, 81250000, &c. Then extracting the square-root thereof as if it were a whole number, it will be found .9013 very near.

XXII. The square root of a mixt number commensurable to its root *To extract the square root of a mixt number.* is found in the same manner as in the 19th Rule of this Chapter, the mixt number being first reduced into an improper fraction by the 10th Rule of the 17th Chapter.

So the square root, of $34 \frac{11}{16}$ will be found $5 \frac{7}{8}$ viz. $34 \frac{11}{16}$ being reduced into the improper Fraction $\frac{555}{16}$, the square root of the Numerator 2209 will be 47 for a new Numerator; also the square root of the Denominator 64 is 8, for a new Denominator; so is found $4 \frac{7}{8}$, which by (the 13th Rule of the 17th Chapter) is $5 \frac{7}{8}$ the square root sought. And here the same Caution is to be observed as in the 19th Rule of this Chapter; viz. The Fractional part of the mixt number, or improper fraction equivalent unto the mixt number, must be in the least terms before any extraction be made.

To find the square root near, of a mixt number incommensurable to its root.

XXIII. When the mixt number given is incommensurable to its square root, prefixing this Character before it, viz. $\sqrt{}$ or $\sqrt{q.}$ So the square root, of $7\frac{1}{3}$ will be thus expressed, $\sqrt{7\frac{1}{3}}$ or $\sqrt{q. 7\frac{1}{3}}$: But if you desire to find the square root near of a mixt number incommensurable to its root, reduce the fractional part of the mixt number into a Decimal of an even number of places, as in the 21 Rule of this Chapter, and annex the Decimal so found unto the whole part of the mixt number; then esteeming the said whole number and Decimal as one entire number, extract the square root thereof according to the foregoing Rules of this Chapter, and from the root found, cut off always to the right hand, so many places as there are points over the Decimal annexed, which number so cut off shall be a Decimal, shewing the fractional part of the root, and that on the left hand shall be the whole part of the root; so the square root of $7\frac{1}{3}$ will be found 2.7688 very near.

C H A P. XXXIII.

The Extraction of the Cube Root.

I. **T**HE Extraction of the Cube Root is that, by which having a number given, we find another number, which being first multiplied by it self, and then by the Product, produceth the number given.

II.

II. In the Extraction of the Cube root, the number propounded is always conceived to be a Cube number, that is, a certain number of little Cubes, comprehended within one entire great Cube, and the root or number required is the side of that great Cube: what a Cube is may well express'd by a Die, which indeed is a little Cube it self; wherefore if you place four Dice in a square form, that is laying two and two in a rank, you shall have a square containing four Dice, upon which if you yet erect such another square of Dice, you shall have a great entire Cube comprehending two times four, that is 8 Dice or little Cubes; and here 8 is the Cube number given, and 2 is the root or number required: In like manner if you rank 25 Dice in a square form, viz. laying 5 in a rank, you have a square containing 25 Dice; now upon this square of Dice if you erect four other such squares one upon another, you shall have a great entire Cube comprehending 5 times 25, this is 125 little Cubes, and in this case 125 is the Cube number propounded, and 5 the root or number required.

III. A Cube number is either single or compound.

IV. A single Cube number is that, which being produced by the multiplication of one single figure first by it self and then by the Product is always less than 1000. So 125 is a single Cube number produced by 5 multiplied first by it self, and then by 25 the product; for 5 times 5 is 25, and 5 times 25 is 125.

V. All the single Cube numbers, and square numbers

bers, together with their respective roots, are expressed in the Table following.

<i>Cubes.</i>	1	8	27	64	125	216	343	512	729
<i>Squares.</i>	1	4	9	16	25	36	49	64	81
<i>Roots.</i>	1	2	3	4	5	6	7	8	9

Here in the uppermost rank of the Table are placed the single *Cube numbers* of the particular figures, 1, 2, 3, 4, 5, 6, 7, 8, 9. in the next the squares of those figures, and in the lowest rank the figures themselves being the respective *roots* of the *Cubes* and *Squares* in the uppermost ranks; and therefore the *Cube root* of 125 being demanded, the *Answer* is 5, and the *Cube root* of 216 being required, the Table will give you 6, and so of the rest.

VI. When a *Cube number* is given, that exceeds not 1000, and yet is none of the *Cube numbers* mentioned in the Table, for this *Root* you are to take the *Root* of the *Cube number*, that being less, yet comes nearest unto it: So 125 being given, the root that belongs unto it is 5.

VII. A compound *Cube number* is that, *A compound cube number.* which being produced by a number (that consists of more places than one) first multiplied by it self, and then by the *Product* is never less than 1000. So 157464 is a *Compound Cube number*, being produced by 54 multiplied first by its self, and then by 2916 the *Product*, for 54 times, 54 is 2916, and then 54 times 2916 is 157464, the *Compound Cube number* propounded.

VIII. To prepare a Cube number for extraction, put a point over the first place thereof towards the right hand (to wit the place of Units;) then passing over the second and third places, put another point over the fourth, and passing over the fifth and sixth put another point over the seventh, and in that order (to wit two places being intermitted between every two adjacent points) place as many points as the number will permit: So 157464

being given, you are to place the points as in the Margin, and so many points as are in that manner placed, of so many figures the *root* demanded will consist.

IX. Having thus prepared your number, you may see it distributed by the points into several Cubes: so in the same example 157 is the *first Cube*, and 464 the *second*. In like manner if this Number 7464 were propounded for extraction, after points are duly placed as before, you will see 7 to be the *first Cube*, and 464 the *second*.

X. Having drawn a crooked line on the right hand of the number propounded to signifie a *quotient*, find the Cube root of the first Cube and place it in the quotient: So I finding (by the sixth Rule of this Chapter) 5 to be the correspondent root of 157, I write 5 in the quotient, and then the work will stand as you see in the Margin.

XI. Subscribe the Cube of the root placed in the quotient, under the first Cube of the number given: So 125 being the Cube of 5 the root (by the

157464

157464

7464

157464

157464

125

fifth

fifth Rule of this Chapter) I write it under 157 the first Cube of the number given, as you see in the example.

XII. Draw a line under the Cube subscribed as aforesaid (to wit, the Cube of the root placed in the quotient) and subtract this Cube from the first

Cube of the number propounded, placing the remainder orderly underneath the line: So 125 the Cube of 5 being subtracted from 157, the remainder is 32, and the work will stand as you see.

157464 (5

125

32

XIII. To the said remainder bring down the next Cube of the number propounded (to wit, the figures or cyphers that stand in the

157464 (5

125

32464 resol.

will be found this number 32464, which may be called the *Resolvend*.

XIV. Having drawn a line underneath the *Resolvend*, square the root in the quotient, that is, multiply by it self, and subscribe the triple of the

157464 (5

125

32464 resol.

75

of is 75, which I subscribe under the *Resolvend*

said square or product under the *Resolvend* in such manner, that the first place (to wit, the place of units, of the said triple square may stand directly under the third place (or place of hundreds) in the resolvend: So the square of the root 5 is 25, the triple where-

end in such manner, that the figure 5 which is in the first place (to wit, the place of units) in the Triple Product 75, may stand under 4, which is set in the third place of the Resolvend, as you see in the Margin.

XV. Triple the root or number in the quotient, and subscribe this triple number in such manner, that the first place thereof (to wit, the place of units) may stand directly under the second place (to wit the place of tens)

of the Resolvend: so the triple

of the root 5 is 15, which I

subscribe in such manner, that

the figure 5 which is in the first

place (to wit the place of units)

of the said triple number, doth

stand directly under 6, which is

set in the second place of

the Resolvend, and the Work

will stand as in the Margin.

XVI. The Triple square of the root, and the

Triple of the root being placed

one under the other, as is di-

rected in the 14 and 15 Rules

pregoing, draw a line un-

derneath, and add them toge-

ther in such order as they are

ranked, and let the sum be e-

stimated as a Divisor: So the

Triple square 75, and the triple

number 15, being added toge-

ther as they are ranked in the

Work, the sum will be 765 for

Divisor.

$$\begin{array}{r}
 157464 \quad (5 \\
 125 \\
 \hline
 32464 \text{ Resolv.} \\
 \hline
 75 \\
 15
 \end{array}$$

$$\begin{array}{r}
 157464 \quad (5 \\
 125 \\
 \hline
 32464 \text{ Resolv.} \\
 \hline
 75 \\
 15 \\
 \hline
 765 \text{ Divisor.} \\
 \hline
 \hline
 \end{array}$$

XVII. Let

XVII. Let the whole *Resolvend*, except the first place thereof towards the right hand (to wit, the place of *units*) be esteemed as a *Dividend*, then demanding how often the first figure (towards the left hand) of the *Divisor* is contained in the correspondent part of the *Dividend*, and observing in that behalf the *Rules* before taught in Division, write the *Answer* in the *Quotient*: So if I ask how often 7 (the first figure of the *Divisor* towards the left hand) is contained in 32 (the correspondent part of the *Dividend* placed above) the *Answer* will be 4; wherefore I write 4 in the *Quotient*, as you see in the Example.

XVIII. Having drawn another line under the *Work*, multiply the triple square before subscribed (as is directed in the fourteenth Rule) by the figure last placed in the quotient, and subscribe this *Product* under the said triple square; (to wit, units under units, tens under tens, &c.) So 75 being multiplied by 4, the *Product* is 300, which I subscribe under 75 (the triple square) and the work will stand as you see in the Margin.

XIX. Multiply

Chap. XXXIII. *the Cube Root.* 277

XIX. Multiply the figure last placed in the quotient first by it self, and then the Product by the triple number before subscribed (as is directed in the 15th Rule of this Chapter;) this done, subscribe the last Product under the said triple number (to wit, units under units, tens under tens, &c.) so 4 being squared or multiplied by it self, the Product is 16, which being Multiplied by the triple number 15, the Product is 240, this therefore I subscribe under the aforesaid triple number 15, and the Work will stand as you see.

$$\begin{array}{r}
 157464 \text{ (54)} \\
 125 \\
 \hline
 32464 \text{ Resolvend.} \\
 \hline
 75 \\
 15 \\
 \hline
 765 \text{ Divisor.} \\
 \hline
 300 \\
 240 \\
 \hline
 \end{array}$$

XX. Subscribe the Cube of the figure last placed in the quotient under the Resolvend, in such manner that the first place of this Cube (to wit, the place of units,) may stand under the place of units in the Resolvend: So 64 being the Cube of 4, I write it under the Resolvend 32464, in such manner that the figure 4, which is in the place of units in the Cube 64, may stand under the figure 4, which is seated in the place of units of the resolvend; Observe the Work in the Margin,

$$\begin{array}{r}
 157464 \text{ (54)} \\
 125 \\
 \hline
 32464 \text{ Resolvend.} \\
 \hline
 75 \\
 15 \\
 \hline
 765 \text{ Divisor.} \\
 \hline
 300 \\
 240 \\
 64 \\
 \hline
 \end{array}$$

XXI. Drawing yet another line under the

157464 (54

125

32464 *Resolvend.*

75

15

765 *Divisor.*

300

240

64

32464

0

Note 1. When the sum of the three last numbers before mentioned is greater than the resolvend the work is erroneous, and then you are to reform it by placing a lesser figure in the quotient.

Note 2. For every one of the particular Cubes (distinguished by the points) except the first Cube on the left hand, a resolvend is to be set apart, by bringing down to the remainder the next Cube (as is directed in the 13 Rule). And as often as a resolvend is set apart, so often is a new Divisor to be found, by adding the *triple* of all the root in the *quotient* (consisting of what number of places soever,) to the *triple* of the *square* of such root after they are orderly placed according to the 14 and 15 Rules.

Note

work, add the three last numbers together in the same order as they are seated, and subtract the sum of them from the Resolvend, placing the remainder orderly underneath: So the sum of the three last numbers, as they are ranked in the Work, is 32464, which if you subtract out of the Resolvend 32464, the remainder is 0. Thus the whole Work being finished, the Cube root of 157464 (the number propounded) is found to be 54.

Chap. XXXIII. *The Cube Root.* 279

Note 3. The work of the 10, 11 and 12 *Rules* for finding of the first figure in the *root* is but once used in the extraction of the root of any number whatsoever, but the Work of all the following *Rules* is to be used for the finding of every place in the *root*, except the first.

The practice of these 3 *Notes* will be seen in the following *Examples*.

Example 2. Let it be required to extract the *Cube root* of 8302348.

Having distributed the number given into several *Cubes* by points, as is directed in the eighth *Rule* of this *Chapter*, I demand the *Cube root* of 8 (the first *Cube* on the left hand) which I find by the fifth *Rule* of this *Chapter*,

to be 2, wherefore placing 2
in the quotient, and 8 the 8302348(2
Cube thereof under 8 the first 8

Cube, I draw a line and —————
subtracting 8 out of 8 the 0
remainder is 0, which I sub-

scribe under the line. This is always the first Work and is no more repeated in the whole extraction (as was intimated in the 3 *Note* aforegoing;) then bringing down the next *Cube* (to wit, the figures standing in the three following places of the number propounded) which is 302, I place it after the remainder 0, so is 302 the *resolvend*; this done, having drawn a line underneath the *resolvend*, I seek for the triple of the square of the root, viz. the root in the quotient is 2, which multiplied by it self produceth the square 4, the triple whereof is 12, this I subscribe under the *resolvend*, in such manner that the figure 2
in

in the units place of this triple square 12, may stand directly under the figure 3, which is seated in the third place of the *Resolvend*, (to wit, the place of hundreds) according to the 14th Rule aforegoing. Again, I triple the root 2, which produceth 6, and subscribe this triple number 6 under the second place (or place of tens) in the *Resolvend*, to wit, under 0 (according to the 15th Rule of this Chapter;) then drawing a line under the Work, and adding together the said two numbers last subscribed, as they are ranked, the sum of them is 126 for a Divisor (according to the 16th Rule aforegoing.)

$$\begin{array}{r}
 \cdot \quad \cdot \quad \cdot \\
 8302348 \quad (2 \\
 8 \\
 \hline
 0302 \text{ Resolvend.} \\
 \hline
 12 \\
 06 \\
 \hline
 126 \text{ Divisor.} \\
 \hline
 \hline
 \end{array}$$

That done, esteeming 30, to wit, all the places except the first or place of units in the *Resolvend*, as a *Dividend*, I demand how often the divisor 126 is contained in 30, and not finding it once contained therein, I write 0 in the *quotient*; and now because the sum of the three numbers which ought to have been Produced (according to the 18, 19, and 20 Rules of this Chapter) by the multiplication of 0 (which was last placed in the *Quotient*) amounts to 0; the *Resolvend* 302, out of which the said sum should have been subtracted, remains the same without alteration, wherefore having drawn a line under the Work, I write down anew the old *Resolvend* 302, and bringing down the next Cube 348, I annex it to the said

Chap. XXXIII. the Cube Root. 281

302; so there will be a new *Resolvend*, to wit, 302348.

Then squaring the root 20 (that is multiplying of it by it self) the Product is 400, which I triple or multiply by

3, and subscribe the Product 1200 underneath the new *Resolvend* in such manner, that the place of units in this triple quadrate 1200 may stand under the place of hundreds, or third place of the *Resolvend* 302348, to wit, under 3 (according to the 14th Rule.)

Again, I subscribe the triple of the root 20, which is 60, in such manner that the place of units in this triple root 60 may stand under the place of tens or second place of the *Resolvend*, to wit, under 4, then adding together the two numbers last subscribed, to wit, 1200 and 60, in such order as they are ranked in the Work, the sum is 12060 for a

8302348 (202

8

0302 *Resolvend.*

12

06

126 *Divisor.*

302348 *Resolvend.*

1200

60

12060 *Divisor.*

2400

240

08

242408 *Ablatitium.*

59940

S

Again,

Again, esteeming the whole Resolvend, except the first place (or place of units) as a Dividend, to wit, 30234, I demand how often 1 (the first figure of the divisor towards the left hand) is contained in 3 the correspondent part of the Dividend; and though it be three times contained in it, yet (according to the first *Note* at the end of the 21 *Rule* of this *Chapter*) I dare take but 2, for if I should take 3, and proceed according to the 18, 19 20, and 21 *Rules* of this *Chapter*, a Number would arise greater than the Resolvend (from which such number arising ought to be subtracted) wherefore I write 2 in the *Quotient*.

Then multiplying the triple square 1200 before subscribed, by 2 (the figure last placed in the quotient,) the Product is 2400, which I subscribe under the said 1200 (to wit, units under units, and tens under tens, &c.) Also multiplying the triple root 60 before subscribed, by 4 (the quadrate of 2 the figure last placed in the quotient) the product is 240, which I subscribe under the said triple root 60; last of all I subscribe 8 the *Cube* of the said new root 2, under the place of units or first place of the *Resolvend*, to wit, under 8, and having added together those three Numbers last subscribed, to wit, 2400, 240 and 8 as they stand in ranks in the Work, the sum of them is 242408, which being subducted from the *resolvend* 302348, there will remain 59940. Wherefore the Work being finished, I find 202 to be the number of units contained in the *Cube root* of 8302348 the number propounded: And because after the extraction is ended there happens
to

I. Chap. XXXIII. *the Cube Root.* 283

to be a remainder, to wit, 59940, I conclude that the *Cube root* sought is *greater* than the said 202, but less than 203; yet how much it is greater than 202, no *Rules* of Art hitherto known will exactly discover, although we may proceed *infinitely* near, as by the next *Rule* will be manifest.

XXII. To find the fractional part of the root very near ternaries of Cyphers, to wit, 000, 000000, or 000000000, &c. are to be annexed to the number first propounded; then esteeming the number propounded with the *cyphers* annexed to be but one entire number, the Extraction is to be made according to the preceeding *Rules* of this *Chapter*, and look how many points were placed over the number first given, so many of the foremost places in the *Quotient* are the Integers or Units contained in the *Cube root* sought, and the rest of the places in the *Quotient* are to be *esteem'd* as the Numerator of a Decimal Fraction, which Numerator consisteth of so many places as there were points over the *Cyphers* first annexed: So if 8302348 were given as before, to find the *Cube root* thereof (according to this *Rule*) annex *Cyphers* in this manner.

8302348,000000 (

And then if you prosecute the extraction according to the *Rules* aforegoing, you shall find the *Cube root* sought to be 202. 48, &c. that is, $202 \frac{48}{100}$ and more; wherefore you may conclude that $202 \frac{48}{100}$ is less than the *true root*, but $202 \frac{49}{100}$ is

greater than it: So that by annexing two ternaries of cyphers, to wit, 6 cyphers, to the number propounded, you will not miss $\frac{1}{1000}$ part of an unit of the true root; also by annexing 3 ternaries of cyphers, to wit, 9 cyphers, you will not miss $\frac{1}{1000000}$ part of an unit of the true root, and in that order you may proceed *infinitely* near, when you cannot obtain the true root. The whole operation of the said Example here followeth, where you may observe, that for the more certain and easie placing, as well of the numbers which constitute the several Divisors, as of those which constitute the ablatitious numbers to be subtracted from the several and respective Resolvends, down-right lines are drawn between the particular *Cubes* of the number propounded, first distinguished by points as before.

8	302	348	000	000 (202. 48, &c)
8				
0	302			Resolvend.
12				
06				
126				Divisor.
302	348			Resolvend.
120	0			
	60			
120	60			Divisor.
240	0			
	240			
		08		
242	408			Ablatitium.
59	940	000		Resolvend.
122	412			
	606			
122	4726			Divisor.
48	964	8		
	96	96		
		64		
49	061	824		Ablatitium.
108	781	176	000	Resolvend.
1228	972	8		
	60	72		
1229	033	52		Divisor.
9831	782	4		
	3886	08		
		512		
9835	668	992		Ablatitium.
1042	507	008		

In like manner the *Cube root* of 2 will be found to be near equal to 1.25992, &c. that is, 1 ²⁵⁹⁹²/₁₀₀₀₀₀₀ and more.

XXIII. The extraction of the *Cube root* is proved by multiplying the root cubically, to wit the root being first multiplied by it self, and then the product multiplied by the root, the number arising or last product (in case there be no remainder after the extraction is finished) will be equal to the number propounded: so in the first example of this *Chapter*, the *Cube root* 54 being multiplied first by it self produceth 2916, which being multiplied again by 54 produceth 157464, to wit, the number whose *Cube root* was inquired. But when after the Extraction is finished, there happeneth to be a remainder, and that the root is found as near as you please in *Integers* and *Decimal parts* (by annexing cyphers as in the 22 *Rule* of this *Chapter*,) then such mixt number expressing the root being multiplied cubically, must produce a mixt number less than the number first propounded, yet so near unto it, that if the figure standing in the last place of the *decimal fraction* in the root be made greater by 1, and the mixt number so increased be multiplied cubically, the Product must be greater than the number first propounded: So in the example of the 22 *rule* of this *Chapter*, if 202.48 be multiplied cubically, it produceth 8301305.49, &c. which is less than the propounded number 8302348; but if 202.49 be multiplied cubically, there will arise 8302535.49, &c. which is greater than the said given number.

XXIV. The *Cube root* of a *Fraction* is found in this manner, viz. extract the *Cube root* of the Numerator

Numerator (according to the foregoing Rules,) which root reserve for a new Numerator; also the Cube root of the Denominator shall be a new Denominator; lastly this new Fraction shall be the Cube root of the Fraction first propounded: so the Cube root of $\frac{8}{27}$ is $\frac{2}{3}$, for the Cube root of 8 is 2 for a new Numerator; also the cube root of 27 is 3 for a new Denominator. In like manner the cube root of $\frac{1}{8}$ is $\frac{1}{2}$. But here note diligently, that the fraction whose cube root is required, must be in its least terms before any Extraction be made; for oftentimes it happens that the fraction first given hath not a perfect root, albeit, when such fraction is reduced into its least terms, the root thereof may be extracted: so in this Fraction $\frac{16}{27}$ neither the numerator nor denominator hath a perfect cube root, yet the said $\frac{16}{27}$ being reduced to its least terms $\frac{8}{27}$ (by the fourth Rule of the 17 Chapter) the cube root of this may be extracted, for the cube root of 8 is 2 for a new numerator, also the cube root of 27 is 3 for a new denominator, so that the cube root of $\frac{8}{27}$ (which is equal to $\frac{16}{27}$) is found to be $\frac{2}{3}$.

XXV. The Cube root of a fraction which hath not a perfect Cube root may be found near in this manner, viz. reduce the Fraction given into a Decimal Fraction, by the third Rule of the 23 Chapter, the more places are in the Decimal, the nearer will the root be found, but the Decimal must consist of ternaries of places, to wit, either of three, six, nine, or twelve, &c. places; then extract the Cube root of the Numerator of that decimal, as if it were a whole number (according to the Rules before given,) which root found shall be a Decimal expressing

expressing near the Cube root of the Fraction propounded.

So if the *cube root* of $\frac{1}{3}$ were required, I reduce the said $\frac{1}{3}$ into a *decimal*, whose *numerator* may consist of *ternaries* of places, to wit into this, 666666666666 &c. then extracting the *cube root* thereof, I find 8735, which is very near the *cube root* of $\frac{1}{3}$.

XXVI. The Cube root of a mixt number commensurable to its root may be found in the same manner as in the 24 Rule of this Chapter, the mixt number being first reduced into an improper Fraction (by the 10 Rule of the 17 Chapter.)

So the *cube root* of $12 \frac{19}{27}$ will be found to be $2 \frac{1}{3}$, viz. reducing $12 \frac{19}{27}$ into this improper fraction $\frac{343}{27}$ the *cube root* of $\frac{343}{27}$ will be found $\frac{7}{3}$ or $2 \frac{1}{3}$. And here the same caution is to be observed as in the 24 Rule of this Chapter, viz. the fractional part of the mixt number, or the improper *fraction* equivalent unto the mixt number, must be expressed by a *Numerator* and *Denominator* in the least terms before any extraction be made.

XXVII. When the mixt number, whose Cube root is required, hath not a perfect cube root, this character, $\sqrt[3]{c}$. is usually prefixed before such mixt number; so the *cube root* of $2 \frac{1}{8}$ is thus expressed, $\sqrt[3]{c} 2 \frac{1}{8}$. Likewise $\sqrt[3]{c} \frac{1}{8}$ denotes the *cube root* of $\frac{1}{8}$ which is a fraction, whose *cube root* is *inexpressible* by any true or rational number: But if you desire to know the *cube root* near of a mixt number which hath not a perfect *cube root*, reduce the *fractional* part of the mixt number into a *decimal* (as in the 25 Rule of this Chapter) and annex the *decimal* so found unto the *Integers* of the mixt number; then esteeming the said *Integers* with the *decimal* so annex-

ed as one entire number, extract the *cube root* thereof and from the *root* found cut off always to the right hand so many places as there were points over the said *decimal* annexed, which places so cut off shall be the fractional part of the *root*, and those remaining on the left hand shall be the Integers of the *root*: So the *cube root* of $2\frac{1}{2}$ will be found 1.334, and more.

XXVIII. I might here proceed to shew the extraction of the *roots* of the *Biquadrate* (or fourth *Power*) the fifth *Power*, &c. but their operations being exceeding tedious, and hardly intelligible without the knowledge of *Algebra*, I shall only in this place touch upon the Extraction of the *Biquadrate root*, because it may be extracted by the Rules delivered in the 32 Chapter, and referr the more curious Arithmetician for further satisfaction in this matter, to my Treatise of the Elements of *Algebra*.

XXIX. A quadrate or square number multiplied by it self produceth a *Biquadrate* number: So 4 multiplied by it self produceth the *Biquadrate* 16. Therefore if a number be propounded and the *Biquadrate root* thereof be required, first extract the *quadrate* or *square root* of the number propounded, and then extract the *square root* of that *root* for the *Biquadrate root* sought. Thus if 20736 be a number propounded, the *Biquadrate root* thereof will be found 12: For the *square root* of 20736 is 144, and the *square root* of 144 is 12. When the number given hath not a perfect *Biquadrate root*, you are to annex *quaternaries* of Cyphers, to wit, either 4, 8, 12, or 16, &c. cyphers, and then proceed as before; so will you find the *root* near, whose fractional part will be a decimal. Thus the *Biquadrate root* of 7 will be found near 1.62.

To extract
the *Biqua-
drate root*.

CHAP.

C H A P. XXXIV.

The Relation of Numbers in Quantity.

I. **T**HUS far single Arithmetick : Comparative Arithmetick insues, which is wrought by numbers, as they are considered to have Relation one to another.

Boetius Ar. II. This Relation consists in quantity, or quality.

III. Relation in quantity is the reference or respect that the numbers themselves have one unto another : As when the *comparison* is made between 6 and 2, or 2 and 6 : 5 and 3, or 3 and 5.

IV. Here the *Terms* or *Numbers* propounded are always two, whereof the first is called the *Antecedent*, and the other the *Consequent* : So in the first Example, 6 is the *Antecedent*, and 2 the *Consequent* : And in the second, 2 is the *Antecedent*, and 6 the *Consequent*.

V. Relation in Quantity consists either in the difference, or else in the *rate* or *reason* that is found betwixt the Terms propounded.

VI. The difference of two numbers is the remainder, which is left after subtraction of the less out of the greater : So 6 and 2 being the terms propounded, 4 is the difference betwixt them ; for if you *subtract* 2 out of 6, the remainder is 4.

VII. The

I.
Chap. XXXIV. *Numbers in Quantity.* 291

VII. The rate or reason betwixt two numbers in the quotient of the Antecedent divided by the Consequent: So if it be demanded what Rate or Reason 6 hath to 2, I answer, triple Reason: For if you divide 6 the Antecedent, by 2 the Consequent, the Quotient is 3, 2 being contained just 3 times in 6. In like manner is the subtriple Reason betwixt 2 and 6, for if you divide 2 by 6, the Quotient is $\frac{2}{6}$, or (which is all one) $\frac{1}{3}$ because 6 being not once found in 2, there remains 2 for the Numerator, 6 the Divisor being the Denominator of the Fraction given you in the Quotient, according to the 9th Rule of the 16 Chapter afore going.

VIII. This Rate or Reason of Numbers is either equal or unequal.

IX. Equal Reason is the Relation that equal numbers have unto one another: as 5 to 5, 6 to 6, 7 to 7, &c.

X. Here the one being divided by the other, the Quotient is always an Unit: For if it be demanded how often 5 is in 5, the answer is 1.

XI. Unequal Reason is the relation that unequal Numbers have one unto another: and this is either of the greater to the less, or of the less to the greater:

XII. Unequal Reason of the greater to the less, is when the greater Term is Antecedent: as of 6 to 2, 5 to 3, and the like.

XIII. Here the Quotient of the Antecedent divided by the Consequent is always greater than an Unit: So 6 divided by 2, the Quotient is

3, and 5 divided by 3, the quotient is $1\frac{2}{3}$.

XIV. Unequal reason of the less to the greater, is when the lesser Term is Antecedent: as of 2 to 6, 3 to 5, &c.

XV. Here the quotient of the Antecedent divided by the consequent is always less than an unit: So 2 divided by 6, the quotient is $\frac{1}{3}$ or $\frac{1}{2}$; and 3 divided by 5, the quotient is $\frac{3}{5}$.

XVI. Each of these kinds of unequal Reason is again subdivided into five other kinds or varieties, whereof the three first are simple, and the other two are mixt.

XVII. The simple kinds of unequal reason are,
1. Manifold. 2. Superparticular. 3. Superpartient.

XVIII. Manifold Reason of the greater to the less is, when the Consequent is contained in the Antecedent divers times without any part remaining: As 4 to 2, 8 to 4, 16 to 8, which is called Double reason, because the less is contained twice in the greater; so 6 to 2 is triple reason, 8 to 2 is fourfold reason, &c.

XIX. Here the quotient of the Antecedent divided by the consequent is always a whole number: So 8 divided by 2, the quotient is 4.

XX. The opposite of this kind, viz. of the less to the greater, is called submanifold: Examples hereof are 2 to 4, 4 to 8, 8 to 16, &c. Likewise 2 to 6, 2 to 8, 2 to 10, &c.

XXI. Superparticular is, when the Antecedent contains the consequent once, and besides an aliquot part of the consequent;

Chap. XXXIV. *Numbers in Quantity.* 293

quent; that is, an half, a third, a fourth, or a fifth part, &c. of the consequent, as 3 to 2, 4 to 3, 5 to 4, 6 to 5, and the like; here three divided by 2, the quotient is $1\frac{1}{2}$ and 4 being divided by 3 the quotient is $1\frac{1}{3}$. In like manner 5 divided by 4, the quotient is $1\frac{1}{4}$, and 6 divided by 5 the quotient is $1\frac{1}{5}$; wherefore I say 2 and half 2 (that is 1) constitute 3: So likewise 3 and one third part of 3 (*viz.* 1.) constitute 4, and so of the rest.

XXII. Here the quotient of the Antecedent divided by the Consequent is a mixt number, whose whole part, as also the Numerator of the fraction annexed, is always an unit: As is observable in the examples last mentioned.

XXIII. The opposite reason of this *Subsuperparticular* kind is Subsuperparticular, as 2 to 3, 3 to 4, 4 to 5, 5 to 6, &c,

XXIV. Superpartient is, when the Antecedent contains the Consequent once, and besides divers parts of the Consequent: *Superpartients.* As 5 to 3, 7 to 5, 7 to 4, 8 to 5, 9 to 5, 11 to 7, &c. here 5 divided by 3, the quotient is $1\frac{2}{3}$, and therefore 5 contains 3 once, and $\frac{2}{3}$ of 3; for 3 and two thirds of 3 (*viz.* 2) constitute 5.

XXV. Here the quotient of the Antecedent divided by the consequent is a mixt number, whose whole part being an unit, hath always for the Numerator of the fraction annexed unto it a number composed of more units than one: So the conference being made betwixt 5 and 3, and 5 the Antecedent being divided by 3 the consequent, the quotient is $1\frac{2}{3}$.

XXVI. The

XXVI. The opposite of this *reason* is Subsuperpartient: Examples hereof are 3 to 5, 5 to 7, 4 to 7, 5 to 8, 5 to 9, 7 to 11, and the like.

XXVII. The mixt kinds of unequal reason are manifold Superparticular, and manifold Superpartient.

XXVIII. Manifold Superparticular reason is, when the Antecedent contains the consequent divers times, and besides an aliquot part of the *consequent*: as 5 to 2, 10 to 3, 17 to 4, 21 to 5, and the like.

XXIX. Here the *quotient* of the *Antecedent* divided by the *consequent* is a mixt number, whose whole part *consisting* of more units than one, hath always an unit for the *Numerator* of the *Fraction* annexed unto it; so 5 divided by 2 the *quotient* is $2\frac{1}{2}$, and 21 divided by 5, the *quotient* is $4\frac{1}{5}$.

XXX. The opposite of this *Reason* is Submanifold Superparticular; as 2 to 5, 2 to 7, 3 to 7, 4 to 9, &c.

XXXI. Manifold Superpartient is, when the antecedent contains the consequent divers times, and besides divers parts of the consequent; as 8 to 3, 17 to 5, 19 to 4, 28 to 5, &c.

XXXII. Here the *quotient* of the *Antecedent* divided by the *Consequent* is a mixt Number, whose whole part as also the *Numerator* of the *Fraction* annexed unto it, is always a Number composed of more units than one: So 8 divided by 3, the *quotient* is $2\frac{2}{3}$, and 28 divided by 5, the *quotient* is $5\frac{3}{5}$.

XXXIII. The

Chap. XXXV. *Numbers in Quality.* 295

XXXIII. The opposite here is *Submanifold Superpartient*: as 3 to 8, 5 to 17, 4 to 19, 5 to 28, and the like.

And these are the several kinds of varieties of the Rates or Reasons that are found amongst Numbers, so that no two Numbers whatsoever can be named, but the Rate or Reason betwixt them is comprehended under one of these five kinds.

C H A P. XXXV.

The Relation of Numbers in Quality, where of Arithmetical and Geometrical Proportion.

I. **R**elation in quality (otherwise called Proportion) is either the reference or respect that the Reasons of Numbers have one unto another, or else which the differences of numbers have one to another.

*Vide Euclid. I.
3 d. 5. & Al-
sted. Arith. c. 5.*

II. Therefore here the Terms propounded ought always to be more than two, for otherwise there cannot be a comparison of *Reasons* or *Differences* in the Plural Number.

III. This proportion is either *Arithmetical*, or *Geometrical*.

IV. *Arith-*

IV. Arithmetical proportion is, when divers numbers differ according to an equal difference, as 2, 4, 6, 8, 10, &c. here 2 is the common difference betwixt 2 and 4, 4 and 6, 6 and 8, 8 and 10, &c. So 1, 2, 3, 4, 5, 6, 7, &c. differ by *Arithmetical Proportion*, 1 being the common difference betwixt them.

V. Arithmetical Proportion is either continued or interrupted.

VI. Arithmetical Proportion continued is, when divers numbers are linked together by a continual progression of equal differences. Such are the examples last propounded, as also these 1, 3, 5, 7, 9, 11, 13, &c. And 100000, 200000, 300000, 400000, &c.

VII. In a rank of *Numbers* that differ by Arithmetical Proportion continued, the sum of the first and last *Terms* being multiplied by half the number of the *Terms*, the Product is the Total Sum of all the *Terms*: So it being demanded, how many strokes the *Clock* strikes betwixt Midnight and Noon; the *Terms* of the Progression in this question are Twelve, viz. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. for in that order the *Clock* strikes, wherefore if I multiply 12 the sum of 12 and 1 (the first and last *Terms*) by 6 (being half the number of the *Terms*) the Product is 78, which is the total sum of all the *Terms* propounded being added together.

VIII. Or thus, Multiply the number of the *Terms* by the half sum of the first and last *Terms*, and then likewise the Product will give you the total
of

Chap. XXXV. Numbers in Quality. 297

of all the Terms: So 13, 11, 9, 7, 5, 3, being given, their total is 48, for 8 the half sum of 13 and 3, the first and last Terms being multiplied by 6, the number of the Terms, the Product is 48.

IX. Three numbers being given that differ by Arithmetical proportion continued, the mean being doubled, is equal to the sum of the extremes: So 5, 6, 7, being given, 6 being doubled, is equal to the sum of 5 and 7, the two extremes.

X. Arithmetical proportion may be continued either upwards or *Upwards* downwards.

XI. Upwards, when the terms of the Progression increase, as these, 2, 4, 6, 8, 10, 12, &c. or these 1, 2, 3, 4, 5, 6, &c. And this last rank is more particularly termed *Natural Progressions*

XII. Here when the first term is also the common difference of the terms, the last term being divided by the number of the terms, the quotient will give you the first term of the rank: Again in this case the first term multiplied by the number of the terms produceth the last term: So this rank 3, 6, 9, 12, 15, 18, 21, being propounded, wherein 3 is both the first term, as also the common difference of the terms; I say 21 the last term being divided by 7 the number of the Terms, the quotient is 3 the first term; contrariwise 3 the first term multiplied by 7 produceth 21 the last term.

XIII. Arithmetical proportion continued downwards is, when the terms of the progression decrease: Such as are 35, *Downwards* 32, 29, 26, 23, 20: And 40, 35, 30, 25, 20, 15, 10, 5.

I

XIV. Here

XIX. Here, when the last term is also the common difference of the terms, the first term being divided by the Number of the terms, the quotient will give you the last term: Again, the last term multiplied by the number of terms, produceth the first term of the rank.

For Example, this rank 40, 35, 30, 25, 20, 15, 10, 5, being propounded, in which 5 is both the last term, and likewise the common difference of the terms, I say, 40 the first term being divided by 8 the number of the terms, the quotient is 5 the last term: On the other side 5 the last term being multiplied by 8, the product is 40 the first term.

XV. Arithmetical Proportion interrupted is, when the Progression is discontinued:
2. Interrupted. as in these numbers 2, 4, 8, 10; here 2 and 4 being compared with 8 and 10 differ according to Arithmetical proportion, but so do not 4 and 8 differ, for 2 is the common difference betwixt 2 and 4, 8 and 10, whereas the difference betwixt 4 and 8, is 4. In like manner 8, 14, 17, 23 differ by Arithmetical proportion interrupted.

XVI. Four numbers being given, that differ by Arithmetical proportion either continued or interrupted, the sum of the two means is equal to the sum of the two extremes: So 5, 6, 7, 8, being given, the sum of 6 and 7, the two mean numbers, is equal to the sum of 5 and 8, the two extremes: and 8, 14, 17, and 23, being propounded the sum of 14 and 17 being added together is equal to the sum of 8 and 23,

XVII. Geo-

Chap. XXXV. *Numbers in Quality*, 299

XVII. Geometrical proportion is, when divers numbers differ according to like Rate or Reason: that is, when the reasons of numbers being compared together are equal. So 1, 2, 4, 8, 16, 32, &c. which differ one from another by double reason are said to differ by *Geometrical* proportion, for as 1 is half 2, so 2 is half 4, 4 half 8, 8 half 16, 16 half 32, &c. *Geometrical Proportion.*

XVIII. Geometrical proportion is either continued or interrupted. *Continued*

XIX. Geometrical proportion continued is, when divers numbers are linked together by a continued progression of the like reason: Of this sort is the example last given: for as 1 is to 2, so is 2 to 4, 4 to 8, 8 to 16, to 32, &c. So likewise the numbers 3, 9, 27, 81, 243, 729, &c. differ by *Geometrical* proportion continued, viz. by triple reason, each of them being contained three times in the next number that follows it.

XX. In numbers continually proportional from 1, the first number from 1 is the root or first power, the second is the square or second power, the third the cube or third power, the fourth the Biquadrate or fourth power, the fifth the fifth power, the sixth the sixth power, &c. So in this rank of numbers, 1, 3, 9, 27, 81, 243, 729, &c. 3 is the root, 9 the square, 27, the cube, 81 the Biquadrate, 243 the fifth power, 729 the sixth power, &c.

XXI. The root being multiplied by it self produceth the square, which being again multiplied by the root produceth the cube, and so each proportional being multiplied by the root, produceth the *Mean proportionals.* *proportionals.*

proportional next above it, then the numbers comprehended betwixt 1, and the last number produced are called mean proportionals: So in this rank of proportional numbers, 1, 2, 4, 8, 16, 32, &c. 2 the root being multiplied by it self produceth 4 the square, which being again multiplied by 2 produceth 8 the cube, then 8 being multiplied by 2, the product is 16 the biquadrate, and so of the rest in their order, and here 2, 4, 8, and 16. are the mean proportion in the rank propounded.

Continual means.
 Briggius Arith Log. 6. XXII. If you multiply the root by it self, and consequently the subsequent numbers by themselves, the numbers intercepted betwixt 1 and the number last produced may not unfitly be called continual means: So 2 being given for the root multiplied by it self, the product is 4, which being again multiplied by it self produceth 16, then 16 in like manner squared produceth 256, which likewise multiplied by it self produceth 65536, I say then that 2, 4, 16, and 256 are continual means betwixt 1 and 65536.

XXIII. The continual means comprehended betwixt any number given and 1, are discovered by a continued extraction of the square roots; for example, 65536, being given, the root thereof extracted is 256 whose root is 16; then the root of 16 is 4, and the root of 4 is 2; so that at last I find 256, 16, 4, and 2 to be continual means intercepted betwixt 65536 and 1 as before.

XXIV. In numbers that increase by Geometrical proportion continued, if you multiply the last term by the quotient of any one of the terms divided

Chap. XXXV. *Numbers in Quality.* 301

divided by another term, which *being* less is next unto it, and then deducting the first term out of that product, divide the remainder by a number that is an unit less than the quotient, the last *quotient* will give you the *total* of all the *terms* propounded in the *progression*; so this rank 2, 6, 18, 54, 162, 486, 1458, being propounded, where in the *proportionals* differ by subtriple proportion, I first take 2 and 6 the two first terms, and dividing 6 by 2, I find the quotient 3, wherefore multiplying 1458 the last term, by 3 the quotient, the product is 4374, out of which if I deduct 2 the first term, the remainder is 4372, which being divided by 2 (*viz.* a number which is an unit less than three the *quotient*) the last *quotient* gives me 2186, which is the total sum of the *proportionals* propounded.

XXV. Three *proportionals* being given, the square of the *mean* is equal to the *product* of the extremes: So 4, 8, and 16 being propounded, 8 times 8 being 64, is equal to 4 times 16, which is likewise 64.

XXVI. Geometrical Proportion interrupted is, when the progression of like reason is discontinued, in such sort that 2. *Interrupted.* four numbers being given, the like reason is not found betwixt the second and third, that is betwixt the first and second, and the third and fourth; of this sort are these numbers 2, 4, 16, 32. here as 2 is to 4, so is 16 to 32, for they differ by double reason; but as 2 is to 4, so is not 4 to 16, for 4 and 16 differ by fourfold reason, 4 being contained 4 times in 16: So likewise 4, 8, 8, 16, differ according to Geometrical proportion interrupted.

XXVII. The numbers of *Multiplication* and *Division* are proportional ; for in *Multiplication*, as 1 is to the *Multiplicator*, so is the *Multiplicand* to the *Product*, or as 1 is to the *Multiplicand*, so is the *Multiplicator* to the *Product* : Again, in *Division*, as the *Divisor* is to 1, so is the *Dividend* to the *Quotient* : Or as the *Divisor* is to the *Dividend*, so is 1 to the *Quotient*.

XXVIII. Four proportional *Numbers* whatsoever being given, the product of the two means is equal to the Product of the two extremes : So 2, 4, 16, 32, being propounded, 4 times 16 (which is 64) is equal to 2 times 32, which is likewise 64.

Here endeth the first Book, which containeth all that is absolutely necessary, for the full understanding of *common or practical Arithmetick*. Such as desire to see how the same is performed by artificial, or borrow'd numbers, called *Logarithms*, may peruse Mr. *Wingate's Second Book*, being a distinct *Treatise of Artificial Arithmerick*.

A N

APPENDIX,

CONTAINING

Choice Knowledge in *Arithmetick*, both *Practical* and *Theoretical*; the Contents whereof are express'd in the following Page.

Composed by *John Kersey*.

Teacher of the

MATHEMATICKS,

Vox audita perit, litera scripta manet.

A N
APPENDIX

CONTAINING

Choice Knowledge in A-
rithmetic, both Practical
and Theoretical; the Con-
tents whereof are ex-
hibited in the following
Page.

Composed by John Kersey

Teacher of the

MATHEMATICKS

For which Part, there is given a Reward

THE CONTENTS OF THE APPENDIX.

CHAP.

1. OF Contractions in the Rule of Three.
2. Of Rules of Practice by aliquot Parts.
3. Of Exchanges of Coins, Weights and Measures.
4. Practical questions about Tare, Tret, Loss, Gain, Barter, Factorship, and measuring of Tapestry.
5. Of Interest of Money, and the construction of Tables to value Annuities, &c.
6. A demonstration of the Rule of Three.
7. A demonstration of the Double Rule of Fellowship.
8. A demonstration of the Rule of Alligation: Where also of the Composition of Medicines.
9. A demonstration of the Rule of False.
10. A Collection of choice Questions to exercise all the parts of vulgar Arithmetick, to which also are added various practical Questions, about the Mensuration of Superficial Figures and Solids, with the Gauging of Vessels.
11. Sports and Pastimes.

An Explication of such Notes or Characters, which for brevity sake are used in this APPENDIX.

THis $+$ is a Note of *Addition*, signifying that the number which followeth such sign is to be added to the number preceeding it; so $3 + 4$ implieth that 4 is to be added to 3: Sometimes also when no number is placed next after the said note, it implieth that the number preceeding is not exactly expressed; so the square root of two is $1.414 +$ or 1.414 &c. and is, 1.414 and somewhat more.

This $-$ is a sign of *Subtraction*, signifying that the number which followeth such sign is to be subtracted from the number preceeding it; so $6 - 2$ signifieth the difference between 6 and 2, or 2 to be subtracted from 6.

This $\times$ is a sign of *Multiplication*, signifying that the number which preceedeth such sign is to be multiplied into, or by the number following the sign; So 3×4 implieth that 3 is to be multiplied by 4; likewise by $3 \times 4 \times 8$ is understood the continual multiplication of the numbers 3, 4, and 8; viz. 3 is to be multiplied by 4, and the Product is to be multiplied by 8. Sometimes also the said sign hath reference to as many of the preceeding or following numbers as have a little line placed over them; so $3 \times 2 \times 6$ or $2 \times 6 \times 3$ signifieth that 3 is to be multiplied by the sum of 2 and 6. Likewise

wise $8 \text{ --- } 5 \times 3$, or $3 \times 8 \text{ --- } 5$ implyeth that 3 is to be multiplied by the difference between 8 and 5. Moreover if A and B represent two numbers, then $A \times B$ or $A B$ implyeth the product of the multiplication of those numbers: Likewise $B = C \times A$ signifyeth the product arising from the multiplication of the excess of the number B above the number C, by (or into) the number A. Again, if A B and A C represent two lines, then $= A B \times A C$ implyeth a rectangular Figure or long square made of the lines A B and A C.

Numbers placed as you see in the 3) 18 (6 Margin, denote a *Divisor*, a *Dividend*, and a *Quotient*, to wit 3 the *Divisor*, 18 the *Dividend*, and 6 the *Quotient*; the like is to be understood of other numbers so placed.

Numbers placed after the manner of a *fraction* denote a *Quotient*, which ariseth from dividing the

$$\frac{2 \times 5 \times 6}{3 \times 4}$$

Numerator by the Denominator; so $\frac{2 \times 5 \times 6}{3 \times 4}$ is equal

to the *Quotient*, which ariseth from dividing the Product of the continual Multiplication of 2, 5 and 6 by the Product of 3 multiplied by 4.

Four numbers placed as you see in 2 4::6.12 the Margin are Geometrical proportionals, viz. As 2 is to 4, so is 6 to 12: or if 2 give 4, then 6 will give 12. Sometimes also they are placed thus, 2....4....6....12.

This $=$ is a note of equality or equation; so by $3 + 4 = 5 + 2$ is signified that the sum of 3 and 4 is equal to the sum of 5 and 2: Also $7 - 3 = 9 - 5$ signifyeth that the difference between 7 and 3 is equal to the difference between 9 and 5; that is, 7 lessened

lessened by 3 leaves the same remainder, as 9 lessened by 5. Also $4 \times 3 = 12$ implieth that the product of the multiplication of 4 by 3 is equal to 12.

➤ This is a sign of *Majority*, signifying that the number on the left hand of such sign is greater than the number on the right hand thereof; so 5

➤ 3 implieth that 5 is greater than 3.

◁ This is a sign of *Minority*, signifying that the number on the left hand of such sign is less than the number on the right hand thereof; so 3 ◁ 5 implieth that 3 is less than 5.

This Character $\sqrt{}$ or $\sqrt{q}$ signifies the square root of the number which follows it, so $\sqrt{144}$ implies the square root of 144, to wit 12.

Also this $\sqrt[3]{}$ signifies the cube root of the number which follows it: So $\sqrt[3]{1728}$ signifies the cube root of 1728, which cube root will be found to be 12.



A N

APPENDIX.

C H A P. I.

Of Contractions in the Rule of Three.

SUCH as are well vers'd in the parts of *Arithmetick*, which have been fully laid open in the precedent Book, and are mindful of the *Notes* or *Symbols* before explained, will find no difficulty in the 1, 2, 3, 5, and 10 Chapters of this *Appendix*, wherein divers compendious operations no less delightful than useful, are methodically handled, and the rest will be as easie to such as are but meanly acquainted with *Geometrical Demonstration*.

II. To repeat the brief ways of *Multiplication* set forth in the 10, 11, and 12 *Rules* of the fifth Chapter, or those of *Division*, in the 11, 15, and 16 *Rules* of the

the sixth Chapter aforegoing, would be a superfluous work, and therefore I shall presuppose the Reader to be thoroughly acquainted with them, as also with competent knowledge in the operations of Fractions both vulgar and decimal.

III. It will be no small advantage to the Practical Arithmetician, to have by heart not only the common Table of Multiplication,

but this also in the Margin, to the end that when a number is given to be multiplied or divided by 12, (which happens in the Reduction of shillings to pence and the converse) the product or quotient may be written down in one line only, as in the Examples following.

2	} $\times 12 =$ }	24
3		36
4		48
5		60
6		72
7		84
8		96
9		108

3472	4736
12	12
12) 41664 (3472	12) 56832 (4736

IV. When a whole number is given to be divided by a Divisor, which is equal to the product of the Multiplication of two single figures, instead of dividing by that Divisor you may first divide by one of those single figures, and then divide the quotient by the other, so will the last quotient be the same as if the Division had been finished by the Divisor first given: Thus if 3466 farthings be given to be reduced to shillings because $8 \times 6 = 48$, I first divide 3466 by 8,

4) 3463 4d. which with the 3 pence
 before remaining make 288
 s. d. s. 7 d. which is the same
 3) 865 (288 7 with the Quotient, when
 3463 pence are divided by
 12, for $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$.

V. In the Rule of Three as well direct as inverse when the Divisor with either of the two given numbers may be severally divided by some common measure, without leaving any remainder, the Quotients may be taken for new terms, and proceeding in like manner as often as is possible, the operation according to the tenth Rule of the eighth Chapter, or the second Rule of the ninth Chapter, will be much contracted: So if it be demanded what 52 yards of Cloth will cost at the rate of 21^l. for 14 Yards; the Answer will be found 78 pounds in manner following.

y.	l.	y
14...	21...	52
2...	3...	52
1...	3...	26... (78.

In the first rank you may observe that the Divisor 14 and the second term 21, being severally divided by their common measure 7, the three new terms (in the second rank) will be 2, 3, 52. Again, in the second rank, the divisor 2 and the third term 52 being severally divided by their common measure 2, the three new terms (in the third rank) will be 1, 3, 26. Lastly working with these according to the Rule of Three direct, the Answer to the question (or fourth term) will be found to be 78.

Another

Chap. I.

The Rule of Three

313

Another Example, if 21 men will finish a work in 16 Days, what time must be allowed to 12 men for the finishing of such a work? *Answer*, 28 days.

men days men

21 . . . 16 . . . 12

7 . . . 16 . . . 4

7 . . . 4 . . . 1 (28 days)

In the first rank you may observe, that the Divisor 12 (for the Rule is inverse) and the first term 21 being severally divided by their common measure 3, the three new terms in the second rank, will be 7, 16, 4. Again in the second rank, the Divisor 4, and the second term 16, being severally divided by their common measure 4, the three new terms in the third rank will be 7, 4, 1. Lastly working with these as the Rule of three inverse requires, the *Answer* to the question (or fourth term) will be found 28.

VI. In the Rule of three, as well direct as inverse. when the Divisor and either of the other two terms are fractions having a common Denominator, the said denominators may be rejected, and their numerators retained as new terms: So if it be demanded what is the value of $\frac{3}{4}$ of an Ell, when $\frac{1}{4}$ of an Ell are worth 66 pence, the *Answer* will be found 154 pence, and the work will stand as you see.

$\frac{3}{4}$. . 66 . . $\frac{1}{4}$

3 . . 66 . . 7

1 . . 22 . . 7 (154)

U

Another

Another Example. 3 $\frac{1}{2}$ yards of Scarlet cloth cost 8l. 15 s. what is the price of one yard at that rate? *Answer* 2l. 6s. 8d.

$$\begin{array}{r} 1\frac{1}{2} \dots 3\frac{1}{2} \dots 1 \\ 15 \dots 35 \dots 1 \\ 3 \dots 7 \dots 1 \dots (2\frac{1}{2} l. \end{array}$$

VII. In the Rule of Three as well direct as inverse when the Divisor only is a fraction either of the other two terms may be reduced to a fraction of the same Denominator, and then the Denominators may be rejected, as before in the sixth Rule. Also when one of the three given terms is a fraction, and is not the Divisor, the Divisor may be reduced to a fraction of the same Denominator with the fraction first given, and then the common Denominators may be likewise cancelled.

An Example of the first Case may be this, if $\frac{1}{4}$ of a yard cost 1 s. what is the price of 1 yard? *Answer* 16 shillings.

$$\begin{array}{r} \text{yard} \quad \text{shill.} \quad \text{yard.} \\ \frac{1}{4} \dots 14 \dots 1 \\ \frac{1}{4} \dots 14 \dots 1 \\ 7 \dots 14 \dots 8 \dots (16 \text{ shill.} \end{array}$$

An Example of the second Case; if of stuff which is $\frac{1}{4}$ of a yard in breadth, 7 yards in length will make a Garment; how much of that stuff which is one yard in breadth will be sufficient for the same purpose? *Answer* 5 $\frac{1}{2}$ yards.

Chap. II. Rules of Pract. by Aliq. part 315

Rules of 3 Inverse

$\frac{3}{4} \dots 7 \dots \frac{1}{4}$ or $\frac{1}{4}$ or $\frac{1}{4}$

CHAP. II.

Rules of Practice by Aliquot parts.

I. **A**N Aliquot part takes its name from the *Latin* word *aliquoties*, for (according to *Euclid*) an aliquot part is of a greater number such a part, which being taken (*aliquoties* or) certain times doth precisely constitute that greater number; so 3 is an aliquot part of 12, for 3 taken 4 times doth exactly make 12 without an excess or defect; in like manner 4 is an aliquot part of 20 because 4 taken 5 times doth precisely make 20; but 7 is not an aliquot part of 20, for 7 taken twice doth want of 20, and being taken thrice doth exceed 20; this kind of part last mentioned is by *Euclid* called *pars aliquanta* of which there will be no use in this place.

II. When the Rule of Three Direct hath 1 or an Integer for the first Term, it is commonly called the Rule of Practice, either from the great use and practice thereof in common affairs, or else for that questions of this nature, may be resolved by operations more speedy and practical than those of the Rule of Three.

III. The choicest of these Rules of Practice may be reduced to 5 Cases, viz.

- When the price of 1 or an Integer consists,
1. Of shillings under 20.
 2. Of pounds and shillings
 3. Of pence under 12:
 4. Of shillings and pence.
 5. Of pounds, shillings, pence, with parts of a penny.

All which cases, with others of the like nature, are handled in their order.

IV. Any even number of shillings is either $\frac{1}{2}$ of a pound (that is 2 shillings,) or else is composed of $\frac{1}{2}$ l. (to wit 2 s.) taken certain times: So 8 s. is composed of $\frac{1}{2}$ l. (or 2 shillings) taken four times, in like manner 18 s. is composed of $\frac{1}{2}$ l. taken nine times.

V. When the price of 1, or an Integer of what name soever, 2 is shillings, the price of as many Integers as one will of that name is discoverable at first sight, to wit by accounting the double of the figure which stands in the first place (towards the right hand) of the said number of Integers, as shillings, and the rest of the said number as pounds:

yard	shill.	yards	so 345 yards at two shillings
1 . . . 2 . . . 345			the yard will cost 34 l. 10 s.

for the double of 5 is 10, which I write down apart as shillings, then esteeming the remaining figures towards the left hand, to wit 34, as an entire number of pounds, the Answer will be 34 l. 10 s. This contraction is nothing else, but dividing the number

ber of Integers, whose price is required by 10.

More Examples hercof are these;

oz. shill. oz.
1 2 2057

l.
Answ. 205 14

yard shill. yards.
1 2 120

l. s.
Answ. 12 0

VI. When the given price of 1 or Integer is any even number of shillings greater than two shillings, multiply the number of Integers, whose price is required, by half the given number of shillings, with this caution, that the double of the figure which ariseth in the first place of the product be written apart as shillings, and the rest of the product as pounds: So if it be demanded what 218 yards at 8 shillings the yard will amount unto, the Answer will be found

87 4 for 1 multiply y. s. y.
218 by 4 (which is half 8 1 8 218
the given number of shil- 4
lings) saying, 4 times 8 is 32, here the double of 2 87 4
(to wit, of that figure which is to possess the first place in the product) is 4, which I set apart as shillings, keeping 3 in mind for the three tens; again 4 times 1 is 4, which

U 3

with

with 3 in mind makes 7; lastly, 4 times 2 makes 8, so I conclude that the Answer to the question is 87l. 4s. The reason of this contraction is evident from the fourth and fifth Rules foregoing. More examples of this Rule are these following.

yard	s.	yards.
1	...	14 ... 436

Ans. 305 ... 4

yard	s.	yards.
1	...	18 ... 230

Ans. 207 ... 0

VII. Any odd number of shillings is either compos'd of $\frac{1}{2}$ l. (or 2s.) and of $\frac{1}{4}$ l. (or 1s.) or else it is compos'd of $\frac{1}{2}$ l. (or 2s.) taken certain times and of $\frac{1}{4}$ l. (or 1s.) So 3s. is compos'd of 2s. and 1s. Also 7s. is compos'd of 2s. taken three times and of 1s. Likewise 13s. is compos'd of 2s. taken six times, and of 1s.

VIII. When the given price of 1 or an Integer is an odd number of shillings, work for the greatest even number of shillings contained in that odd number, according to the fifth or sixth Rule foregoing; then for the odd shillings remaining, take $\frac{1}{2}$ of the number of Integers, whose price is required (by the 16th Rule of the sixth Chapter of the preceeding Book.) These two results added together give the Answer to the question

Question : So if it be demanded what 2344 ounces at 13 s. the ounce will cost, the answer will be found 1523 l. 12 s. For if (according to the sixth Rule of this Chapter)

I multiply 2344 by 6, oz. shill. oz.
(to wit, by half the remainder, when one is abated from 13 the

given number of shillings) there will arise

1406 l. 8 s. Then taking $\frac{1}{2}$ of 2344, there

will arise 1172 l. 4 s. which being added to the former product

gives 1523 l. 12 s. for the answer to the question.

Note, When 5 shillings is the given price of 1 or an Integer, the briefest way will be to take $\frac{1}{4}$ of the number of Integers, whose value is required for such quotient will give the pounds and shillings, which answer the question : so 2347 ounces at 5 s. the ounce amount unto 586 l. 15 s. for $\frac{1}{4}$ of 2347 is 586 $\frac{3}{4}$ or 586 l. 15 s. But when the given price of 1 is any other odd number of shillings, this eighth Rule will be as compendious as any other whatsoever.

More Examples of this Rule are these following

yard shill. yards.

1 ... 19 ... 739

l. s.

665 ... 2

36 ... 19

Ans. 702 ... 1

4

yard

yard shill. yards
1...17...348

l. s.
276...0
17...5

Ans. 293...5

+ IX. When the given price of 1 or an Integer consists of pounds and shillings, first multiply the number of Integers whose price is required, by the number of pounds in the said given price, and subscribe the product as pounds; then proceed with the shillings in the said given price, according to the sixth or eighth Rule of this Chapter, and having subscribed that which ariseth under the afore-said product of pounds, add them all together for the answer of the question: So if it be demanded what 328 hundred weight will amount unto at 2 l. 17 s. per C. (or one hundred weight) the answer will be found to be 934 l. 16 s. as by the operation is evident.

C. l. s. C.
1...2 : 17...328

l. s.
656...0
262...8
16...8

Ans. 934 : 16

More

More Examples to illustrate this Rule are these following:

C. l. s. C.
1...7 : 12...504

l. s.
3528..0
302..8

Answ. 3830..8

C. l. s. C.
1...5 : 7...129

l. s.
645..
38..14
6..9

Answ. 690..3

X. Any number of pence under 12 is either an Aliquot part of a shilling, or else compos'd of Aliquot parts thereof; so 3 pence is an Aliquot part, to wit, $\frac{1}{4}$ of a shilling. Likewise 4 is $\frac{1}{3}$ of 12; moreover 5 pence are compos'd of 2 Aliquot parts, to wit, of 3 pence, which is $\frac{1}{4}$ of a shilling, and of 2 pence which is $\frac{1}{6}$ of a shilling; all which will readily appear by the following Table.

Pence

Pence.	Aliquot parts of a Shilling.
1	$\frac{1}{20}$ (or $\frac{1}{2}$ of $\frac{1}{10}$)
$1\frac{1}{2}$	$\frac{1}{15}$
2	$\frac{1}{10}$
3	$\frac{1}{8}$
4	$\frac{1}{6}$
5	$\frac{1}{4} + \frac{1}{20}$
6	$\frac{1}{5}$
7	$\frac{1}{4} + \frac{1}{10}$
8	$\frac{1}{3} + \frac{1}{10}$
9	$\frac{1}{3} + \frac{1}{12}$
10	$\frac{1}{2} + \frac{1}{20}$
11	$\frac{1}{2} + \frac{1}{10} + \frac{1}{20}$

XI. When the given price of 1 or an Integer is an Aliquot part of a shilling, divide the number of Integers whose value is required by the denominator of such aliquot part; so will the quotient be the number of shillings which answer the question, which number of shillings (when there is occasion) may be reduced to pounds by the brief way of dividing by 20: So if it be required to know what 2686 ounces at 4 pence the ounce will amount

amount unto; the answer will be found 44 l. 15 s. 4 d. for since 4 d. is an aliquot part, to wit, $\frac{1}{3}$ of a shilling, I divide 2686 by 3, so will the quotient be 895 $\frac{1}{3}$ or 895 s. 4 d. which shillings being divided by 20, give 44 l. 15 s. 4 d. for the answer to the question, as you see by the following operation.

$$\begin{array}{r}
 02. \quad \frac{1}{3} \text{ d.} \quad 02. \\
 2686 \\
 \hline
 895 \text{ s.} \quad 4 \text{ d.} \\
 \hline
 \text{Answ. } 44 \text{ l. } 15 \text{ s. } 4 \text{ d.}
 \end{array}$$

More Examples of this Rule are these following.

$$\begin{array}{r}
 \text{yard} \quad \text{d} \quad \text{yards.} \\
 20 \times 37 \mid 9 \dots 6 \\
 \hline
 \text{Answ. } 18 \quad 19 \quad 6
 \end{array}$$

$$\begin{array}{r}
 \text{yard} \quad \text{d} \quad \text{yards.} \\
 1 \times 204 \dots 204 \\
 \hline
 \text{Answ. } 17 \text{ shillings.}
 \end{array}$$

XII. When the given price of an Integer is compos'd of aliquot parts of a shilling, divide the number of Integers, whose price is required, by the several denominators of the aliquot parts contained in the given number of pence, then add the quotients

ents together, and the sum shall be the number of shillings which answer the question. So if it be demanded what 2347 yards of linnen cloth will cost at 9 pence the yard, the answer will be found 88l.

or. 3d. For since 9d. is compos'd of 6d. and 3d. to wit, of aliquot parts $\frac{1}{2}$ and $\frac{1}{4}$ of a shilling, I first divide 2347 by 2 (the denominator of the aliquot

yard d. yards.

1 ... 9 ... 2347

—————

s. d.

1173 : 6

586 : 9

—————

20) 176 | 0 : 3

l. s. d.

Answ. 88 . 0 : 3

part $\frac{1}{2}$) so there ariseth

1173 $\frac{1}{2}$, or 1173 s. 6d.

Again, dividing the said 2347 by 4 (the denominator of the other a-

liquot part) there will

arise 586 $\frac{1}{4}$, or 586 s. 9d.

which two quotients be-

ing added together give

1760 s. 3d. or 88l. or.

3d. which is the answer

of the question. More

Examples to illustrate this Rule are these.

yard d. yards

1 ... 8 ... 782

—————

s. d.

260 ... 8

260 ... 8

—————

20) 52 | 1 ... 4

l. s. d.

Answ. 25 . 1 . 4

X
fists
of l
num
shill
by t
den
num
Que
ling
num
if i
cost
7 s.
the
fou
for
mu
giv
shil
242
div
and
cau

$$\begin{array}{r}
 540 \\
 180 \\
 180 \\
 135 \\
 \hline
 20) 4915 \text{ s. d.} \\
 \text{Answ. } 24 \dots 15 : 0
 \end{array}$$

XIII. When the given price of an *Integer* consists of *shillings* and *pence*, first multiply the *number* of *Integers* whose value is required by the said given number of *shillings*, and subscribe the product as *shillings*, then divide the said number of *Integers* by the several *Denominators* which are correspondent to the aliquot parts contained in the given number of *Pence*, and subscribe the Quotient or Quotients underneath the aforesaid Product of *shillings*, all which being added together give the number of *shillings* which answer the Question: So if it be demanded what 347 yards of cloth will cost at the rate of

7 s. 10 d. the yard, yard s. d. yards.

the answer will be 1..7:10..347

found 135l. 18s. 2d.

for first 347 being

multiplied by 7 (the

given number of

shillings) produceth

2429 shillings, then

dividing 347 by 2

and 3 severally, (be-

cause 10 d. is com-

$$7 \times 347 = 2429$$

$$2) 347 (.. 173$$

$$3) 347 (.. 115$$

$$20) 2718$$

$$10) 2718$$

$$10) 2718$$

$$\text{Answ. } 135 : 18 : 2$$

pos'd

pos'd of $\frac{1}{2}$ and of $\frac{1}{3}$ a *shilling*) the quotients will be 173 $\frac{1}{2}$ and 115 $\frac{2}{3}$, that is 173s. 6d. and 115s. 8d. Lastly the sum of all is 2718s. 2d. or 135l. 18s. 2d.

More Examples of this kind are these.

$$\begin{array}{r} \text{yard} \quad s. \quad d. \quad \text{yards,} \\ 1 \dots 17 : 9 \dots 540 \end{array}$$

$$\begin{array}{r} 17 \times 540 = \\ 2 \} 540 (\dots \\ 4 \} 540 (\dots \end{array}$$

$$\begin{array}{r} s. \\ \{ 3780 \\ 540. \\ 270 \\ 135 \end{array}$$

$$20 \overline{) 95815}$$

$$l. \quad s. \quad d.$$

$$\text{Answ. } 479 : 5 : 0$$

$$\begin{array}{r} y. \quad s. \quad d. \quad y. \\ 1 \dots 14 : 6 \dots 313 \end{array}$$

$$\begin{array}{r} 14 \times 313 = \\ 2 \} 313 (\dots \end{array}$$

$$\begin{array}{r} s. \\ \{ 1252 \\ 313. \\ 156 : 6 \end{array}$$

$$20 \overline{) 45318}$$

$$\text{Answ. } 226 : 18 : 6$$

XIV. When the price of an Integer consists of shillings and pence, and that such shillings and pence jointly considered do make an aliquot part of a pound, it will oftentimes be a *brief* way than that in the last Rule, to divide the number of Integers, whose value is required, by the *denominator* of such aliquot part, so will the quotient give the answer

answer to the question in pounds and known parts of a pound. Thus if it be demanded what 767 yards will cost at the rate of 6s. 8d. the yard, the answer will be found 255 l. 13 s. 4 d. For since 6s. 8d. is an aliquot part, to wit,

$\frac{1}{3}$ of a pound, I divide y. s. d.

767 by 3, so there arith-

feth in the quotient

255 $\frac{2}{3}$ or 255 l. : 13 s. 4 d. which is the an-

swer of the question.

Note that the Aliquot parts of a pound convenient for this Rule are these expressed in the following Table.

sh. d.	Aliquot parts of a Pound.
6 . . 8	$\frac{1}{3}$
3 . . 4	$\frac{1}{6}$
2 . . 6	$\frac{1}{4}$
1 . . 8	$\frac{1}{12}$
1 . . 4	$\frac{1}{24}$
1 . . 3	$\frac{1}{36}$

XV. When the given price of 1 or an Integer consists of pounds, shillings and pence, reduce the said pounds and shillings all into shillings, then proceed according to the 12 Rule of this Chapter: So 517 C. at 3l. : 17s. 5d. per C. will be found to amount unto 2001l. 4s. 5d. for having reduced 3l. 17s. into 77s. I multiply 517 by 77, and write down the particular

particular products; then for the 5 pence which is compos'd of the aliquot parts $\frac{1}{4}$ and $\frac{1}{8}$ of a shilling, I take $\frac{1}{4}$ and $\frac{1}{8}$ of 517, and subscribe the quotients orderly underneath the aforesaid products: Lastly, adding all together the sum 40024s. 5d. or 2001l. 4s. 5d. for the answer of the question.

$$\begin{array}{r} C \quad L \quad s. \quad d. \quad C. \\ 1 \dots 3 : 17 : 5 \dots 517 \end{array}$$

$$\begin{array}{r} 77 \times 517 = \left\{ \begin{array}{l} 3619 \\ 3619 \end{array} \right. \\ 4) 517 \dots 129 : 3d. \\ 6) 517 \dots 86 : 2 \end{array}$$

$$20) 40024 : 5$$

$$\text{Answ. } 2001 : 4 : 5$$

More Examples of this Rule are these following.

$$\begin{array}{r} C \quad L \quad s. \quad d. \quad C. \\ 1 \dots 5 : 13 : 8 \dots 108 \end{array}$$

$$114 \times 108 = \left\{ \begin{array}{l} 324 \\ 108 \\ 108 \end{array} \right.$$

$$3) 108 \dots 36$$

$$36$$

$$20) 12276$$

$$\text{Answ. } 613 : 16 : 0$$

N
fists
be co
ber o
of an
Then
tegers
take
s; be
corre
follow

$$\begin{array}{r} \text{C.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{C.} \\ 1 \dots 2 : 10 : 6 \dots 84 \end{array}$$

$$50 \times 84 = 4200$$

$$\begin{array}{r} 82 \overline{) 84} (42 \\ 164 \\ \hline 20 \overline{) 424} (212 \end{array}$$

$$\begin{array}{r} \text{C.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{C.} \\ 1 \dots 1 : 12 : 4 \frac{1}{2} \dots 306 \end{array}$$

$$32 \times 306 = \left\{ \begin{array}{l} 612 \\ 918. \end{array} \right.$$

$$\begin{array}{r} 3 \overline{) 306} (\dots 102 \\ 48 \overline{) 306} (\dots 6 : 4 \frac{1}{2} \end{array}$$

$$20 \overline{) 990} : 4 \frac{1}{2}$$

$$\text{Answ. } 495 : 0 : 4 \frac{1}{2}$$

Note, When the given price of an Integer consists of certain Pence together with $\frac{1}{2}d.$ or $\frac{3}{4}d.$ it will be convenient to take due aliquot parts of the number of Integers propounded for all the given price of an Integer, except 1 d. and the said $\frac{1}{2}d.$ or $\frac{3}{4}d.$ Then for that Penny, and $\frac{1}{2}d.$ take $\frac{1}{2}$ of the said Integers propounded, and if there be yet a farthing, take $\frac{1}{4}$ of the said quotient which ariseth by taking $\frac{1}{2}$; both which quotients give the value in shillings correspondent to $1 \frac{1}{2}d.$ This will be evident by the following Examples.

yard d. yards
1 ... 8 $\frac{1}{2}$... 326

		s.	d.
3)	326 (..	108	.. 8
4)	326 (..	81	.. 6
8)	326 (..	40	.. 9
6)	40 (..	6	.. 8
6)	9 (..	0	.. 1 $\frac{1}{2}$

20) 23 | 7 .. 8 $\frac{1}{2}$

l. s. d.
Answ. 11 : 17 : 8 $\frac{1}{2}$

s. d.
1 ... 3 : 6 $\frac{1}{2}$... 720

	s.
3 x 720 =	2160
4) 720 (::	180
6) 720 (::	120
8) 720 (::	90

l. s. d.
20) 255 | 0 (127 : 10 : 0

XVI. When the price of an Integer is given, and the price of many Integers of the same name together with $\frac{1}{2}$ or $\frac{1}{4}$ or $\frac{3}{4}$ of an Integer is required, the value of those Integers may be first found by some of the precedent Rules, and then for the price of $\frac{1}{2}$ of an Integer, take $\frac{1}{2}$ of the given price of an Integer; likewise for $\frac{1}{4}$ of an Integer, take

Chap. II.

by Aliquot parts.

331

of the said given price ; also for $\frac{1}{2}$ of an *Integer* take the composed of $\frac{1}{2}$ and $\frac{1}{4}$ of the said given price : So if it be demanded what 34 C. 3 qu. (to wit, 34 hundred weight, and $\frac{3}{4}$ of an hundred weight) of Sugar will cost at 4 l. 16 s. 3 d. per C. the Answer will be found 167 l. 4 s. 8 d. as by the subsequent operation is manifest.

C. l. s. d. C. q.
1...4 : 16 : 3...34 : 3.

$$96 \times 34 = \begin{cases} 204 \\ 306 \end{cases}$$

4) 34 C... 8...6

The Quotients $\left\{ \begin{array}{l} \frac{1}{2} C. \{ 48...1\frac{1}{2} \\ \frac{1}{4} C. \{ 24...0\frac{3}{4} \end{array} \right.$
for

20) 334 1/4 ... 8 1/4.

l. s. d.
Ans^r. 167...4...8 1/4.

An Example of *Avoirdupois* greater Weight; where the quantity whose Price is sought consists of entire Hundred Weights, quarters of an Hundred, and of some number of Pounds, which is not an Aliquot Part of 28 or $\frac{1}{4}$ C.

$218 \times 218 =$		1000		
		218		
		218		
2) 218 (..		109	d.	far.
8) 218 (..		27	: 3	: 0
$\frac{1}{8}$ of 27s. 3d.		4	: 6	: 2
$\frac{1}{2}$ C.		57	: 9	: $3\frac{1}{2}$
$\frac{1}{4}$ C.		28	: 10	: $3\frac{3}{4}$
The quotients arising for		14	: 5	: $1\frac{1}{4} +$
7 lb.		7	: 2	: $2\frac{1}{4} +$
3 lb.		3	: 1	: 0 +
		20) 253212	: 3	: 2 +
			s.	d.
Answ.		1266	: 2	: $3\frac{1}{2} +$

The example last mentioned being (of those questions which ordinarily happen in Trade) one of the hardest to be resolved by the Rule of Practice, I shall touch upon the foregoing operation, where you may observe the price of 218 C. 3 qu. to be found after the manner of former Examples; then the 14 lb. part of the 24 lb. in the question, I take $\frac{1}{2}$ of the price of $\frac{1}{2}$ C. Likewise for 7 lb. I take half the price of 14 lb. and so there yet remains 3 lb. whose price is found by taking $\frac{1}{3}$ of the price of 7 lb. viz. the price of 7 lb. being very near 7s. $2\frac{1}{2}$ d. or $86\frac{1}{2}$ d. I multiply $86\frac{1}{2}$ by 3, and divide the quotient by 7, so there ariseth 37d. or 3s. 1d. very near. Lastly, all being added together, the sum is found to be

Chap. II.

by Aliquot parts.

333

be very near 15322 s. 3 $\frac{1}{2}$ d. or 1266 l. 2 s. 3 $\frac{1}{2}$ d.
Note, that a quarter of a farthing (or $\frac{1}{4}$ of a Penny) is the smallest money express in the example; and where any thing ariseth less than a quarter of a Farthing it is omitted, but it is supposed to follow this note +, for which surpluses some respect ought to be had in adding all together: Now albeit, in resolving questions after this practical manner there will be some error, yet the loss for the most part will be less than a Farthing which is inconsiderable.

XVII. When the price of 1 or an Integer consists of divers Denominations, as Pounds, shillings, Pence; and the price of a certain number of Integers, which exceeds not a single figure, is required, work as in the following Example, viz. If it be required to find what 8 C. must cost at 3l. 13s. 7 $\frac{1}{2}$ l. per C. it is evident that 8 C. must cost 8 times 3l.

C.	l.	d.	C.
1	3	13	8
		7	
		8	

Ans 29 : 9 : 0

13s. 7 $\frac{1}{2}$ l. therefore I multiply $\frac{1}{2}$ by 8, saying, 8 half Pence make 4 Pence, which I reserve in mind; again, 8 times 7 Pence make 4s. 8d. (to wit, 8 six-Pences make 4s. and there are 8 Pence besides) to which adding 4 pence in mind, there will arise 5s. which I reserve in mind, and subscribe a cypher under the place of Pence; again, I say 8 times 13 Shillings make 5l. 4s. (to wit, 8 Angels make 4l. and 8 times 3 s. make 1l. 4s.) to which adding 5 s.

X 3

in

in mind, the sum will be $5\text{ l. } 9\text{ s.}$ wherefore I subscribe 9 s. (the excess above the pounds) under the shillings, and keep 5 l. in mind; lastly, I say 8 times 3 pounds make 24 pounds, which with 5 pounds in mind make 29 pounds; so that the total product or answer of the question is found to be $29\text{ l. } 9\text{ s.}$

More Examples of this kind are these.

C. l. s. d. C.

1. 17. 15. 5. 7

7

Ans. 124 : 8 : 04

C. l. s. d. C.

1. 18. 12. 6. 8

8

Ans. 149 : 00 : 6

XVIII. When the price of 1 lb. weight is known, and the price or value of 1 C. (to wit 112 lb.) is required, the answer may sometimes be given more speedily than by any of the former Rules, by this Rule which follows, viz. Find the number of farthings contained in the given price of 1 lb. weight, then take twice the number of shillings, and once that number of groats, and having added them together, the sum will give the value of 1 C. to wit, 112 lb. weight: So if it be demanded what 1 C. or 112 lb. weight of Cheese will cost at the rate of $3\frac{1}{4}$ pence the pound weight, the answer will be $1\text{ l. } 10\text{ s. } 4\text{ d.}$

For

For according to the said Rule, the number of farthings contained in 3 $\frac{1}{4}$ l. (the price of 1 pound weight) is 13, therefore the double of 13 shil-

lings is...

13 Groats make ...
Therefore the sum (which is the price of 1 C, or 112 lb. weight) is ...

$$\begin{array}{r} 13 \\ 26 \\ \hline 39 \end{array} \begin{array}{r} s. \\ d. \\ \hline 6 \\ 4 \end{array}$$

The reason of this Rule is evident, for if 1 lb. weight cost 13 farthings, then 112 lb. must necessarily cost 112 times 13 farthings, or (which is the same) 13 times 112 farthings; but 13 times 112 farthings are equal to twice thirteen shillings together, with once thirteen groats, because 112 farthings are composed of twice 48 farthings (or two shillings) and of 16 farthings (or one groat;) wherefore the truth of the said Rule is evident.

Another Example, when Sugar is at 5 $\frac{1}{2}$ the pound weight, what is the value of 1 C (or 112 l. weight?)

Ans^r. 2l. 11 s. 4 d. For in 5 $\frac{1}{2}$ l.

are contained 22 farthings, therefore the double of 22 shillings is...

22 groats make ...

Which added together give the price of 1 C, or 112 l. to wit.

$$\begin{array}{r} 22 \\ 44 \\ \hline 66 \end{array} \begin{array}{r} l. \\ s. \\ d. \\ \hline 2 \\ 11 \\ 4 \end{array}$$

XIX. When the gain of (or allowance for) 100 Integers consists of some number of pounds not exceeding 10, the gain of as many like Integers and known parts of an Integer as one will, may be found very briefly by the following method, viz. If 100 l. gain 3 l. what is the gain

*Compendious
ways of compu-
ring Interest
and Factors
allowances.*

gain of 246*l.* 18*s.* 10*d.*) *Answer* 7*l.* 8*s.* 4*d.*
First, I multiply 246*l.* 18*s.* 10*d.* by 3 (the second term) after the manner delivered in the 17 Rule of this Chapter, and write down the *Product* which is 746*l.* 16*s.* 6*d.* Then I divide the said *Product* by 100 (the first term in this *Rule of Three*) in this manner, *viz.* I divide 740 pounds by 100, which is performed by cutting off towards the right hand,

Chap. II.

by Aliquot parts.

337

tient gives 1 penny, and there will remain 98 pence; so the exact quotient or answer of the question is found to be 7l. 8s. 1 $\frac{82}{100}$ d.

More Examples of this Rule are these following.

$$\begin{array}{r} \text{l.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \\ 100 \dots 6 \dots 793 : 12 : 7 \\ \hline 6 \end{array}$$

$$\begin{array}{r} \text{l.} \quad 47 \mid 61 : 15 : 6 \\ \hline 20 \end{array}$$

$$\begin{array}{r} \text{s.} \quad 12 \mid 35 \\ \hline 12 \end{array}$$

$$\begin{array}{r} \text{d.} \quad 4 \mid 26 \\ \hline 4 \end{array}$$

$$\begin{array}{r} \text{l.} \quad 100 : 78 : 43 : 14 : 3 \\ \hline 100 : 78 : 43 : 14 : 3 \end{array}$$

$$\begin{array}{r} \text{s.} \quad 3 \mid 49 : 14 : 0 \\ \hline 26 \end{array}$$

$$\begin{array}{r} \text{s.} \quad 9 \mid 94 \\ \hline 12 \end{array}$$

$$\begin{array}{r} \text{d.} \quad 11 \mid 28 \\ \hline 11 \end{array}$$

After the same manner may this following question and such like be resolved, viz. When 100 Ells of Linnen cloth cost 30l. 18s. 9d. what is the price of 1 Ell? Answer 6s. 2d. 1 farth.

Ells

Ells. l. s. d. Ell.
 100 : 30 : 18 : 9 : 1
 20

Shil. 6 | 18
 | 12

Pence 2 | 25
 | 4

Farth. 1 | 00

XX. When the given gain of (or allowance for) 100 Integers consists of some number of Pounds not exceeding 10, together with some Aliquot part or parts of a pound, the operation will be little different from the last mentioned Examples, as may appear by the resolution of the subsequent question, viz. What must be allowed for 2156 l. 13 s. 4 d. at the rate of 6 l. 15 s. for 100 l.? *Answer* 145 l. 11 s. 6 d. thus found; I first multiply the said 2156 l. 13 s. 4 d. by 6 (the number of Pounds in the given allowance 6 l. 15 s.) after the manner of the last examples, and subscribe the Product which is 12940 l. underneath the line as you see; then since 15 s. are equal to $\frac{3}{4}$ l. together with $\frac{1}{4}$ l. I take $\frac{3}{4}$ of 2156 l. 13 s. 4 d. which is 1078 l. 6 s. 8 d. likewise $\frac{1}{4}$ of the said 2156 l. 13 s. 4 d. to wit, 539 l. 3 s. 4 d. and having subscribed these quotients underneath the Product first found, and added them all together, I find 14557 l. 10 s. 0 d. for the total product with which I proceed as in the former Examples, and so at length the *Answer* is found to be 145 l. 11 s. 6 d. View diligently the operation

$$\begin{array}{r} 1. \quad 1. \quad 1. \quad 1. \quad 1. \quad 1. \\ 100 : .6\frac{1}{2} :: 2156 : 13 : 4 \end{array}$$

$$12940 : 00 : 0$$

$$1078 : 06 : 8$$

$$539 : 03 : 4$$

$$\begin{array}{r} 1. \quad 145 \quad | \quad 57 : 10 : 0 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 1. \quad 11 \quad | \quad 50 \\ \hline 12 \end{array}$$

$$d. \quad 6 \quad | \quad 00$$

CH A P. III.

Concerning Exchanges of Coins, Weights, and Measures.

TH E rate or proportion between Coins, Weights, &c. of different kinds being known, either from some good Author, or rather by experience; it will not be difficult, to such as understand the Rule of Three, to know how to exchange a given quantity of one kind, for a quantity of the same value in another kind. But since in some cases the common way of working may be much contracted,

tracted, I shall endeavour to shew the most compendious ways to perform this business.

II. In exchanging of things of different kinds (whether they be *Coins* or *Weights*, &c.) when two things of different kinds are compared together, the question may be resolved by one single *Rule of Three*, as will be evident by the subsequent Examples; viz.

Quest. 1. How many *Riders* at 21s. 2 $\frac{1}{2}$ d. sterling the piece ought to be received for 251 l. 6 s. 4 d. of sterling Money? *Answer*, 237 *Riders*. For the first and third terms in the *Rule of Three*, which arise from this question, being converted into half pence, the proportion will be thus,

$$509. 1 :: 120633. 237$$

Quest. 2. If 100 *Ells* of *Antwerp* make 75 yards of *London*, how many yards of *London* measure will 27 *Ells* of *Antwerp* make? *Answer* 20 $\frac{1}{4}$ yards.

$$100. 75 :: 27. 20 \frac{1}{4}$$

III. When more than two different *Coins*, *Weights*, *Measures*, &c. are compared together, viz. when one kind of *Coin* is compared with a second of another kind, that second with a third; the third with a fourth; the fourth with a fifth, &c. two different cases are ordinarily raised from such comparison, viz.

1. How many pieces of the first *Coin* are equal in value to a given number of pieces of the last *Coin*: Or, It may be required to know.
2. How many pieces of the last *Coin* are equal in value to a given number of pieces of the first kind of *Coin*? *An*

An Example of the first Case.

If 35 ells of *Vienna* make 24 ells of *Lyons*; 3 ells of *Lyons* 5 ells of *Antwerp*; and 100 ells of *Antwerp* 125 ells of *Frankfort*; how many ells of *Vienna* are equal unto 50 ells at *Frankfort*? Answer, 35 ells of *Vienna*.

For the more easie understanding of the resolutions of this question and others of like nature, Let *a* represent an ell at *Vienna*; *b* an ell at *Lyons*; *c* an ell at *Antwerp*, and *d* an ell at *Frankfort*; then may the given terms in the question be stated in the following order.

$$\begin{array}{lcl}
 \text{Suppositions} & \left\{ \begin{array}{l} 35 a = 24 b \\ 3 b = 5 c \\ 100 c = 125 d \end{array} \right. \\
 \text{The question} & 50 d = ? a
 \end{array}$$

Which order of placing the said given numbers (or terms) being observed, it appears that if 35 *a* be accounted to stand in the first place: 24 *b* in the second: 3 *b* in the third; 5 *c* in the fourth; 100 *c* in the fifth, &c. then all the terms which stand in odd places, to wit, the first, third, fifth, and seventh places, will necessarily fall under the first row or column on the left hand, and all the terms which stand in even places, to wit, in the second, fourth, and sixth places, will fall under the latter column.

These things premised, all questions which fall under Case 1 before mentioned, may be resolved by this Rule, viz.

Rule

Rule I.

Multiply all the given terms which stand in odd places (to wit, in the first column) according to the Rule of continual Multiplication, and reserve the last Product for a Dividend: Again, multiply continually all the terms which stand in even places, so shall the Product be a Divisor, and the Quotient arising from the said Dividend and Divisor shall be the answer of the Question.

So in the last mentioned Question, if all the Numbers in the first Column, to wit, 35, 3, 100, and 50 be multiplied continually, the Product will be 525000 for a Dividend; also if all the Numbers in the latter Column, viz. 24, 5 and 125 be multiplied continually, the last Product will be 15000 for a Divisor, and the Quotient arising from the said Dividend and divisor will be 35, which is the number of Bills of Vienna required.

$$\begin{array}{r|l}
 35 & 24 \\
 3 & 5 \\
 100 & 125 \\
 50 & \\
 \hline
 \end{array}$$

$$525000 : 15000 \text{) } 525000 \text{ (35}$$

The reason of the said Rule, will be manifest by solving the question propounded by three single Rules of Three, thus,

$$I. 24 b. 35 a :: 3 b. \frac{35 \times 3}{24} a (= 5 c.)$$

$$II. \frac{5}{1} c. \frac{35 \times 3}{24} a :: \frac{100}{1} c. \frac{35 \times 3 \times 100}{5 \times 24} a (= 125 d.)$$

$$III. 125 \frac{35 \times 3 \times 100}{5 \times 24} d. \frac{50}{1} a :: \frac{50}{1} d. \frac{35 \times 3 \times 100 \times 50}{125 \times 5 \times 24} a.$$

which fourth proportional last found, to wit,
 $35 \times 3 \times 100 \times 50$
 $\frac{125 \times 5 \times 24}{\text{---}}$ being well viewed and compared
 with the before mentioned order of placing the
 terms given in the question gives the very *Rule I.*
 before exprest in words.

An Example of the latter of the two Cases before mentioned.

If 10 lb. of *Avoirdupois* weight at *London* be equal
 to 9 lb. of *Amsterdam*; 45 lb. at *Amsterdam*, 49 lb.
 at *Bruges*; and 98 lb. at *Bruges* equal to 116 lb. at
Dantzick; how many lb. of *Dantzick* are equal to
 112 lb. of *Avoirdupois* weight at *London*? *Answer,*
 129. 92 lb. of *Dantzick*.

That the operation may be the more clear, let *a*
 represent one pound of *Avoirdupois* weight; *b* one
 lb. of *Amsterdam*; *c* one lb. of *Bruges*, and *d* one lb.
 of *Dantzick*; then let the question be stated after
 the order in the first Case, viz.

Suppositions

$$\text{Suppositions } \begin{cases} 10a = 29b \\ 45b = 49c \\ 98c = 116d \end{cases}$$

$$\text{The question } 112a = ?d$$

These things premised, all questions which fall under Case 2. before mentioned, may be resolved by this Rule, viz.

Rule II.

Multiply all the given terms which stand in even place (to wit in the latter column) and the last odd term in the first column according to the Rule of continual Multiplication, and reserve the last product for a Dividend; again multiply continually the rest of the terms which stand in odd places (to wit in the first column) for a Divisor, so shall the quotient arising be the answer of the question.

Or in this latter case if you place the last of the given terms in the same column with the even terms, the rule for solving questions, which fall under the latter case will be this which followeth, viz.

Multiply continually all the numbers in the latter column for a Dividend; also multiply continually all the numbers in the first column for a Divisor, so shall the quotient arising be the answer of the question. Thus the answer of the last mentioned question will be found 129.92, to wit, 192 $\frac{1}{2}$ lb. of *Danish*, as is evident by the subsequent operation.

then let the question be stated after the order in the first case, viz.

10	9
45	49
98	116
	112

44100 5729472 (129.92

The Reason of the said Rule II. will be manifest by solving the question propounded by three single Rules of three, thus,

$$9b. 10 a :: 45 b .. 45 \times 10$$

$$\text{—————} a (= 49 c.$$

$$II. 49 \cdot 45 \times 10 \cdot 98 \cdot 45 \times 10 \times 98$$

$$\text{---}c\text{---}a::\text{---}c\text{---}\text{---}a (=116d.)$$

1 9 1 49 x 9

$$\text{II. } \begin{array}{ccccccc} 45 & \times & 10 & \times & 98 & 116 & 112 \\ \hline & & & & n. & d. & \\ 49 & \times & 9 & & & & \end{array}$$

which fourth proportionul last found, to wit,

9 x 9 x 116 x 112

~~_____~~ being well viewed and compa-

5 x 10 x' 98

ed with the forementioned order of placing the
terms given in the question discovers the very Rule
I. before exprest in words.

Note, When the same Numbers happen to be Multipliers in the Dividend, and also in the Divisor, such Multipliers may be cancelled in both, and thereby much labour will oftentimes be spared.

Such which have much practice in calculating *Exchanges* and do exactly know the rate or proportion between two [different weights or measures or coins, which they would compare together, may by the *Rule of Three* frame Tables of proportions for the more speedy reducing of a given quantity of one kind of Weight, Measure, &c. into a quantity of the same value in another kind of weight, &c. In the expressing of which proportions it will be very convenient that the first Number or Antecedent of each Proportion be made 1 or unity, and the second term or consequent a Decimal, or else a mixt number whose Fractional part is a Decimal, for then the Coin, Weight, &c. of the one place (whose term is 1) may be reduced into that of the other place by help of those Tables and of Multiplication of Decimals without sensible error: For example, it hath been observed by some ingenious Merchants that 100 lb of *Avoirdupois* weight at *London*, are equal unto 89 lb. in *Paris* by the King's Beam, and consequently 1 lb *Avoirdupois* is equal to $\frac{89}{100}$ lb or 89lb at *Paris*; (for if 100 give 89, then 1 will give 89;) therefore any number of Pounds *Avoirdupois* being multiplied by 89 (with respect unto Multiplication of Decimals, explained in the 26 Chapter of the preceeding Book) will produce pounds of *Paris*. Again, if 89 lb. of *Paris* be equal to 100lb. *Avoirdupois* then 1lb. of *Paris* will be near equal to 1.1235 lb. of *Avoirdupois*; therefore any number of Pounds of *Paris* being multiplied by 1.1235 will produce pounds *Avoirdupois* very near.

Upon this ground I have collected the proportions in the following Tables, wherein I would not have

Chap. III. *Weight and Measures.*

347

have any to confide further than they shall know them to be agreeable to truth, for I have only derived them from those delivered by Mr. Lewis Roberts, Merchant, in his *Map of Commerce*, and do herein only aim at the *Instruction* of ingenious Merchants and Factors in the briefest ways of calculating their *exchanges*, the rate or proportion being truly known; in which Practice, *Decimal Arithmetick* (which hath no Enemy but the Ignorant) will be very serviceable.

Y 2

A

A Table for the Reduction of Avoirdupois Weight at London, to the Weights of divers Foreign Cities and remarkable Places.

	lb.
Antwerp	.9615
Amsterdam	.9
Abeville	.91
Acona	1 :282
Avignon	1 :12
Burdeaux	.91
Burgoigne	.91
Bollonia	1 :25
Bruges	.98
Callabria	1 :3698
Callais	1 :07
Constanti- nople	3 .8474
Deep	.91
Dantzick	1 :16
Ferrara	1 :3333
Florence	1 :282
Flanders in general	3 1 :06
Geneva	.9345

One Pound
of Avoirdu-
pois weight
at London,
makes at

Genoa

The use of the precedent Table will be manifest by the subsequent Example.

	lb.	
Geneva }	1 .4084	fine
Hamburg }	1 .4285	gross
Holland }	.92	
Lisbon }	.95	
	.881	
Lyons }	1 .87	common weight.
	.98	silk weight.
	.9	customers weight.
Leghorn }	1 .3333	
Millan }	1 .4285	
Mirandola }	1 .3333	
Norimberg }	.88	
Naples }	1 .4084	
Paris }	.89	
Prague }	.83	
Placentia }	1 .3888	
Rotchel }	1 .12	
Rome }	1 .27	
Revan }	.875	by vicont.
	.9017	com. weight.
Sevil }	1 .08	
Thoulusa }	1 .12	
Turin }	1 .2195	
Venetia }	1 .5625	subtil
	.9433	gross
Vienna }	.813	

One Pound
of Avoirdupois weight
at London,
makes at

The use of the precedent Table will be manifest by the subsequent-*Example*, viz!

How much weight at *Dantzick* do 320 lb. *Avoirdupois* make? *Answer*. 371 .2 lb. Seek in the precedent *Table* for *Dantzick*, and right against it you shall find 1 .16. which shew that 1 lb *Avoirdupois* is equal to 1 .16 lb. at *Dantzick*, therefore multiply 320 by 1 .16, so will the product be 371 .2 lb. of *Dantzick*, as by the Operation is manifest.

Avoir. Dantz. Avoir. Dantz.

1 : 1 . 16 :: 320 : 371 .2
1 . 16

1920
320
320

371|20

A Table for the Reduction of the Weights of divers Foreign Cities and remarkable Places to Avoirdupois Weight at London.

One Pound weight in		makes at London of Avoirdupois weight	lb.
	Antwerp		1.94
	Amsterdam		1.1111
	Abbeville		1.0989
	Ancona		.78
	Avignon		.8928
	Burdeaux		1.0989
	Burgoyne		1.0989
	Bollonia		.8
	Bruges		1.0204
	Callabria		.73
	Callais		.9345
	Deep		1.0989
	Dantzick		.862
	Ferrara		.75
	Florence		.78
	Flanders in general		.9433
	Geneva		1.07
	Genoa subtil		.17
	Genoa gross,		.70

One Pound weight in		makes at London of Avoirdupois weight	lb.
Hamburg			1 .0865
Holland			1 .0526
Lisbon			1 .135
Lyons	{ com: weight.		.9345
	{ silk weight.		1 .0204
	{ custom weight		1 .1111
Leghorn			.75
Millan			.7
Mirandola			.75
Norimberg			1 .1363
Naples			.71
Paris			1 .1235
Prague			1 .2048
Placentia			.72
Rotchel			.8928
Rome			.7874
Roban	{ by Vicont.		1 .1428
	{ com. weight.		1 .1089
Sevil			.9259
Thoulousa			.8928
Turin			.82
Venetia	{ futtle,		.64
	{ gross,		1 .06
Vienna			1 .23

The

The use of the last mentioned Table, will be manifest by this example, viz.

In 224 lb. weight at *Hamburg*, how many pounds *Avoirdupois*?

Answ. 243. 376 lb.

Seek in the Table for *Hamburg*, and right against it you will find 1.0865, which sheweth that 1 lb. of *Hamburg* makes 1.0865 lb. *Avoirdupois*, therefore if 1.0865 be multiplied by 224 the product will be pounds *Avoirdupois*.

1.0865	224	
224		
43469		
21730		
21730		
243	3760	

*A Table for the Reduction of English Ells
to the Measures of divers Foreign Cities,
and remarkable Places.*

One Ell at London makes at	Amsterdam	1 .6949	} Ells
	Antwerp	1 .6666	
	Bruges	1 .64	
	Arras	1 .65	
	Norimberg	1 .74	
	Colen	2 .08	
	Lisle	1 .66	
	Mastricht	1 .57	
	Frankfort	2 .0866	
	Dantzick	1 .3833	
	Vienna	1 .45	} Aulnes
	Paris	.95	
	Rouan	1 .03	
	Lions	1 .0166	
	Callais	1 .57	
	Venice	1 .8	} Braces
	linen	1 .96	
	filk	2	
	Lucea	2 .04	
	Florence	2 .3	
	Milan	2	
	Leghorn	1 .0328	
	Madera		
	Istis		

Sevil

One Ell at London makes at	Sevil	I .35	}	Vares
	Lisbon	I .		
	Castilia	I .3875		
	Andaluzia	I .3625		
	Granado	I .3625	}	Palmes
	Genoa	4 .8083		
	Saragofa	.55	}	Canes
	Rome	.56		
	Barcelona	.7125		
	Valentia	I .2125		

The use of the aforesaid Table will be manifest by the subsequent Example, viz.

In 325 ells of *London*, how many ells at *Antwerp*?
Answ. 541 .645 ells: Seek in the Table for *Antwerp*, and right against it you shall find 1 .6666 which being multiplied by 325 produceth 541 .645 ells of *Antwerp*, as by the operation is manifest.

1 . . . 1 . 6666 . . . 325

325

83330

33332

49998

54116450

*A Table for the Reduction of the Measures
of divers Foreign Cities and remarkable
Places to English Ells.*

One Ell at	Amsterdam	makes at London	.59
	Antwerp		.6
	Bruges		.6097
	Arras		.606
	Norimberg		.5747
One Auln at	Colen		.4807
	Lisle		.6024
	Mastricht		.6369
	Frankfort		.4792
	Dantzick		.7228
One Brace at	Vienna		.6896
	Paris		1 .0526
	Rouan		.9708
	Lions		.9836
	Callais		.6369
One Brace at	Venice		.5555
	Lucques		.5
	Florence		.4901
	Milan		.4347
	Leghorn		.5
	Madera Isles		.9681

Servil

One Cane at One Vane at	{	Sevil	makes at London	{	.7407	Ells
		Lisbon			I. .7407	
		Castilia			.7339	
		Andaluzia			.7339	
		Granado			.2079	
	{	palm at Genoa		{	I .8181	
		Saragosa			I .7875	
		Rome			I .4035	
		Barcelona			.8247	
		Valentia				

The use of the said Table will be manifest by the subsequent Example, *viz.*

In 730 Aulns at *Lions*, how many Ells at *London*.

Answ. 718.028. Seek in the Table for *Lions*, and right against it you shall find .9836, which being multiplied by 730 produceth 718.028 Ells of *London*, as by the operation is manifest.

1 : . . . 9836 . . . 730

730

295080

68852

718|0280

Note

Note, That one and the same kind of Weight or Measure doth seldom or never alter from its peculiar quantity, in the Kingdom or Common-wealth, where such Weight or Measure was first established; but one and the same kind of Money doth often rise and fall in its value in foreign Parts: For which cause I have spared the pains of calculating *Decimal Tables for Coins*; yet to give some light to such as read modern Relations, and want experimental knowledge in this matter, I shall here insert a *Table* in the same estate as I find it in the aforesaid *Map of Commerce*, and refer the Reader for further satisfaction, to the *Table in Rider's Dictionary*, concerning *Coins, Weights and Measures*, both Ancient and Modern.

Of

Chap

Of

re

London doth Exchange with

Of Exchanges of London, with divers Foreign Cities.

Pence

London doth Exchange with

Placentia sterh	64	for	1	Crown
Lyons	64	for	1	Crown
Rome	66	for	1	Ducat
Genoa	65	for	1	Crown
Milan	64 $\frac{1}{4}$	for	1	Crown
Venice	50	for	1	Ducat
Lucca	53 $\frac{1}{2}$	for	1	Ducaton
Naples	50	for	1	Ducat
Lecchia in 2	50 $\frac{1}{2}$	for	1	Ducat
Calabria 3	51	for	1	Ducat
Barri	57 $\frac{1}{2}$	for	1	Ducat
Palermo	56 $\frac{1}{2}$	for	1	Ducat
Mosina	57 $\frac{1}{2}$	for	1	Ducat
Antwerp 3	for 34 $\frac{1}{2}$			shill.
and Colen 3				stem.
Valentia	57 $\frac{1}{2}$	for	1	Ducat
Saragosa	59	for	1	Ducat
Barcelona	64	for	1	Ducat
Lisbon	53 $\frac{1}{2}$	for	1	Ducat
Bollonia	53 $\frac{1}{3}$	for	1	Ducaton
Bergamo	52	for	1	Ducaton
Frankfort	59 $\frac{1}{2}$	for	1	Florin
Genoa	65	for	1	Crown

London

London exchangeth in the denomination of Pence sterling with all other Countries, *Antwerp* and those neighbouring Countries of *Flanders* and *Holland* excepted, with which it exchangeth by the entire Pound of 20 Shillings English (or sterling.)

CHAP. IV.

Practical Questions about various Things : viz. Tare, Tret, Loss, Gain, Barter, Factorship, and Measuring of Tapestry.

Of Abatements and Allowances in Traffick, viz. T. of Tare.

IN the Trade of Merchandize there are in use various Allowances, and Abatements, known by the names of *Tare, Tret* &c. concerning which I shall give a few examples, whereby the practical *Arithmetician* will easily see, that there is more difficulty in the name than in the thing; for the rate, or proportion agreed upon, in any Allowance or Abatement (be it called by what name soever) being once known, the Arithmetical work will quickly be dispatch'd by the *Rule of Three*, or else by that and some of the former rules mixtly used, as will partly appear by the following questions.

Gross weight is composed of the Neat weight of the Commodity, and also the Tare, to wit, the Chest, Bag, But, &c. which containeth the Commodity.

Gross weight is as followeth.

Quest. 1. A Factor buyeth 4 Chests of Sugar marked A. B. C. D. The gross weight of each Chest in Avoir-

A

C.	11	81211	19
B.	10	3	20
C.	11	2	13
D.	10	17	17

The total gross weight 44 C. 0 q. 4 lb.

Now supposing the Tare or weight of each Chest, when it is empty, to be 37 lb. the question is what neat weight of Sugar will remain, when the total Tare is subtracted? *Ans.* 43 C. 0 q. 4 lb.

C.	q.	lb.
from 44	0	13
Subtr. 1	0	8
Rem: 43	0	5

Quest. 2. If from 990 C. 3 q. 2 lb. gross weight, Tare is to be subtracted after the rate 14 lb. per C. (or 12 lb.) of gross weight, how many C. neat will remain? *Ans.* 867 C. 0 q. 7 lb.

The gross weight being converted into pounds by the sixth Rule of the 7th Chapter of the preceding Book, will give 110985 lb.

Then by the Rule of Three,

112	14	110985	13873
or 8	1	110985	13873

III. From

lb.

III. From 110985 the gross weight:

Subtr. 13873 $\frac{1}{2}$ the total Tare.

Rest neat 97111 $\frac{1}{2}$ = 867 . . 0 . . 7 $\frac{1}{2}$

Note, When the number of lb. to be abated per C. for Tare is an aliquot part of 112, as in the last mentioned Example, where $14 = \frac{1}{8}$ of 112, the operation may be thus:

C. C. C. q. lb. C. qu. lb.
1 : $\frac{1}{8}$: : 990 : 3 : 21 : (123 : 3 : 13 $\frac{1}{2}$

$\frac{1}{8}$ of { 990 c. = 123 : 3 : 00
3 q. = 00 : 0 : 10 $\frac{1}{2}$
21 lb. = 00 : 0 : 02 $\frac{1}{2}$

Total Tare 123 : 3 : 13 $\frac{1}{2}$

Rest neat 867 : 0 : 07 $\frac{1}{2}$

Quest. 3. Suppose at some City, there is a Custom in selling of certain Merchandize by weight, to allow or cast in as Overplus to the Buyer, 4 lb. weight for every 100 lb. weight that is bought, and in that proportion for a greater or lesser quantity. Now if a Merchant buy 1175 lb. weight of some commodity, and is to be allowed thereupon after the aforesaid rate, the question is, how many lb. weight ought he to receive, in all? Answ. 1222 lb. weight.

100. 104 : : 1175 : 1222;

This

This kind of allowance is commonly called *Tret*

Quest. 4. Suppose a Merchant hath 1222 lb weight of a certain commodity, part whereof he bought at a certain rate *per lb.* and the rest was allowed to him or cast in as an Overplus, after the rate of 4 lb. weight for every 100 lb. weight which he bought; the question is, to know how many pounds neat weight he bought? *Ans.* 1175 lb. weight.

$$104. 100 :: 1222. 1175$$

This question is the converse of the former, and sheweth how to make abatement for *Tret*.

Quest. 5. If from 55 C. 1 qu. of gross weight *Tare* is to be subtracted after the rate of 16 lb. *per C.* and from the remainder *Tret* is to be abated after the rate of 4 lb. *per 104 lb.* the Question is, what the neat weight is worth in money after the rate of 848 s. for every C. (or 112 lb.) *Ans.* 382 ½ l.

I. The gross weight in lb. is 6188 l.

$$II. 112. 16 :: 6188. 884$$

$$or 7. 1 :: 6188. 884$$

$$III. 6188 - 884 = 5304$$

$$IV. 104. 00 :: 5304. 5100$$

$$V. 112. 8\frac{1}{2} :: 5100. 382\frac{1}{2}$$

Quest. 6. A Merchant hath bought Linen Cloth at 11 s. *per Ell*, which proving worse than he expected, he is willing to sell it at such a price that he may lose precisely after the rate of 1 ½ l. for every 20 l. that he laid out; the question is to know at what price he ought to sell the ell, that the proportion in the

*Of Loss
and Gain*

Z 2

faid

aid loss may be observed? *Ans.* 10 s. 1 d. per ell.

$$I. 20 : 18 \frac{1}{2} = 18 \frac{1}{2}$$

$$II. 20 : 18 \frac{1}{2} :: 11 : 10 \frac{1}{2} \text{ shillings.}$$

Otherwise,

$$I. 20 : 18 \frac{1}{2} :: 11 : 10 \frac{1}{2}$$

$$II. 11 - 1 \frac{1}{2} = 10 \frac{1}{2}$$

Quest. 7. If 100 lb. weight of any commodity cost 30 s. at what price must 1 lb. weight of that commodity be sold to gain after the rate of 10 l. for every 100 laid out? *Ans.* 3 $\frac{3}{4}$ d. per lb. weight.

$$I. 100 : 110 :: 30 : 33$$

$$II. 100 : 33 :: 1 : 1 \frac{3}{4} \text{ s. (or } 3 \frac{3}{4} \text{ d.)}$$

Quest. 8. A Merchant selleth a parcel of Jewels which cost him 250 l. ready Money, for 559 l. payable at the end of 6 Months; the *Question* is (his security being supposed to be good) what his gain was worth in ready Money upon rebate of Interest at the rate of 6 l. for 100 l. for a year? *Ans.* 300 l.

$$559 - 250 = 309$$

$$103 : 100 :: 309 : 300$$

Quest. 9. How much Sugar at 8 d. per lb. weight may be bought for 20 C. of Tobacco at 3 l. per C.?
Ans. 1800 lb. weight of Sugar.

Of Barter.

Quest. 9. How much Sugar at 8 d. per lb. weight may be bought for 20 C. of Tobacco at 3 l. per C.?

Ans. 1800 lb. weight of Sugar.

are w
barter
rate ta
C. wh
the qu
for th
gainer
for th

I.

II.

So i
which
only o
fore B

Que
600 l.
for ac
ney, a
ging t
have
questi
Factor

Ans.

I. T
chant
is

II.

III.

$$I. 3 : : 20 : 60$$

$$II. 1 : : 60 : 1800$$

Quest. 10. A. hath 100 pieces of Silk, which are worth but 3 *l.* per piece in ready money, yet he barterers them with B. at 4 *lb.* per piece, and at that rate takes their value of B. in Wools at 7 *l.* 10 *s.* per C. which are worth but 6 *l.* per C. in ready money, the question is to know what quantity of Wool pays for the Silk, and which of the two A. or B. is the gainer, and how much? *Ans.* 53 $\frac{2}{3}$ C. of Wool pays for the Silk, and A. gaineth 20 *l.* by the barter.

$$I. 7 \frac{1}{2} : 1 : : 400 : 53 \frac{1}{3}$$

$$II. \begin{cases} I. 6 : : 53 \frac{2}{3} : 320 \\ \text{or } 7 \frac{1}{2} : 6 : : 400 : 320 \end{cases}$$

So it is evident that the true worth of the Wool which B. delivered was 320 *l.* for which he received only of A. the worth of 300 *l.* in Silks, and therefore B. loseth 20 *l.* by the barter.

Quest. 11. A Merchant delivered to his Factor 600 *l.* upon condition that if the Factor add to it 250 *l.* of his own money, and bestow his pains in managing the whole stock, he shall then have $\frac{2}{3}$ parts of the total gain. The question is to know what stock the Factor's service was estimated at? *Ans.* 150 *l.*

Of Factorship. See brief rules for computing of Factors allowances in the 19, and 20 Rules of the second Chap. of this Appendix.

I. The Factor's part of the gain being $\frac{2}{3}$ the Merchant must necessarily have the remainder, which is $\frac{1}{3}$.

$$II. \frac{2}{3} : \frac{1}{3} : : 600 : 400$$

$$III. 400 = 250 = 150$$

Z 3

Quest.

Quest. 12. A Merchant delivereth to his Factor 320 l. and permitteth him to add to it 64 l. of his own money, to be employed in traffick, and by agreement between them the Factor's service is estimated equivalent to a certain stock; which is such, that if the total gain be divided proportionable according to those three Stocks, the Factor is to receive $\frac{1}{3}$ of the total gain, in consideration of the said imaginary stock (being the value of his service;) the question is to know the full part of the gain belonging to each, and what stock the Factor's service was valued at? *Answ.* The Merchant $\frac{2}{3}$ of the gain, and the Factor $\frac{1}{3}$, whose service was valued at 96 l. stock.

$$I. \quad 320 + 64 = 384$$

$$II. \quad \frac{4}{5} : \frac{1}{5} :: 384 : 96$$

$$III. \quad \left\{ \begin{array}{l} 320 \\ 64 \\ 96 \end{array} \right. : \frac{1}{5} :: 480 : 1$$

$$\left\{ \begin{array}{l} 320 : \frac{2}{3} \\ 160 : \frac{1}{3} \end{array} \right.$$

Quest. 13. If a piece of Arras Hangings, in the form of a long square, hath for its length $6\frac{1}{2}$ yards *English*, and breadth 4 yards; how many square ells, or ticks *Flemish* are contained in that piece, when the length of a *Flemish* ell is equal to $\frac{3}{4}$ yard *English*? *Answer* 44 $\frac{1}{2}$ square ells or ticks *Flemish*.

Forasmuch as by supposition, a *Flemish* ell in length, hath such proportion to an *English* yard in length, as 3 to 4, and consequently the square of the one to the square of the other, as 9 to 16.

Therefore

Therefore in a direct proportion, as 9 is to 16; so is any given number of square yards *English* to a number of square ells *Flemish*, which will take up equal space with the said square ells *English*. Also in a direct proportion, as 16 is to 9, so is any given number of square ells *Flemish* to a number of square yards *English*, which will take up an equal space with the said *Flemish* ells: Therefore to resolve the aforesaid question, first find the number of square yards *English* contained in the said piece of Arras, by multiplying the length and breadth in yards mutually one by the other, then proceed according to the aforesaid proportion; so the work will stand thus,

I. $6\frac{1}{4} \times 4 = 25$ square yards *English*.

II. $9 \cdot 16 :: 25 \cdot 44\frac{2}{3}$ square ells *Flemish*.

Otherwise.

$6\frac{1}{4}$ yards *English* in length give $8\frac{1}{4}$ length.
by the Rule of Three in *Flemish* ells—

Also 4 yards *English* give in *Flemish* ells— $5\frac{1}{4}$ breadth.

Therefore the product of the said $8\frac{1}{4}$ multiplied by $5\frac{1}{4}$ gives for the superficial content as before— $44\frac{2}{3}$

Quest. 14. If a piece of Tapestry in the form of a long square be in length $15\frac{1}{4}$ ells *Flemish*, and in breadth $4\frac{1}{3}$ ells *Flemish*, how many square yards *English* are contained in that piece, when 4 ells *Flemish* in length are equal to 3 yards *English*?

Ans. $37\frac{11}{14}$ square yards *English*.

I. $15\frac{1}{4} \times 4\frac{1}{3} = 66\frac{1}{12}$.

II. $16 \cdot 9 :: 66\frac{1}{12} \cdot 37\frac{11}{14}$.

CHAPTER IV.

Concerning the Interest of Money, and the Construction of Tables to that purpose.

I. IN resolving questions concerning interest of money, four things are to be well observed, to wit, First, the Principal, or money lent for gain or interest; Secondly, the time for which the said Principal is lent; Thirdly, the rate or proportion which the Principal bears to the sum of the Principal and Interest; and Fourthly, the Interest it self: So if 100 *l.* be lent upon condition that 106 *l.* shall be repaid at the end of a year, the said 100 *l.* is called Principal; the time for which the said principal is lent is one year; the proportion which the principal bears to the sum of the principal and interest is such as 100 hath to 106; Lastly, the interest is self is 6 *l.*

II. Interest is either Simple or Compound.

III. Simple Interest is that which ariseth or is computed from the principal only: So if 100 *l.* be lent for two years, the simple Interest thereof after the rate of 6 pounds for 100 pounds for 1 year will be 12 pounds, viz. 6 pounds due at the first years end, and 6 pounds due at the second year end.

IV. Compound Interest is that which ariseth from the Principal, and also from the interest thereof, and therefore it is called interest upon interest: So if 100 pounds be lent and forborn 3 years, and compound interest thereof it to be computed

puted after the rate of 6 pounds for 100 *l.* for one year; there will arise besides the simple interest of the principal for three years, the interest of 6 pounds (due at the first years end) for 2 years, and the interest of 6 pounds (due at the second years end) for one year following.

V. Rebate or discompt of money is, when a sum of money due at any time to come, is satisfied by the payment of so much present money, which if it were put forth at a certain rate of interest for the said time, would become equal to the sum first due: So if 100 pounds be due at the end of two years, and is to be satisfied by the payment of present Money upon rebate, after the rate of 6 pounds *per centum, per annum, simple interest*, there ought to be so much ready Money paid, which in two years after the said rate of interest would be augmented unto 100 *l.* In like manner if the rebate or discompt were to be made after any rate of compound interest, so much ready Money ought to be paid, which at such rate of compound interest, for the time agreed on, would become equal to the sum first due. *Examples* of the manner of computation by rebate may be seen in the tenth and fourteenth Rules of this Chapter.

VI. In the taking of interest, or use Money, for the loan or forbearance of Money lent, respect must be had to the rate limited by Act of Parliament, which now restraineth all Persons from taking more than 6 *l.* for the interest or use of 100 *l.* lent for a year, but what part of 6 *l.* may be taken for the interest of 100 *l.* lent for half a year, a quarter

The Foundation upon which the Rules for Computing simple Interest are grounded.

of

of a year, a month, or any other part of a year, is not exprest in the Act; In this case therefore we must observe custom and daily practice, so we shall find that 3*l.* is usually taken for half a years Interest of 100*l.* and 30*s.* for a quarter of a year, &c. by which practice, this following Analogy (which is the ground or reason of the common Rules for computing simple Interest) seems to be assumed for a safe Exposition of the Statute, *viz.* That such proportion as the whole year (supposed to consist of 365 days) hath to any propounded space of time more or less than a year, such proportion any Interest (not exceeding the rate limited by the Act) for any principal lent for a year, ought to have to the Interest of the same Principal for the time propounded: This Analogy being granted the manner of computing simple Interest, for any Principal lent and forborn any time propounded, will be such as is exprest in the two next Sections.

VII. The Interest or Gain of 100*l.* principal Money forborn for a year being known, the Interest of any other principal Money for the same time may be found out by one single *Rule of Three*, for as 100*l.* principal is in proportion to the Interest thereof, so is any other principal to its Interest: So if it be demanded what 270*l.* will gain in a year at the rate of 6*l.* for 100*l.* for one year, the *Answer* will be found to be 16*l.* 4*s.* For,

l. l. l. l. l. s. d.

100 : 6 :: 270. 16, 2 (or 16 : 4 : 0

A second Example. What is the Interest of 175*l.* 18*s.* 11*d.* for a year, at the rate of 6*l.* for 100*l.*
for

for a year? *Answ.* 10 l. 11 s. 1 $\frac{6}{100}$ d. as by the following operation (which is performed after the practical manner delivered in the nineteenth Rule of the second Chapter of this *Appendix*) is evident.

l.	l.	l.	s.	d.	l.	s.	d.		
100	:	6	::	175	:	18	:	11	
				(	10	:	11	:	1
				$\frac{6}{100}$					
multiply by .. 6									

l.		10		55	:	13	:	6
				20				

s.		:	:	11		13
				12		

d.		:	:	1		62
----	-------	---	---	---	--	----

VIII If the Interest of 100 l. principal for one whole year, or 365 days be known, the simple interest of any other principal, for any number of days more or less than 365, may be found out by the following Rule, *viz.*

Multiply these three numbers according to the Rule of continual Multiplication, to wit, the given interest of 100 l. for a year, the principal, whose interest is required, and the number of days prescribed, reserving the last product for a Dividend: Also multiply 365 by 100, and reserve this product for a Divisor; Lastly finish Division, so shall the quotient be the interest or gain sought.

A Rule for computing simple interest for any number of days.

Note here, that the two principals, to wit 100 l. and the other propounded, are supposed to be of one and the same denomination: Also the interest required

required will be of the same denomination with the given interest of 100*l*.

For an example of this Rule, let it be required to find out the interest of 400*l*. for a week, or 7 days at the rate of 6*l*. for 100*l*. for a year, or 365 days: First multiplying these three numbers 6, 400, and 7 continually (*viz.* multiplying 6 by 400, and the product thence arising by 7) the last product will be 16800 for a Dividend; also multiplying 365 by 100, the product is 36500 for a Divisor; Lastly, dividing 16800 by 36500 (after cyphers at pleasure are added to 16800) the quotient (according to the fourth Rule of the 27th Chapter of the preceeding Book) will be discovered to be this decimal .4602, which is equal to 9*s*. 2*d*. 1*farth*. (as will appear by the brief way of valuing a decimal fraction in the fourth Rule of the 26th Chapter.

The reason of the above mentioned rule of the computing of interest for days, will be manifest by this following way of solving the same question by two single *Rules of Three*, *viz.*

$$I. 100 . 6 :: 400 . \frac{6 \times 400}{100}$$

$$II. 365 \quad \frac{6 \times 400}{100} \quad 7 \quad \frac{6 \times 400 \times 7}{365 \times 100} \\ \text{---} : \text{---} :: \text{---} : \text{---}$$

Which fourth proportional in the latter *Rule of Three*, to wit, $\frac{6 \times 400 \times 7}{365 \times 100}$ being well viewed the

truth of the rule before delivered will be manifest.

Hence

Hence one vulgar error in computing interest is discovered, for some argue thus, 6 *l.* is the interest of 100 *l.* for a year, *therefore* 10 *s.* (or $\frac{1}{12}$ of 6 *l.*) is the interest for a month, and consequently 2 *s.* 6 *d.* for a week or seven days, and so the interest of 400 *l.* for 7 days, computed after that manner would be 10 *s.* which exceeds the answer found by the preceeding Rule by 9 $\frac{3}{4}$ *d.* very near, which fallacy hath its rise from the taking, (or rather mistaking) of 28 days for $\frac{1}{12}$ part of the number of days, in a year, when indeed the just $\frac{1}{12}$ part of 365 days consists of 30 $\frac{5}{12}$.

Moreover, by the help of this decimal fraction of a pound, to wit, .000164383, which is very near the interest of one pound for a day at the rate of 6 per cent. per annum (as will appear by the preceeding rule) the interest of any principal (supposed to be pounds or decimal parts of a pound) for any number of days propounded at the said rate of interest, may be found out by multiplication only, viz. First multiply the said decimal .000164383, by the principal whose interest is required, then multiply that product by the number of days propounded, so shall the last product be the interest required; (but in these multiplications respect must be had to the cutting off of places in the products, according to the second and third Rules of the 26th Chapter of the preceeding Book;) for example, if it be required to find the interest of 1000 *l.* for 131 days, at the rate of 6 per cent. per annum, the *Answ.* will be found 21.534 +; or 21 *l.* 10 *s.* 8 *d.* + for according to the rule last given.

*Another Rule
for computing
simple Interest
for days.*

.000164383

$$.000164383 \times 1000 \times 131 = 21.534 +$$

But at another rate of interest, a *peculiar* decimal instead of the said .000 164383 (which serves only for 6 per cent. per annum) must be found out by the first rule foregoing before the latter rule can take place, the reason of which latter rule doth also evidently arise from two single rules of three.

IX. When an Annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount unto, simple interest being computed for each particular yearly payment, from the time it became due, until the end of the term of years, the work will be as in this following example, *viz.* If an Annuity, or yearly rent of 134*l.* 10*s.* 6*d.* be all forborn till the end of 4 years, what will it then amount unto, simple interest being allowed at the rate of 6 per cent. per annum for each years rent, from the time on which, it was due, until the end of the said term of four years? *Answ.* 586*l.* 10*s.* 6*d.*

It is evident by the question, that at the rate of interest propounded, there must be computed the interest of 134*l.* 10*s.* 6*d.* (due at the third years end) for one year (to wit, the fourth year;) also the interest of the like sum due at the second years end, for two years (to wit, the third and fourth years;) likewise the interest of the same sum due at the first years end, for three years (to wit, the second, third and fourth years:) all which interest being added to the sum of the four years rent, the total sum will shew what the said Annuity will amount

The manner
of summing
up Annuities
in arrear
with allowan-
ces of simple
interest.

mount unto at the end of the said term of 4 years.

Explication.

	years	l.	s.	d.
The interest of 134 l.	1 is ...	8	1	5. 16
10 s. 6 d. at 6 per cent. per annum, for	2 is ...	16	2	10. 32
	3 is ...	24	4	3. 48

The sum of the 4 years	}	is ...	538	2	0
rent (to wit, 4 times 134 l. 10 s. 6 d.)					

All which added together give the Answer of	}	... 586	10	6	96
the question, to wit,					

X. When it is required to find out how much ready Money will satisfy a Debt due at the end of any space of time to come, by rebating or discounting at a given rate of simple interest, it may be effected by this rule, viz.

Of rebate or discount of money at simple interest.

First, find out the interest of 100 l. at the given rate of interest, for the time which the ready money is to be paid beforehand, then adding the interest so found to 100 l. make always the sum of that addition the first term in a rule of three; 100 l. the second term; and the debt propounded to be satisfied the third term; lastly, the fourth proportional found out by the said Rule of Three shall be the ready money which ought to be paid in satisfaction of the debt propounded.

Example 1. If a debt of 100 l. be payable at the end of a year to come, how much ready money will discharge that debt by rebating or discounting at the rate of 6 per cent. per annum? *Ans.* 94 l. 6 s.

6 s. 9 d. 2 f. very near; for by the Rule of Three,

$$106 : 100 :: 100, 94, 3396 +$$

That is to say, if 106 l. (which is compos'd of 100 l. principal and 6 l. interest) proceeds from 100 l. principal forborn for a year, from what principal forborn for a year doth. 100 l. (compos'd of principal and interest) proceed from? *Answ.* 94. 3396 l. + (or 94 l. 6 s. 9 $\frac{1}{2}$ d. very near) principal money: therefore 94 l. 6 s. 9 $\frac{1}{2}$ d. in ready money, is of equal value with 100 l. due at the end of the year to come; for if the said 94 l. 6 s. 9 $\frac{1}{2}$ d. be put forth at interest for a year, at the rate of 6 per cent. per ann. it will gain 5 l. 13 s. 2 $\frac{1}{2}$ d. very near, which together with the said 94 l. 6 s. 9 $\frac{1}{2}$ d. makes the 100 l. the debt first propounded to be discharged by rebate.

Example 2. If 150 l. 10 s. be payable at the end of 73 days to come, how much present money will discharge the said debt, by rebating after the rate of 6 per cent. per annum? *Answ.* 148 l. 14 s. 3 $\frac{1}{2}$ d. + as by the following operation is manifest.

$$\begin{array}{cccc} \text{days} & \text{l.} & \text{days} & \text{l.} \\ \text{I.} & 365.6 & :: & 73 : 1,2 \end{array}$$

$$\begin{array}{cccc} \text{l.} & \text{l.} & \text{l.} & \text{l.} \end{array}$$

$$\text{II. } 101, 2 \text{ } 100 :: 150, 5. \text{ } 148, 7154 +$$

That is to say, First, I seek by a single Rule of Three the interest of 100 l. for 73 days, at the rate of interest propounded, saying, if 365 days (or a year) gain 6 l. what will 73 days gain? *Answ.* 1 $\frac{1}{2}$ l. or 1.2. l. Then adding the said 1.2 to 100, I say, by

by a second Rule of Three, if 101. 2*l*. principal and interest, payable at the end of 73 days to come, be equivalent to 100 *l*. ready money, what ready money is 150 *l*. 10 *s*. (or 150. 5) payable at the end of 73 days to come equivalent unto? So by multiplying and dividing (according to the rules of decimal Multiplication and Division explained in Chapter 26 and 27 of the preceding Book) the quotient or answer of the question will be found 148.7154+, that is, 148 *l*. 14 *s*. 3*l* 4. + for the decimal .7154 being valued according to the brief way at the end of the fourth rule of the 26th Chapter, will by inspection only be discovered to be 4 *s*. 3*l* 4. which rule I shall here once for all, advise the Learner to be well acquainted with.

The Proof.

Seek (by the Rule of Three) what the ready money found as aforesaid will gain, in so much time as it is paid beforehand at the rate of interest, propounded: then having added this gain to the said ready money, if the sum be equal to the debt first propounded to be satisfied by rebate, the ready money was rightly found out. So the last example will be thus proved,

$$100. \quad 1. \quad 2 \text{ s.} : 148.7154 : (1.7845.$$

Which fourth proportional 1.7845 being added to 148.7154, the sum will be 150.4995+, which doth not want a farthing of 150 *l*. 10 *s*. the debt first propounded.

XI. When it is required to find the present worth of an annuity, by rebating or discounting at a given rate of simple interest, the operation will be as in the following example, viz. How much present Money is equivalent to an annuity or rent of 100 *l.* per annum, to continue five years, rebate being made at the rate of 6 *l.* for 100 *l.* for one year at simple interest? *Ans.* 425 *l.* 18 *s.* 9 $\frac{1}{2}$ *d.* very near.

It is manifest that there must be computed the present worth of 100 *l.* due at the first years end, also the present worth of 100 *l.* due at the second years end, and in like manner for the third, fourth and fifth years; all which particular present worths being added together, the aggregate or sum will be the total present Worth of the Annuity, to wit 82861 $\frac{1}{2}$ in the example above propounded, 425 ——— 882126 $\frac{1}{2}$ that is, 425 *l.* 18 *s.* 9 $\frac{1}{2}$ *d.* very near.

The operation by decimals (which will come near enough to the truth) will be as followeth viz.

l. *l.* *l.* *l.*

- | | | |
|----|--|--------------------------------|
| 1. | | 106 . 100 :: 100 . 94, 33962 + |
| 2. | | 112 . 100 :: 100 . 89, 28571 + |
| 3. | | 118 . 100 :: 100 . 84, 74576 + |
| 4. | | 124 . 100 :: 100 . 80, 64516 + |
| 5. | | 130 . 100 :: 100 . 76, 92307 + |

Ans. 425, 93933 +

Her

Here by the way, from the manner of resolving the last mentioned *Question*, that *Rule* commonly called *Equation of Payments*, which is insisted on by divers *Arithmetical Writers*, will be found erroneous, which I thus prove.

1. Since that *Rule* aims at the reducing of several *Days of Payment*, upon which particular Sums of *Money* are due, unto a mean time upon which the aggregate or total of those particular Sums ought to be paid, without damage to the *Debitor* or *Creditor*, there must be necessarily some rate of *Interest* implied; for otherwise why may not any Day at pleasure be assigned for one intire payment?

2. If some rate of *Interest* be implied, then equity requires that the present worth of the total Sum payable at one intire payment, rebate or discount being made according to that rate of interest, may be equal to the Sum of the present worths of the particular Sums of *Money*, rebate being made at the same rate of *Interest*.

3. In regard the said *Rule* doth mention no particular rate of interest, it ought to be true at any rate of interest whatsoever.

4. Let us therefore examine the said *Rule* according to the rate of 6 per cent. per annum, simple interest, by taking the last mentioned *Question* for an example, (which according to the accustomed manner) will be thus stated, viz. If 500 l. ought to be paid by five equal yearly payments, to wit, 100 l. at each years end, what time ought to be given for the payment of the said 500 l. at one entire payment, without loss either to the *Debitor* or *Creditor*.

5. By proceeding according to the said *Rule* of *Equation of Payments* (which saith, if the sum of the

Products, arising from the multiplication of each particular sum of money by its respective time, be divided by the sum or aggregate of the said particular sums of money, the quotient will be the mean time to be assigned for one entire payment) there will be found three years, which time (according to the said rule) ought to be given for the payment of the whole 500 l.

6. Now it 500 l. due at the end of three years to come, be worth as much in present money, as is the present worth of an *Annuity* of 100 l. to continue five years, then the said *Rule of Equation* is true; otherwise false; but the present worth of 500 l. due at the end of three years to come, rebate being made at the rate of 6 per centum, per annum, simple interest, will be found (by the tenth rule of this Chapter) to be 423 l. 14 s. 6 d. 3 f. very near; also the present worth of the said *Annuity*, rebate being made as before, is found, (as appeareth by the resolution of the last mentioned question) to be 425 l. 18 s. 9 $\frac{1}{2}$ d. very near; wherefore it is evident that the *Creditor* loseth 2 l. 4 s. 2 $\frac{1}{2}$ d. very near, by receiving the whole 500 l. at the three years end: Moreover, at 6 per centum, per annum, compound interest, he would lose 1 l. 8 s. 6 d. very near; as will be manifest by the *Tables of compound interest* hereafter expressed: So that the loss will be either more or less according as the rate of interest doth differ: And therefore I conclude the said Rule (as also all other rules or resolutions of questions which have dependance thereon) to be erroneous.

Although questions of this nature seldom come into practice, yet he that will take the pains, may find out such a mean time as is required by the said

Rule of Equation of Payments, at any rate of simple interest by this following Rule, viz.

First, By the preceeding tenth Rule of this Chapter find out the present worth of every particular Sum in the Question payable at a time to come, by rebating at the rate of interest agreed on; then find in what time the Sum of those present worths will be augmented unto the total of all the particular Sums payable at times to come, according to the first agreement, so shall the time found out be the mean time for the payment of the whole Debt: Thus the mean or equated time in the last example will be found to be 2.8979, &c. years (not three years, as the said Rule of Equation of Payments would have it) for by rebating at 6 per cent. per annum, simple interest, 300 l. payable at the end of 2.8979, &c. years to come (that is 2 years and 328 days very near) is worth in ready Money 425 l. 18 s. 9½ d. very near, and the same ready Money is also the present value of 100 l. Annuity for five years, at the same rate of interest, as before hath been manifested. But to return to the path from which I have made a digression.

From the preceeding tenth Rule of this Chapter, the following Tables I. and II. are deduced, whose construction and use are afterwards declared.

A 2 3

Years

Table I.		Table II.	
Years	Which sheweth in decimal parts of a pound, the present worth of one pound due at the end of any number of years to come, not exceeding 7 years, at the rate of 6 per centum, per annum, simple interest.	Years	Which sheweth in pounds and decimal parts of a pound, the present worth of one pound Annuity, to continue any number of years not exceeding 7, at the rate of 6 per centum, per annum, simple interest.
1	.943396	1	.943396
2	.892857	2	1.836253
3	.847457	3	2.683710
4	.806451	4	3.490162
5	.769230	5	4.259393
6	.735294	6	4.994687
7	.704225	7	5.698912

The Construction of Table I.

The numbers in the first Table which are placed right against the numbers of Years 1, 2, 3, 4, 5, 6, and 7, are decimal *Fractions*, one pound of English Money being the *Integer*, and are thus found (according to the preceeding tenth Rule of this Chapter,) viz.

$$106 : 100 :: 1 .,943396 +$$

$$112 : 100 :: 1 .,892857 +$$

$$118 : 100 :: 1 .,847457 +$$

whereby

whereby it appears, that 1 *l.* due at the end of a year to come, is worth in ready money .943396+, that is, 18 *s.* 10 *d.* 1 *f.* and some what more. Also 1 *l.* due at the end of two years to come, is worth in ready money .892857+, or 17 *s.* 10 $\frac{1}{4}$ *d.* rebate being made at the rate of 6 per centum, per annum, simple interest; the like is be understood of the rest, of the numbers in Table I. which may be continued to more years, and other Tables also of rebate may be framed upon the same ground, for months, or days, by the ingenious Artist.

The use of Table I.

The practical use of the said first Table will be manifest by solving this following question; viz. How much ready money will discharge 345 *l.* 15 *s.* 6 *d.* due at the end of five years to come, by rebating simple interest at the rate of 6 per centum per annum? *Ans.* 265 *l.* 19 *s.* 7 $\frac{1}{4}$ *d.* which is thus found out; viz. In the preceding Table I. right against 5 years, I find the decimal .76923, which shews that 1 *l.* due at the end of five years to come is worth in ready money .76923 (that is, 15 *s.* 4 $\frac{1}{2}$.) then instead of 15 *s.* 6 *d.* mentioned in the question propounded, taking the decimal .775 which is equal to 15 *s.* 6 *d.* (the same being reduced according to the fifth Rule of the 23d. Chapter of the preceding Book) I say, by the Rule of Three.

$$1 \cdot .76923 :: 345.775 \cdot (265.9805+)$$

That is to say, if 1 *l.* give .76923 *l.* what will 345.775 *l.* give? *Ans.* 265.9805 *l.* for multiplying 345.775 by .76923, according to the second Rule of the 26 Chapter of the preceding Book, the product will be 265.9805, that is, 265 *l.* 19 $\cdot$ 7 $\frac{1}{4}$ *d.*

The Construction of Table II.

The numbers in the second Table are found out by the addition of those in the first, *viz.* the first number in the latter Table is the same with the first number in the former; the second in the latter is the sum of the first and second in the former; the third in the latter is the sum of the first, second and third in the former, and in that manner the rest are found; (the reason of which composition is manifest from the example of the eleventh rule foregoing,) otherwise the numbers in Table II. may be found more easily thus, *viz.* the first number in the said Table II. is the same with the first number in Table I. the second number in the latter Table is compos'd of the second number in the former and the first in the latter, the third number in the latter Table is compos'd of the third number in the former, and the second in the latter, the fourth in the latter is compos'd of the fourth in the former, and the third in the latter; the like is to be understood of the rest of the numbers in Table II. which might be continued to more years, and fitted to other rates of interest, but I shall spare that labour, in regard a more equal way of finding out the present worth of an Annuity, agreeable to the accustomed and practical rates of buying and selling Annuities of Rents for terms of years, is grounded upon a computation of interest upon interest as will hereafter be made manifest; for at simple interest an Annuity will be overvalued.

The use of Table II.

The use of Table II. will appear by this following

ing

ing example; viz. What is the present worth of an Annuity of 100 *l.* per annum payable yearly during the term of five years, discount or rebate being made at the rate of 6 per centum, per annum, simple interest? Answer, 425 *l.* 18 *s.* 9 $\frac{1}{2}$ *d.* very near, which is thus found out, viz. In the preceding Table II. right against five years, I find this number 4.259393, which shews that an Annuity of 1 *l.* payable yearly during five years, is worth in ready money 4.259393 *l.* (that is 4 *l.* 5 *s.* 2 *d.* and somewhat more) therefore, I say, by the Rule of Three;

$$\begin{array}{ccccccc} l. & l. & & l. & l. & & l. \\ 1 & . & 4.259393 & :: & 100 & . & (425.9393. \end{array}$$

That is to say, if 1 *l.* give 4.259393 *l.* what will 100 *l.* give? Answer. 425 *l.* 18 *s.* 9 $\frac{1}{2}$ *d.* very near, for by multiplying 4.259393 by 100, the product (according to the second rule of the 26th Chapter of the preceding Book) is 425.9393, that is, 425 *l.* 18 *s.* 9 $\frac{1}{2}$ *d.* very near. Which operation being compared with the manner of solving the same question before mention'd in the eleventh Rule of this Chapter, the great benefit of Tables of this kind in Point of expedition will be apparent.

XII. When it is required to know, unto what sum of money any proportioned principal forborn any number of years will at the end of such term be augmented unto, interest upon interest being computed at a given rate, there must be found a rank of continual proportionals, more in number by one than is the number of years in the question; of which proportionals the first the principal assigned, the second must increase

*Of the forborne
value of Mo-
ney at com-
pound interest*

or

or proceed from the first, the third from the second, &c. in such manner or rate, as 106 proceeds from 100 (or as 180 from 100, if the rate of interest be 8 *per centum*) then will the last proportional be the Answer of the question: So if 300 pounds principal money be put forth at interest upon interest, at the rate of 6 *l.* for 100 *l.* for one year, and all forborn until the end of 4 years, there will then be due 378.743088, or 378 *l.* 14 *s.* 10 $\frac{1}{2}$ *d.* very near, as by the four following *Rules of Three* is manifest.

$$100. : 106 : : \begin{cases} 300 & . 318 \\ 318 & . 337.08 \\ 337.08 & . 357.3048 \\ 357.3048 & . 378.743088 \end{cases}$$

For the said 300 *l.* will at the first years end be augmented unto 318 *l.* which 318 *l.* being put forth as a *principal* for 1 year, will (at the second years end) be augmented unto 337.08, again this 337.08 being put forth as a *principal* for 1 year, will (at the third years end) be augmented unto 357.3048, in like manner 357.3048 being put forth as a *principal* for 1 year, will (at the fourth years end) be augmented unto 378.743088, which is the number required by the question. And if the work be well examined, it will appear (as was before declared) that the *principal* first assigned, to wit 300 *l.* and the numbers resulting successively at the ends of the several years are *continual proportionals*, viz. these five numbers are so qualified, that if the

$$300 | 318 | 337.08 | 357.3048 | 378.743088$$

second

second be multiplied by it self, the product will be equal to the product of the first and third; also if the third be multiplied by it self, the product will be equal to the product of the second and fourth; in like manner, if there were more *continual proportionals* in a rank, if any one proportional which is placed *between* two next on each side of such one be multiplied by it self, the product will be equal to the product of those two extreame (which is a property peculiar to continual proportionals.)

Note here by the way, that if any two numbers be propounded, suppose 300 and 318, and it be required to find to them a third, a fourth, a fifth, &c. in continual proportion, multiply the second proportional 318 by it self, and divide the product 101.124 by the first proportional 300, so shall the quotient 337.08 be a third in continual proportion: In like manner if you multiply the third proportional 337.08 by it self, and divide the product 113622.9264 by the second proportional 318, the quotient 357.3048 shall be a fourth in continual proportion, and after the same manner a fifth, a sixth, or as many as you please may be found out.

From what hath been said by way of Explication of the preceeding twelfth Rule, the following Table III. is deduced, the construction and use whereof is afterwards declared.

*Two numbers
being given to
find a third,
a fourth, a
fifth, &c. in
continual pro-
portion.*

TABLE

TABLE III.

Which sheweth what one Pound will amount unto, being forborne unto the end of any term of Years under 31, compound interest being computed Yearly, at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum.

Years	4	5	6	7	8	9	10	11	12
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	1.08160	1.10250	1.12360	1.14490	1.16640	1.18810	1.21000	1.23216	1.25440
3	1.12486	1.15762	1.19101	1.22504	1.25971	1.29502	1.33100	1.36762	1.40492
4	1.16985	1.21550	1.26247	1.31079	1.36048	1.41158	1.46410	1.51807	1.57351
5	1.21665	1.27628	1.33823	1.40255	1.46932	1.53862	1.61051	1.68505	1.76234
6	1.26531	1.34009	1.41851	1.50073	1.58687	1.67710	1.77156	1.87041	1.97381
7	1.31593	1.40710	1.50363	1.60578	1.71382	1.82803	1.94871	2.07616	2.21068
8	1.36856	1.47745	1.59384	1.71818	1.85093	1.99256	2.14358	2.30453	2.47596
9	1.42331	1.55132	1.68947	1.83845	1.99900	2.17189	2.35794	2.55803	2.77207
10	1.48024	1.62880	1.77908	1.96715	2.15892	2.36736	2.59374	2.83942	3.10584
11	1.53945	1.71033	1.89829	2.10485	2.33163	2.58042	2.85314	3.15175	3.47844
12	1.60103	1.79385	2.01219	2.25219	2.51817	2.81266	3.13842	3.49849	3.89597
13	1.66507	1.88564	2.13292	2.40984	2.71962	3.06580	3.45227	3.88328	4.36349
14	1.73167	1.97993	2.26090	2.57853	2.93719	3.34172	3.79749	4.31044	4.88711
15	1.80084	2.07892	2.39625	2.75902	3.17216	3.64248	4.17724	4.78438	5.47256

A continuation of the preceding Table III.

Years.	4	5	6	7	8	9	10	11	12
16	1.87298	2.18287	2.54035	2.95216	3.42549	3.97030	4.59497	5.31089	6.13039
17	1.94790	2.29001	2.69277	3.15881	3.70061	4.31763	5.05447	5.89309	6.86604
18	2.02581	2.40661	2.85433	3.37993	3.99601	4.71712	5.55991	6.54355	7.68996
19	2.10684	2.52695	3.02559	3.61652	4.31570	5.14166	6.11590	7.26234	8.61276
20	2.10112	2.65329	3.20713	3.89168	4.66095	5.60441	6.72749	8.06231	9.64629
21	2.27876	2.78596	3.39956	4.14056	5.03383	6.10880	7.40024	8.94916	10.80384
22	2.36991	2.92529	3.60353	4.43040	5.43654	6.65860	8.14027	9.93357	12.10031
23	2.46471	3.07162	3.81975	4.74033	5.87146	7.25787	8.95430	11.02626	13.55234
24	2.56330	3.22509	4.04893	5.07236	6.34118	7.91108	9.84973	12.23915	15.17862
25	2.66583	3.38635	4.29187	5.42743	6.84847	8.62308	10.83470	13.58546	17.00606
26	2.77246	3.55567	4.54938	5.8073	7.39635	9.39915	11.91817	15.07986	19.04007
27	2.88336	3.73345	4.82234	6.2138	7.98806	10.24508	13.10999	16.73864	21.32488
28	2.99870	3.92012	5.11168	6.64883	8.62710	11.16713	14.42099	18.57990	23.88386
29	3.11865	4.11613	5.41838	7.11425	9.31727	12.17218	15.86309	20.62369	26.74993
30	3.24330	4.32194	5.74349	7.61225	10.06265	13.26767	17.44940	22.89220	29.55903

The Construction of the preceeding Table III.

The numbers 1, 2, 3, 4, &c. to 30, in the first Column on the left hand signifie years; the numbers, 4, 5, 6, 7, 8, 9, 10, 11, and 12, placed at the head of the rest of the Columns signifie rates of interest, for 100*l.* lent for a year, and the numbers placed in the several Columns underneath those rates of interest, are found out by the *Rule of Three* in decimals, in manner following, *viz.*

I.	100 : 104 :: 1	:(1.04
II.	100 : 104 :: 1.04	:(1.0816
III.	100 : 104 :: 1.0816	:(1.12486

That is to say, First if 100*l.* put forth at interest for a year be augmented to 104*l.* at the years end, what will 1*l.* be then augmented unto at the same rate? *Answ.* 1.04*l.* (that is 1*l.* 0*s.* 9*d.* 2*f.* and somewhat more) which 1.04 (or 1.04000, the Cyphers after the 4 being of no value in decimals) is the first number in the second Column belonging to 4 per cent. and is placed right against 1 year in the first Column.

Secondly, say if 100*l.* lent for a year be augmented to 104*l.* at the years end, what will 1.04*l.* be then augmented unto at the same rate? *Answ.* 1.0816*l.* (that is 1*l.* 1*s.* 7*d.* 2*f.* +) which 1.0816 is the second number in the said Column of 4 per cent. and is placed right against 2 years in the first Column.

Thirdly,

Thirdly, as 100 is to 104, so is 1.0816 to 1.124864 (or *1l. 2s. 5d. 2f. +*) which 1.124864 is the third number in the *Column* of 4 per cent. and is placed right against 3 years in the first *Column*. Hence it appears, that 1 *l.* at 4 per cent. per annum, compound interest, will at the end of 3 years be augmented unto 1.124864 *l.* (that is, 1 *l.* 2 *s.* 5 *d.* 2 *f.* and somewhat more.)

After the same manner the rest of the numbers in the second *Column*, as also in the other *Columns* are found out (*mutatis mutandis.*)

The Use of the preceeding third Table.

Quest. 1. What will 136 *l.* 15 *s.* 6 *d.* be augmented unto, being forborn 20 years, interest upon interest being computed at the rate of 6 per cent. per annum? *Answ.* 438 *l.* 13 *s.* 1 *d.* very near, which is thus found out.

First, looking into the fourth *Column* of the said third *Table*, to wit, that *Column* which hath the figure 6 placed at the head of it, I find right against 20 years the number 3.20713, which shews that 1 *l.* being continued 20 years at 6 per cent. per annum, compound interest, and all forborn untill the end of the said term will be augmented unto 3.20713 *l.* (that is 3 *l.* 4 *s.* 1 *d.* 2 *f.* and somewhat more) therefore after the 15 *s.* 6 *d.* in the *Question* is reduced to the decimal .775 (by the sixteenth Rule of the 23 Chapter of the preceeding Book) I multiply the said tabular number 3.20713 by 136.775 (the sum propounded in the *Question*) according to the second Rule of the 26th Chapter, so the product is found

found to be 438:665, &c. that is, 438 l. 13 s. 1 d. for the answer of the question. View the operation here following.

$$\begin{array}{r}
 1 \cdot 320713 : : 136.775 : (438.665 + \\
 136.775 \\
 \hline
 1603565 \\
 2244991 \\
 2244991 \\
 1924278 \\
 962139 \\
 320713 \\
 \hline
 438165520575
 \end{array}$$

Quest. 2. If 320 l. be forborn 11 years, at interest upon interest at 5 per centum, per annum, what will be due at the end of those eleven years for principal and interest? *Ans.* 547 l. 6 s. 1 d. + For in the third Column of the third Table, under the figure 5 at the head of the column and right against 11 years you will find this number 1.71033, which shews that 1 l. at the end of 11 years will at five per centum, per annum, compound interest, be augmented to 1.71033 (that is 1 l. 14 s. 2 d. 1 f. and somewhat more) wherefore by multiplying the said 1.71033 by 320 the number of pounds propounded in the question) the product will be 547.305, &c. that is 547 l. 6 s. 1 d. + for the answer of the question. See the following operation:

After
any o
the thir
XIII.
early
years,
that th
mpoun
r each
e time
the t
llowing
o l. pa
unpai
what
g comp
r each y
e to th
nswer, r
It is ev
mputed
ill be au
arth year
cond ye
ars, (to

$$1 : 1.71033 :: 320 \cdot (547.305 +$$

320

30420660

5130099

547130560

After the same manner the numbers belonging to any of the other rates of interest mentioned in the third Table are to be used.

XIII. When an Annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount unto, compound interest being computed for each particular Annuity from the time it became due until the end

The manner of summing up Annuities in arrear with allowances of interest upon interest.

of the term of years, the work will be as in the following example; viz. Suppose an Annuity of 300*l.* payable at yearly payments be forborn, and unpaid until the end of four years, the question is what will then be due, compound interest being computed at the rate of 6 per cent. per annum, for each yearly payment from the time it becomes due to the end of the said term of four years?

Answer, 1312*l.* 7*s.* 8*d.* very near.

It is evident by the question, that there must be computed what 300*l.* due at the third years end will be augmented unto in one year (to wit, the fourth year) at 6 per cent. Also what 300*l.* due at the second years end will be augmented unto in two years, (to wit, the third and fourth years; like-

B b

wise

wise what 300 *l.* due at the first years end, will be augmented unto, in the three following years (to wit the second, third and fourth years) all which sums being added to 300 *l.* (the payment due at the end of the fourth year, which is incapable of any improvement) the aggregate or sum will be the total money in Arrear at the end of the fourth year, to wit, 1312, $\frac{3848}{1000}$ *l.* as may appear by the following operation, viz.

The last payment of the Annuity due at the end of the fourth year is } 300.

Again, the 300 *l.* due at the third years end, will in one year after the rate of 6 *per centum*, be augmented unto } 318.

Also 300 *l.* due at the second years end, will in two years at the rate of 6 *per centum*, *per annum*, compound interest, be augmented unto (as appears by the first example of the twelfth Rule aforegoing.) } 337.08.

In like manner; 300 *l.* due at the first years end, will in three years be augmented unto } 357.304

The sum due at the four years end } 1312.384

The invention of the numbers before-mentioned being well examined, it will appear, that if an Annuity or Rent payable at yearly payments be improved

improved to the utmost as interest upon interest, and all forborn or respited unto the end of certain years, the total then due will be the sum of a rank of continual proportionals as many in number as there are yearly payments, the first of which proportionals is the first (or any one) years rent, and the second proportional proceeds from the first in the same rate as 106 proceeds from 100, if the rate of interest be 6 per centum, (or as 108 proceeds from 100, if the rate of interest be 8 per centum, &c.) and so likewise the third from the second, the fourth from the third, &c. after the manner of the operation in the first example of the twelfth Rule of this Chapter.)

Otherwise.

Find a principal which may have such proportion to 300 as 100 hath to 6, and say by the *Rule of Three*,

$$6 \cdot 100 :: 300 \cdot 5000.$$

That is to say, as 6 *l.* interest hath 100 *l.* for a principal, so 300 *l.* interest hath 5000 *l.* for a principal; then seek what 5000 *l.* will be augmented unto, being forborn four years at 6 per centum, *per annum*, compound interest (after the manner of the first *Example* of the 12th *Rule* aforegoing :) so will you find 6312.3848, from which subtracting the said principal 5000 *l.* the remainder (as before) is 1312.3848 *l.* being the sum which 300 *l.* Annuity will be augmented unto at the end of four years, according to the said rate of interest, the Annuity being payable at yearly payments.

The Reason of the latter Rule.

If a principal be put forth at interest upon interest payable by yearly payments, and all be forborn untill the end of certain years, the total then due is equal to the aggregate or sum of these three numbers, to wit, the said principal first put forth, the sum of the annual simple interest of that principal; and the utmost improvement of those simple interests by computing interest upon interest. wherefore if from the said aggregate the first principal be subtracted, the remainder must necessarily consist of the sum of the annual simple interests, (which are in the nature of an Annuity) and the utmost improvement of those simple interests (or Annuity) by computing Interest upon Interest.

The Construction of the following Table IV.

Upon the aforesaid grounds, the following Table IV. is calculated, to shew what one pound Annuity, payable at yearly payments, and forborn any number of years under 31, will amount unto by computing interest upon interest at any of the rates exprest at the head of the said Table.

But the same Table may be more easily composed by the addition of the numbers in the preceeding Table III. in this manner, viz. the first number in each of those columns in the following Table IV. at the head whereof are placed the numbers 4, 5, 6, 7, 8, 9, 10, 11, and 12, signifying rates of interest.

Interest per centum, is 1 or unity, the second number in each of these columns in the latter Table is compos'd of 1 or unity, and the first number in the respective columns of the said preceeding Table III.

Also the third number in each of the said columns of this latter Table is compos'd of 1, and the sum of the first and second number of the respective columns of the former Table, and in that order the rest are found out; or more easily thus: The third number in the latter Table is compos'd of the second number in the latter, and of the second in the former; the fourth number in the latter is compos'd of the third in the latter, and of the third in the former, &c. But you are to observe that according to either of these ways of composing the fourth Table by Addition, the numbers in the preceeding Table III. ought to be continued to more places than are there express'd to prevent error, which may happen by adding of defective decimal fractions.

B b 3

TABLE

TABLE IV.

Which sheweth what one Pound Annuity, payable by yearly payments, and forborn any number of years under 31, will amount unto, at the end of the term, compound interest being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum.

Years.	4	5	6	7	8	9	10	11	12
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.04000	2.50000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440
4	3.24646	4.31012	4.37461	4.43994	4.50611	4.57312	4.64100	4.70973	4.77932
5	5.41632	5.52563	5.63709	5.75073	5.86660	5.98471	6.10510	6.22780	6.35284
6	6.63297	6.80190	6.97531	7.15329	7.33592	7.52333	7.71561	7.91285	8.11518
7	7.89829	8.14200	8.39383	8.65402	8.92280	9.20043	9.48717	9.78327	10.08901
8	9.21422	9.64900	9.89746	10.25980	10.63662	11.02847	11.43588	11.85943	12.29969
9	10.58279	11.02656	11.49131	11.97798	12.48755	13.02103	13.57947	14.16397	14.77565
10	12.00610	12.55789	13.18079	13.81644	14.48656	15.19292	15.93742	16.72200	17.54873
11	13.48635	14.20678	14.97164	15.78359	16.64548	17.56029	18.53116	19.56142	20.65458
12	15.02580	15.91712	16.86994	17.88845	18.97712	20.14071	21. 8428	22.71318	24.13313
13	16.62683	17.71298	18.88213	20.14064	21.49529	22.95338	24.52271	26.21163	28.02910
14	18.29191	19.59863	21.01506	22.55048	24.21492	26.01918	27.97498	30.09491	32.39266
15	20.02358	21.57856	22.27596	25.12902	27.15211	29.36091	31.77248	34.40535	37.27971

15 20.02358 21.57856 22.27596 25.12902 22.55048 24.21492 26.01918 27.57498 30.09491 32.39260
 16 21.57856 23.27596 25.12902 27.55048 29.21492 31.77248 34.40535 37.27971

A continuation of the preceding Table IV.

Years	4	5	6	7	8	9	10	11	12
16	21.82453	23.65749	25.67252	27.88805	30.32428	33.00339	35.94972	39.18993	42.75328
17	23.69751	25.84036	28.21287	30.84021	33.75022	36.97370	40.54470	44.50084	48.88367
18	25.64541	28.13238	30.90565	33.99903	37.45024	41.30133	45.59917	50.39592	55.74971
19	27.67120	30.53900	33.75999	37.37896	41.44626	46.01845	51.15009	56.93948	63.43968
20	29.77807	33.06595	36.78559	40.99549	45.76196	51.16011	57.27499	64.20283	72.05244
21	31.96920	35.71925	39.99272	44.86517	50.42292	56.76453	64.00249	72.26514	81.69873
22	34.24796	38.50521	43.39228	49.00573	55.45675	62.87333	71.40274	81.21430	92.50258
23	36.61788	41.43047	46.99582	53.43614	60.89329	69.53193	79.54302	91.14788	104.60289
24	39.08260	44.50199	50.81557	58.17667	66.76475	76.78981	88.49732	102.11741	118.15524
25	41.64590	47.72709	54.86451	63.24903	73.10599	84.70087	98.34705	114.41330	133.33387
26	44.31174	51.11345	59.15638	68.67646	79.95441	93.32397	109.18176	127.99877	150.33393
27	47.08421	54.66912	63.70576	74.48382	87.35076	102.72313	121.09994	143.07863	169.37400
28	49.96758	58.40258	68.52810	80.69769	95.33882	112.96821	134.20993	159.81728	190.69888
29	52.96628	62.32271	73.63979	87.34652	103.96593	124.13535	148.63092	178.39718	214.58275
30	56.08492	66.42384	79.55818	94.46078	114.28321	136.30752	164.49402	199.02087	241.33268

The use of the preceeding Table IV.

The use of the said fourth Table will be manifest by the manner of solving this question, viz. If an Annuity of 20 *l.* payable by yearly payments for 15 years; be all forborn or unpaid until the end of the said term, what will it then amount unto, upon a computation of interest upon interest, at the rate of 6 per centum, per annum? *Answ.* 465 *l.* 10 *s.* 4 *d.* 2 *f.* very near, as by the following operation is evident; for in the column belonging to 6 per centum (to wit, that column which hath the figure 6 placed at the head of it) right against 15 years, you will find 23.27596, which shews that an Annuity of 1 *l.* payable at yearly payments for 15 years, will at the end of the said term (compound interest being computed at 6 per cent. per annum) amount unto 23.27596 *l.* (or 23 *l.* 5 *s.* 6 *d.* +) Therefore multiplying the said tabular number 23.27596 by 20. (because the Annuity propounded is 20 *l.*) the product will be 465.519 + that is 465 *l.* 10 *s.* 4 *d.* 2 *f.* which is the answer of the question; view the following operation.

$$\begin{array}{r}
 1 \cdot 23.27596 : : 20 : (465.519+ \\
 \quad \quad \quad 20 \\
 \hline
 465\,51920
 \end{array}$$

In the same manner the numbers in the other columns are to be used.

XIV. When a sum of money is due of rebate at at a time to come, and it is required Compound in- to know what it is worth in ready terest money, rebate being made at a given rate

rate of compound interest, the work will not be much different from the 12 Rule of this Chapter, viz. there must be found a series or rank of continual proportionals more in number by one, than is the number of years in the question; of which proportionals, the first is the money propounded to be rebated, the second must decrease or lessen from the first, the third from the second, &c. in such manner or rate as 100 decreaseth from 106 (or as 100 from 108, if the rate of interest be 8 per cent.) then will the last proportional be the answer of the question: So if 378 ⁷⁴³⁰⁸⁸/₁₀₀₀₀₀₀ l. be due at the end of four years wholly to come, it will be found to be worth in ready money 300 l. rebate being made at compound interest at 6 per cent. as by the four following Rules of Three is manifest, which may be proved by the preceding twelfth Rule, where it will appear that 300 l. being forborn four years, will at the said rate of compound interest be augmented unto 378.743088 l.

	l.		l.
106 : 100 ::	{ 378.743088	.	357.3048
	{ 358.3048	.	337.08
	{ 337.08	.	318.
	{ 318.	.	300.

Upon this ground the following Table V. is calculated, to shew what one pound due at the end of any number of years to come, is worth in present money, rebate being made at the rates of compound interest, mentioned in the said Table; by the help whereof and of *Multiplication*, questions of rebate for any sum propounded may be performed without considerable error.

TABLE

TABLE V.

Which sheweth what one pound, payable at the end of any term of what years is to come under 31, is worth in ready money, discount or rebate being yearly computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per ann. compound interest.

Years	4	5	6	7	8	9	10	11	12
1	.961538	.952381	.943396	.934579	.925925	.917431	.909090	.900900	.892857
2	.924556	.907029	.889996	.873438	.857338	.841680	.826446	.811622	.797193
3	.888996	.863837	.839619	.816297	.793832	.772183	.751314	.731191	.711780
4	.854802	.822702	.792095	.762895	.735029	.708425	.683013	.658731	.635518
5	.821927	.783526	.747258	.712986	.680583	.649931	.620921	.593451	.567426
6	.790314	.740215	.704960	.666342	.630169	.596267	.564447	.534640	.506631
7	.759917	.710681	.665057	.622749	.583490	.547034	.513158	.481658	.452349
8	.730690	.676839	.627412	.582009	.540268	.501866	.466507	.433926	.403883
9	.702586	.644608	.591898	.543933	.500248	.460427	.424097	.390924	.360610
10	.675564	.613913	.558391	.508349	.463193	.422410	.385543	.352184	.321973
11	.649580	.584679	.526787	.475092	.428882	.387532	.350494	.317283	.287476
12	.624596	.556837	.496989	.444012	.397113	.355534	.318630	.285840	.256675
13	.600573	.530321	.468839	.414964	.367697	.326178	.289664	.257514	.229174
14	.577474	.505067	.442300	.387817	.340461	.299246	.263331	.231994	.204619
15	.555264	.481017	.417266	.362446	.315241	.274538	.239392	.209004	.182696

A continuation of the preceding Table V.

Years.	4	5	6	7	8	9	10	11	12
16	.533908	.438111	.393646	.338734	.291860	.251869	.217629	.188292	.163121
17	.513373	.436296	.371364	.316374	.270269	.231073	.197844	.169632	.145644
18	.493628	.413520	.350343	.295864	.250249	.211993	.179858	.152822	.130039
19	.474642	.395733	.330512	.276508	.231712	.194489	.163508	.137677	.116106
20	.456386	.376889	.311804	.258419	.214548	.178430	.148643	.124034	.103666
21	.438833	.358942	.294153	.241513	.198655	.163698	.135130	.111742	.092559
22	.421955	.341849	.277505	.225713	.183940	.150181	.122840	.100668	.082642
23	.405726	.325571	.261797	.210947	.170315	.137781	.111678	.090692	.073787
24	.390121	.310067	.246978	.197146	.157699	.126405	.101525	.081705	.065882
25	.375116	.295302	.232998	.184249	.146018	.115967	.092296	.073608	.058823
26	.360689	.281240	.219810	.172195	.135201	.106392	.083905	.066313	.052520
27	.346816	.267848	.207367	.160930	.125186	.097607	.076277	.059742	.046893
28	.333477	.255093	.195630	.150402	.115913	.089548	.069343	.053821	.041869
29	.320651	.242946	.184556	.140562	.107327	.082154	.063039	.048487	.037383
30	.308318	.231377	.174110	.131367	.099377	.075371	.057308	.043682	.033377

The Construction of the preceeding Table V.

The numbers 1, 2, 3, 4, &c. to 30, in the first column on the left hand, signify years; the numbers 4, 5, 6, 7, 8, 9, 10, 11 and 12, placed at the head of the rest of the columns signify rates of interest for 100*l.* lent for a year, and the numbers placed in the several columns underneath those rates of interest are found out by the *Rule of Three* in decimals, in manner following, *viz.*

- | | | | |
|------|--|--------------------------|---------------------|
| I. | | 104.100 :: ,1 | . (,9615384615, &c. |
| II. | | 104.100 :: ,9615384615 + | (,9245562, &c. |
| III. | | 104.100 :: ,9245562, &c. | (,888996 + |

That is to say, First, if 104 decrease to 100, or if 104*l.* payable at the end of a year to come be worth 100*l.* ready Money, what ready Money, is 1*l.* due at the end of a year to come worth?
Ans. .9615384615 + (or 19*s.* 2*d.* 3*f.* very near.) So that .961538 is the first decimal in the second column belonging to 4 per centum, in Table V. and is placed right against 1 year in the first column.

Secondly, say in like manner if 104 decrease to 100, what will .9615384615, &c. (the decimal found by the first *Rule of Three*) decrease unto?
Ans. .9245562, &c. the first 6 places whereof, to wit, 924556 are the second decimal in the said column of 4 per cent. which is placed right against two years.

Thirdly,

Thirdly, as 104 is to 100; so is, 9245562, &c. (the decimal found by the second Rule of Three) to .888996 $\frac{1}{4}$ (or 17s. 9d. 1 f. $\frac{1}{4}$) which is the third decimal in the column of 4 per centum. Hence it appears, that 1 l. due at the end of 3 years to come is worth .888996 $\frac{1}{4}$ (or 17s. 9d. 1 f. and somewhat more) in ready Money, rebate being made at the rate of 4 per centum, per annum,) compound interest.

After the same manner the rest of the decimal fractions in the said second column, as also in the other columns are found out (*mutatis mutandis.*)

The use of the preceeding Table V.

To exemplify the said fifth Table, let it be required to find out how much ready Money will discharge a debt of 356 l. payable at the end of seven years to come, by rebating at the rate of 7 per cent. per annum, compound interest? *Ans.* 221 l. 13 s. 11 d. 3 f. very near. For in the fifth column, at the head whereof is placed 7, signifying 7 per centum, right against 7 years, I find .622749, which shews that 1 l. due at the end of 7 years to come is worth in present Money .622749 decimal parts of a pound, rebate being made at the said rate of compound interest. Therefore multiplying the said tabular number .622749 by the said 356 l. (the debt propounded) the product (according to the second Rule of the 26th Chapter) will be 221.698, &c. that is, 221 l. 13 s. 11 d. 3 f. which is the Answer of the question. See the subsequent operation.

1. . 622749 :: 356 : (221.698 +

356

3736494

3113745

1868247

2211698644

In the same manner the numbers in the other columns are to be used.

To find the present worth of Annuities by a computation of compound interest. XV. The finding out the present worth of an Annuity is grounded upon this Foundation, to wit, if the present Money which is paid for the purchase of an Annuity, to continue any term of Years, be put forth at any rate of compound interest, and all forborn until the end of the said term, and that the total Money then due be put into one Scale: Also if the total Sum of the utmost improvements of the annual payments of the Annuity, put forth at the same rate of compound interest, from the time those annual payments become due until the end of the term, be put in the other Scale, the Scales must be even, viz. the said two total sums of Money must be equal one to the other.

Now to find out such a present worth of an Annuity, there are divers ways, some of which I shall here explain by examples :

First therefore let it be required to find the present worth of an Annuity of 378.743088 l. to continue three years *compound interest* being computed at 6 per cent. per ann. *Ans.* 1212.3848 l.

It

It is evident by the question, that there must be computed (after the manner of the Example upon the fourteenth Rule foregoing) the present worth of 378.743088 ~~1000000~~ $l.$ due at the first years end, also the present worth of the like sum due at the second years end, and in like manner for the third year; all which particular present values being added together, the aggregate or sum will be the total present worth of the Annuity propounded, viz.

378.743088 $l.$ payable at the end of } $l.$
 one year is worth in ready money }
 (as is evident in the fourteenth Rule } 357.3048
 foregoing.) }

Also the like sum payable at the }
 end of 2 years to come, is worth in } 337.08
 ready money }

Again, the like sum payable at the }
 end of three years to come, is worth } 318
 in ready money }

Therefore the total present worth }
 of an Annuity of 378.743088 $l.$ to } 1012.3848
 continue 3 years is }

Otherwise,

Find a Principal which may be in such proportion to the propounded Annuity 378.743088 $l.$ as 100 is to 6. Which will be exactly 6312.3848 $l.$ for

$$6 : 100 :: 378,743088 . 6312, 3848$$

Then supposing this principal so found to be a sum due at the end of three years to come, find what it will be worth in ready money, by diminishing it according to the fourteenth Rule of this Chapter, so will you find 5300 $l.$ for the ready money equivalent to the said 6312.3848 $l.$ due at the end

end of three years, which ready money 5300 *l.* being substracted from the said 6312.3848 *l.* leaves (as before) 1012.3848 *l.* for the present worth of the said Annuity of 378.743088 *l.* to continue three years, compound interest being allowed at 6 per centum, per annum.

The Reason of the latter Rule.

It will not be difficult to apprehend, that if 6312.3848 *l.* ready money be put forth as a Principal at interest upon interest, it will at three years end be augmented unto an Aggregate or sum compos'd of these three numbers, to wit, the said Principal 6312.3848; the sum of the annual simple interests of that Principal, and the utmost improvement of those simple interests by interest upon interest: And because (by the operation aforesaid) 5300 *l.* ready money (part of the said ready money 6312.3848.) will at three years end be augmented unto 6312.3848 *l.* part of the said Aggregate, therefore 1012.3848 *l.* the complement or remaining part of the said ready money 6312.3848 *l.* must necessarily be augmented unto the complement or remaining part of the said Aggregate, which remaining part last mentioned is composed of the sum of the aforesaid simple interests, and of their utmost improvement at interest upon interest, that is, the said remainder is the utmost improvement of an Annuity of 378.743088 *l.* to continue three years, compound interest being allowed at 6 per centum, per annum.

The

The Construction of the following Table VI.

Upon the aforesaid grounds the following Table VI. is calculated to shew how much ready money an *Annuity* of one pound to continue any number of *years* under 31. and payable at yearly payments, is worth, upon a *computation* of compound interest at any of the rates *per centum*, mentioned at the head of the said Table. But the said Table VI. may more easily be compos'd by the help of the preceding Table V. in this manner, *viz.* the first number in every of the *Columns* (except the *Column* of *years*) in the following Table VI. is the same with the first number in the like *Columns* respectively in the preceding Table V. the second number in each of the said *Columns* of the sixth Table is the sum of the first and second numbers in the respective *Columns* of the fifth Table; the third number in the said *Columns* of the fifth Table is the sum of the first, second and third numbers in the respective *Columns* of the fifth Table: Or yet more easily thus, the third number in the sixth Table, is compos'd of the third in the fifth Table and of the second in the sixth; the fourth number in the sixth Table is compos'd of the fourth in the fifth and of the third in the sixth; the like is to be understood of the rest. But you are to observe that according to this way of composing the sixth Table by Addition, the numbers of the fifth Table must be continued to more places than are there express'd, to prevent error arising by the addition of defective decimal Fractions.

TABLE VI.

Which sheweth the present worth of one Pound Annuity, to continue any term of year under 31, and payable by yearly payments, compound interest being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum.

Years.	4	5	6	7	8	9	10	11	12
1	.96153	.95238	.94339	.93457	.92592	.91734	.90909	.90090	.89285
2	1.88609	1.85941	1.83339	1.80801	1.78326	1.75911	1.73553	1.71252	1.69005
3	2.77509	2.72324	2.67301	2.62431	2.57709	2.53129	2.48685	2.44371	2.40183
4	3.62989	3.54595	3.46510	3.38721	3.31212	3.23971	3.16986	3.10244	3.03734
5	4.45182	4.32947	4.21236	4.10019	3.99270	3.88965	3.79078	3.69589	3.60477
6	5.24213	5.07569	4.91732	4.76653	4.62287	4.48591	4.35526	4.23053	4.11140
7	6.00205	5.78637	5.58233	5.38928	5.20636	5.03295	4.86841	4.71219	4.56375
8	6.73274	6.46321	6.20979	5.97129	5.74663	5.53481	5.34492	5.14612	4.96763
9	7.43333	7.10782	6.80169	6.51523	6.24688	5.99524	5.75901	5.53704	5.32824
10	8.11089	7.72173	7.36008	7.02358	6.71008	6.41765	6.14456	5.88923	5.65022
11	8.76047	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	6.20651	5.93769
12	9.38507	8.86325	8.38384	7.94268	7.53607	7.16072	6.81369	6.49235	6.19437
13	9.98964	9.39357	8.85268	8.35765	7.90377	7.48690	7.10335	6.74987	6.42354
14	10.56312	9.89864	9.29498	8.74546	8.24423	7.78614	7.36668	6.98186	6.62816
15	11.11838	10.37965	9.71224	9.10791	8.53947	8.06068	7.60608	7.19087	6.81086

A continuation of the preceding Table VI.

Tem

A. continuation of the preceding Table VI.

Years	4	5	6	7	8	9	10	11	12
16	11.65229	10.83776	10.10589	9.44664	8.85136	8.31255	7.82371	7.37916	6.97398
17	12.16560	11.27406	10.47725	9.76322	9.12163	8.54363	8.02155	7.54879	7.11962
18	12.65929	11.68958	10.82760	10.05908	9.37188	8.75562	8.20141	7.70161	7.24966
19	13.13393	12.08531	11.15811	10.33559	9.60359	8.95011	8.36492	7.83929	7.46577
20	13.59032	12.46220	11.46992	10.59401	9.81814	9.12854	8.51356	7.96332	7.46944
21	14.02915	12.82115	11.76407	10.83557	10.01680	9.29224	8.64869	8.07507	7.56200
22	14.45111	13.16300	12.04158	11.06124	10.20074	9.44242	8.77154	8.17574	7.64464
23	14.85683	13.48857	12.30337	11.27218	10.37105	9.58020	8.88322	8.26643	7.71843
24	15.24696	13.79864	12.55035	11.46933	10.52875	9.70661	8.98474	8.34813	7.78431
25	15.62207	14.09394	12.78335	11.65358	10.67477	9.82257	9.07704	8.42174	7.84313
26	15.98276	14.37518	13.00316	11.82577	10.80997	9.92897	9.16094	8.48805	7.89505
27	16.32958	14.64303	13.21053	11.98671	10.93516	10.02657	9.23722	8.54780	7.94255
28	16.66305	14.89812	13.40616	12.13711	11.05107	10.11612	9.30656	8.60162	7.98441
29	16.98371	15.14107	13.59071	12.27767	11.15840	10.19828	9.36960	8.65011	8.02180
30	17.29292	15.39244	13.76482	12.40004	11.25778	10.27365	9.42691	8.69379	8.05518

The Use of the preceding Table VI.

The use of the said sixth Table will appear by the manner of solving these two subsequent questions, viz.

Quest. 1. What is the present worth of an Annuity or rent of 56 l. per annum payable by yearly payments for 21 years, accompting interest upon interest at the rate of 6 per centum, per annum?

Ans. 658 *l.* 15 *s.* 9 *d.* very near, thus found out; In the fourth Column of the preceding Table IV. under the figure 6 at the head, and right against 21 years, I find 11.76407, which shews that an Annuity of 1 *l.* payable by yearly payment for 21 years, is worth in present money 11.76407 *l.* or 11 *l.* 15 *s.* 3 *d.* 1 *f.* (and somewhat more) interest upon interest being computed on both sides at the rate of 6 per centum, per annum; therefore multiplying the said tabular number 11.76407 by 56, (56 because the Annuity propounded is 56 pound) the product (according to the second rule of the 26th Chapter of the preceding Book) will be found to be 658.787, &c. that is, 658 *l.* 15 *s.* 9 *d.* very near: Wherefore I conclude that the Answer of the Question is 658 *l.* 15 *s.* 9 *d.* View the following Operation.

$$1 : 11.76407 :: 56 : (658,787 +)$$

$$\begin{array}{r} 7058442 \\ 5882035 \\ \hline \end{array}$$

$$658178792$$

Quest. 2. What is the present worth of an Annual Rent of 45 l. payable by yearly payments for 21 years, interest upon interest being computed at 10 per centum, per annum? *Ans.* 389 l. 3 s. 10 d. very near; for in the Column of 10 per centum, in the said sixth Table, right against 21 years, and under 10, at the head I find this number 8.64869; which shews that at 10 per centum, compound interest, an Annuity or Rent of 1 l. payable by yearly payments for 21 years, is worth in ready money 8.64869 l. that is, 8 l. 12 s. 11 d. 3 f. therefore multiplying the said tabular number 8.64869, by 45 (the Rent propounded) the Product will be 389.191 +, that is 389 l. 3 s. 10 d. very near, which is the Answer of the Question.

$$1 : 8.64869 :: 45 : (389 191 +)$$

$$\begin{array}{r} 45 \\ \hline 4324345 \\ 3459476 \\ \hline \end{array}$$

$$38919105$$

In the same manner the numbers in the other Columns of Table VI. are to be used.

Moreover, the numbers in the said sixth Table will at first sight shew how many years purchase an Annuity to continue any number of years under 31 is worth, to be sold for present money, compound interest being computed on both sides, at any of the said rates, 4, 5, 6, 7, 8, 9, 10, 11 and 12 *per centum*: So if you desire to know how many years purchase an Annuity issuing out of Lands for 21 years, to begin presently, is worth, if it were to be sold for ready money, when the current rate of interest is 6 *per centum*; seek in the first Column of Table VI. for 21 years, and carry your eye from thence equidistant to the head-line of the Table till you come under 6, which (as before hath been said) signifies 6 *per centum*. So in the fourth Column you will find 11.76407, whereof you need only consider 11.76, which shews that the said Annuity is worth 11 years purchase, (or 11 times one years rent whatever it be) and 76 parts of one years purchase divided into 100 parts, or a $11\frac{3}{4}$ years purchase and a little more. The same Annuity when money was at 8 *per centum* was worth 10 years purchase and about $\frac{1}{100}$ part of a years purchase more, as the number in the Column of 10 *per centum* right against 21 years will discover.

In like manner supposing 10 *per centum* to be a fit rate to be allowed in the valuation of Leases of houses, the Lease of a house for 21 years will be found by the said Table to be worth 8 years purchase and $\frac{64}{100}$ parts of a years purchase, or 8 years purchase

Chap.
purcha
purcha
that i
be set
of the
tainty
XV
and it
nuity
years,
rate)
presup
the te
of the
fifteen
then
As
ad y
t
So
gin
in pr
being
Answ
Fo
wit,
valu
Rule
The

purchase and an half, and half a quarter of a years purchase and somewhat more: But note here that in valuing of Leases, the rate *per centum* is to be set higher or lower according to the goodness of the thing leased, and the certainty or uncertainty of the rent.

XVI. When a sum of money is propounded, and it is required to know what Annuity (to continue any number of years, and according to any given rate) that sum will buy, you may presuppose at pleasure an Annuity for the term of years propounded, and find the value of that Annuity in ready money (according to the fifteenth Rule aforegoing) at the rate assigned; then will the proportion be as followeth.

As the value found is in proportion to the supposed Annuity; so is the sum of Money propounded, to the Annuity required.

So if it be required to find what Annuity to begin presently, and to continue three years 500*l.* in present money will purchase, compound interest being computed at 6 *per centum*, *per annum*: The Answer will be 187. *l.* 1 *s.* 1 *d.* very near.

For presupposing an Annuity at pleasure, to wit, 378.743088 *l.* payable yearly for 3 years, the value thereof in present money will (by the 15th. Rule of this Chapter) be found to be 1012.3848*l.* Therefore by the Rule of proportion, say,

$$1012.3848 \cdot 378,743088 :: 500 \cdot 187,054.$$

That is to say, if 1012.3848 l. in ready money will buy an Annuity of 378;743088 l. (to continue three years, then 500 l. in present money will purchase an Annuity (to continue the same term of years, and at the same rate of interest) of 187.054, &c. that is, 187 l. 1 s. 1 d. very near.

The Construction of the following Table VII.

Upon this ground the following Table VII. is calculated to shew what Annuity (to continue any term of years under 31, and at any rate of interest mentioned at the head of that Table) one pound will purchase, by which Table, and by the help of *Multiplication*, questions concerning the *Purchase of Annuities, Rents or Pensions*, by any sum of ready money propounded, may be resolved without considerable Error. But a more ready way to make the said Table VII. may be this following, viz.

Forasmuch as it is evident by the construction of the third Table, foregoing, that one pound ready money is equivalent unto 1.06 l. payable at the end of a year to come, at the rate of 6 per centum, per annum; therefore this 1.06 is to be the first number in the Column intitled 6 per centum in the subsequent Table VII. Again, the present value of one Pound Annuity to continue two years at the said rate will be found by the preceding Table VI. to be near 1.83339 l. Therefore by the Rule of Proportion, say,

1.83339

I.83339 : I :: I . 54543, &c.

That is, if 1.83339 *l.* ready money will purchase an Annuity of 1 *l.* (to continue two years; what Annuity to continue the same time will 1 *l.* in present money purchase? *Answer*, an Annuity of 54543 *l.* that is 10*s.* 11*d.* very near, to continue two years; therefore the said Decimal 54543 *l.* is to be placed as the second number in the fourth Column of the subsequent *Table VII.* Hence it follows, that if 1 or unity be divided by every one of the numbers in all the Columns of *Table VI.* except the first Column of years, the quotients will give the respective numbers to be placed in the like Columns of the following *Table VII.* in which operation it will be requisite, that the numbers in the preceding *Table VI.* be continued to more places than are there express'd, to prevent error that will arise by adding of defective Decimals.

TABLE	
13280	10791
14455	11871
15074	12421
15280	13041
15328	13681
15330	14291
15332	14871
15334	15471
15336	16071
15338	16671
15340	17271
15342	17871
15344	18471
15346	19071
15348	19671
15350	20271
15352	20871
15354	21471
15356	22071
15358	22671
15360	23271
15362	23871
15364	24471
15366	25071
15368	25671
15370	26271
15372	26871
15374	27471
15376	28071
15378	28671
15380	29271
15382	29871
15384	30471
15386	31071
15388	31671
15390	32271
15392	32871
15394	33471
15396	34071
15398	34671
15400	35271
15402	35871
15404	36471
15406	37071
15408	37671
15410	38271
15412	38871
15414	39471
15416	40071
15418	40671
15420	41271
15422	41871
15424	42471
15426	43071
15428	43671
15430	44271
15432	44871
15434	45471
15436	46071
15438	46671
15440	47271
15442	47871
15444	48471
15446	49071
15448	49671
15450	50271
15452	50871
15454	51471
15456	52071
15458	52671
15460	53271
15462	53871
15464	54471
15466	55071
15468	55671
15470	56271
15472	56871
15474	57471
15476	58071
15478	58671
15480	59271
15482	59871
15484	60471
15486	61071
15488	61671
15490	62271
15492	62871
15494	63471
15496	64071
15498	64671
15500	65271
15502	65871
15504	66471
15506	67071
15508	67671
15510	68271
15512	68871
15514	69471
15516	70071
15518	70671
15520	71271
15522	71871
15524	72471
15526	73071
15528	73671
15530	74271
15532	74871
15534	75471
15536	76071
15538	76671
15540	77271
15542	77871
15544	78471
15546	79071
15548	79671
15550	80271
15552	80871
15554	81471
15556	82071
15558	82671
15560	83271
15562	83871
15564	84471
15566	85071
15568	85671
15570	86271
15572	86871
15574	87471
15576	88071
15578	88671
15580	89271
15582	89871
15584	90471
15586	91071
15588	91671
15590	92271
15592	92871
15594	93471
15596	94071
15598	94671
15600	95271

TABLE VII.

Which sheweth what Annuity, payable by yearly payments, to continue any term of years under 31, one Pound will purchase, compound interest, being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum.

Years	4	5	6	7	8	9	10	11	12
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	.53019	.53780	.54543	.55309	.56076	.56846	.57619	.58393	.59169
3	.36034	.36720	.37411	.38105	.38803	.39505	.40211	.40921	.41634
4	.27549	.28209	.28859	.29519	.30192	.30866	.31547	.32232	.32932
5	.22462	.23097	.23739	.24389	.25045	.25709	.26379	.27057	.27740
6	.19076	.19701	.20336	.20979	.21631	.22291	.22960	.23637	.24322
7	.16660	.17281	.17913	.18555	.19207	.19869	.20545	.21221	.21911
8	.14852	.15472	.16103	.16746	.17401	.18067	.18744	.19432	.20130
9	.13449	.14069	.14702	.15348	.16007	.16679	.17364	.18060	.18767
10	.12329	.12950	.13586	.14237	.14902	.15582	.16274	.16980	.17698
11	.11414	.12038	.12679	.13335	.14007	.14694	.15396	.16112	.16841
12	.10655	.11282	.11927	.12590	.13269	.13965	.14676	.15402	.16140
13	.10010	.10645	.11296	.11965	.12652	.13356	.14077	.14815	.15567
14	.09466	.10102	.10758	.11434	.12129	.12843	.13574	.14322	.15087
15	.08924	.09563	.10226	.10912	.11622	.12353	.13107	.13886	.14682

A Continuation of the preceding Table VII.

Years	4	5	6	7	8	9	10	11	12
16	.08581	.09226	.09895	.10585	.11298	.12009	.12781	.13551	.14339
17	.08219	.08869	.09544	.10242	.10962	.11704	.12466	.13247	.14056
18	.07899	.08554	.09235	.09941	.10670	.11421	.12192	.12984	.13793
19	.07613	.08274	.08962	.09675	.10412	.11173	.11954	.12756	.13576
20	.07358	.08024	.08718	.09439	.10184	.10954	.11745	.12557	.13387
21	.07128	.07799	.08500	.09228	.09983	.10761	.11562	.12383	.13224
22	.06919	.07597	.08304	.09040	.09803	.10590	.11400	.12231	.13081
23	.06739	.07413	.08127	.08871	.09642	.10438	.11257	.12097	.12955
24	.06655	.07347	.07967	.08718	.09497	.10302	.11126	.11978	.12846
25	.06401	.07095	.07822	.08581	.09367	.10180	.11016	.11874	.12749
26	.06256	.06956	.07690	.08456	.09250	.10071	.10915	.11781	.12665
27	.06123	.06829	.07569	.08342	.09144	.09973	.10825	.11698	.12590
28	.06001	.06712	.07459	.08239	.09048	.09885	.10745	.11625	.12524
29	.05887	.06604	.07357	.08144	.08961	.09805	.10672	.11565	.12465
30	.05782	.06496	.07264	.08058	.08882	.09733	.10607	.11502	.12414

.14322
.13906
.14087
.14682.13574
.13147
.13793
.14387.12843
.12405
.13098
.13692.12129
.11682
.12375
.12969.11434
.10979
.11672
.12266.10758
.10296
.10989
.11583.10102
.09634
.10327
.10871.09466
.08994
.09687
.10281.08581
.08113
.08806
.09399.07899
.07431
.08124
.08718.07358
.06890
.07583
.08177.06919
.06451
.07144
.07738.06739
.06271
.06964
.07558

The use of the preceeding Table VII.

Quest. 1. What Annuity or yearly rent issuing out of Lands, to begin presently, and to continue 14 years, will 320 *l.* purchase, compound interest being reckoned on both sides, at the rate of 6 per centum, per annum? *Ans.* 34 *l.* 8 *s.* 6 *d.* very near, which is thus found out, *viz.* In the 4th. Column of the preceding Table VII. under 6 at the head of that Column, and right against 14 years, you will find this Decimal .10758, which shews that 1 *l.* ready money will purchase an Annuity of .10758 *l.* (that is 2 *s.* 1 *d.* 2 *f.* +) therefore multiplying the said Decimal .10758 by the said 320; the product (according to the second Rule of the 26th. Chapter of the preceding Book) will be found to be 34.425, &c. that is 34 *l.* 8 *s.* 6 *d.* very near, which is the *Answer* of the *Question*.

$$1 \cdot ,10758 : 320 : (34.425 + \\ 320$$

215160

32274

3442560.

In like manner, if 10 per centum be thought a fit rate of interest to be allowed in purchasing Leases of Houses, 500 *l.* will buy a present yearly rent of 63 *l.* 18 *s.* 1 *d.* payable for 16 years out of a house. For underneath 10. at the head of the 8th Column, and right against 16 years (in the preceding Table VII.) you will find this decimal .12781, which being

Chap
being
propo
will b
1 d. +

XV
been l
Rules
king
paym
half y
the in
likew
we su
year,
bers r
ner to
as the
to for
rate o
100 l.
to the
at int
the su
actly
cal m
tream
will (
near
102.9

being multiplied by 500, (the number of pounds propounded to purchase the Lease) the product will be found to be 63.90500, that is, 63*l*, 18*s*. 1*d*. $\frac{1}{4}$, as by the subsequent operation is manifest.

$$1 : 12781 :: 500 . (63.905.$$

500

63190500

XVII. Upon the same foundations which have been laid in the 12, 13, 14, 15 and 16

Rules of this Chapter for the making of Tables which respect yearly payment, Tables may be made for half yearly and quarterly payments, the interest of 100*l*. for $\frac{1}{2}$ year, and

The making of Tables for half yearly and quarterly payments.

likewise for $\frac{1}{4}$ year being first agreed upon: For if we suppose that at the rate of 6*l*. for 100*l*. for a year, the interest of 100*l*. for $\frac{1}{2}$ year is 3*l*. the numbers 100 and 103 are to be used in the same manner to calculate Tables for half yearly payments, as the numbers 100 and 106 have been before used to form Tables for yearly payments. But if at the rate of 6 per centum, per annum, the interest of 100*l*. for $\frac{1}{2}$ year ought to be such, that being added to the said principal 100*l*. and the whole put forth at interest for the next half year, at the said rate, the sum then due (to wit, at the years end) must exactly amount unto 106*l*. In this case a Geometrical mean proportional number between the extremes 100 and 106 must be sought, which mean will (by the following 18th. Rule) be found to be near 102.956301 $\frac{1}{4}$, and then the numbers 100 and 102.956301, &c. are to be used instead of the num-

numbers 100 and 106 in manner aforesaid. In like manner, if it be supposed that at the rate of 6 per centum, per annum, the interest of 100 l. for $\frac{1}{4}$ year is 1 l. 10 s. or 1.5 l. the numbers 100 and 101.5 are to be used for the calculating of Tables for quarterly payments, in the same manner as the numbers 100 and 106 for yearly payments. But if at the rate of 6 per cent. per ann. the interest of 100 l. for $\frac{1}{4}$ year ought to be such, that being added to the said 100 l. and the whole put forth at the same rate of interest for the next $\frac{1}{4}$ of a year, and in that manner for the third and fourth quarters, and that the sum due at the years end must exactly amount unto 106 l. In this case a series or rank of five numbers in Geometrical proportion continued must be considered, viz. the principal 100 l. (which is the lesser of the two extream proportionals;) the three sums (composed of principal and interests) due at the end of the first, second and third quarters of the year, (which are the three mean proportionals) and 106 l. due at the years end (which is the greater of the two extream proportionals;) now between the said extreams 100 and 106, the first, (to wit the least) of the said three mean proportionals is to be sought, which (by the following 20th. Rule of this Chapter) will be found to be near 101.4673† And then the numbers 100 and 101.4673, &c. are to be used instead of the numbers 100 and 106 in manner aforesaid.

To find a Geometrical mean proportional number between two numbers given

XVIII. Two numbers being given to find a Geometrical mean proportional between them; multiply the two given numbers one by the other, and extract the square root of the pro-

Cha
prod
onal
num
num
mult
squa
so th
metr
prop
ner a
the
102.9
XI
of tw
tiona
given
fer ex
tract
so is f
two n
for ex
for t
will b
squa
multi
1728,
sough
easily
so tha
propo
these
contin

product, so is such square root the mean proportional sought: For *Example*, if 8 and 18 are two numbers given, and it is required to find a mean number Geometrically proportional between them, multiply 18 by 8, 10 is the product 144, whose square root is 12 for the mean proportional sought, so that 8, 12 and 18, are three numbers in Geometrical proportion continued, viz. As 8 is in proportion to 12, so is 12 to 18. In like manner a Geometrical mean proportional between the extremes 100 and 106 will be found near 102.956301 +.

XIX. Two numbers being given, to find the first of two Geometrical mean proportional numbers between the extremes

given, multiply the square of the lesser extremum by the greater, and extract the cube root of the product, so is such cube root the lesser of the two mean proportionals required: for *example*, if 8 and 27 are assigned for two extremes, the lesser mean

To find the first of the Geometrical mean proportional numbers between two extreme numbers given.

will be found 12; for according to the rule, the square of 8 the lesser extremum is 64, which being multiplied by 27 (the greater extremum) produceth 1728, whose cube root is 12 the lesser mean sought, then may the greater mean be found more easily by the *Rule of Three*, for $8 : 12 :: 12 : 18$, so that 12 and 18 are two means Geometrically proportional between the extremes 8 and 27, viz. these four numbers are in Geometrical proportion continued, to wit, 8. 12. 18 and 27.

XX. Two

To find the first of three Geometrical mean proportionals between two extrem numbers given.

XX. Two numbers being given to find the first of three Geometrical mean proportionals between the extremes given, multiply the cube of the lesser extrem by the greater, and extract the Biquadrate root of the product, so is such Biquadrate root the first (to wit, the least) of the three mean proportionals required: For Example, if 2 and 32 are two extremes given, the first and least of three Geometrical mean proportionals will be found to be 4, for (according to Rule) the cube of 2 (the lesser extrem given) is 8, which being multiplied by 32 (the greater extrem) produceth 256, the Biquadrate root whereof being extracted (according to the 29th Rule of the 33d Chapter of the preceding Treatise) gives 4 for the first and least of the three means sought, the other means may be easily found by the Rule of Three; for,

$$2 : 4 :: 4 : 8 :: 8 : 16 :: 16 : 32.$$

So that these five numbers will appear to be in Geometrical proportion continued, to wit,

$$2 : 4 : 8 : 16 : 32.$$

In like manner the first and least of three Geometrical mean proportionals between the extremes 100 and 106, will be found to be near 101. 4673, &c. Thus have I shewed the most easie ways (raised from clear grounds) to make Tables for the resolution of the usual questions, which depend upon the computation of interest, by the help of Multiplication only.

Quest.

Questions to exercise the precedent Tables, with their use in solving Questions of the same nature, when the number of years exceeds 30.

Quest. 1. If the Lease of an House be worth 153*l.* Fine, and 16*l.* yearly rent, payable yearly for 21 years, and the Lessee be desirous to bring down the Fine to 50*l.* and so to pay the more Rent, the question is, what rent the Tenant shall pay according compound interest at the rate of 10 per centum, per annum? Answ. 27*l.* 18*s.* 1*½d.* near.

First, find the difference between the Fines, which is 103*l.* Then after the manner of the examples of use of the preceding Table VII. seek what Annuity or Rent to continue 21 years, 103*l.* ready money will purchase at 10 per cent. so will you find 11*l.* 18*s.* ½*d.* which being added to the old rent 16*l.* gives 27*l.* 18*s.* ½*d.* which the Tenant must pay to the Landlord that the Fine may be diminished unto 50*l.*

Quest. 2. There is a Lease of certain Lands to be had for 14 years for 250*l.* Fine, and 44*l.* Rent per annum, payable yearly, but the Tenant is desirous to pay less Rent, viz 20 pounds per annum, and to have a greater Fine; the question is what Fine ought to be paid to bring down the Rent to 20*l.* per annum, computing compound interest, at the rate of 6 per centum, per annum? Answ. 473*l.* 1*s.* 7*d.*

First find the difference between the Rents, which will be 24 pounds per annum. Then by the help of the preceding Table VI. seek what Annuity or Rent of 20*l.* per ann. to continue 14 years, is worth in ready money at 6 per centum, per annum, so will you

find 223 l. 1 s. 7 d. which being added to the first Fine 250 pounds, gives 473 l. 1 s. 7 d. which the Tenant must pay, to the end the Rent may be brought down to 20 l. per annum.

Quest. 3. There is a Lease of certain Lands worth 32 l. per annum, more than the Rent paid to the Lord for it, of which Lease seven years are yet in being, and the Lessee is desirous to take a Lease in reversion for 21 years, to begin when his old Lease is expired, the Question is what Sum of money is to be paid for this Lease in reversion, accompting compound Interest at the rate of 6 per cent. per annum.

Ans. 250 l. 7 s. 2 d. + 28 l. 1 s. 7 d. which Summ First, by adding the 7 Years of the Lease in being to the 21 years you would have in reversion after those 7 are expired, the sum is 28. Then by the preceding Table VI

The present worth of 1 l. Annuity for 28 years at 6 per centum compound interest, is 13.40616

Likewise the present worth of 1 l. Annuity for 7 years is 5.58233

Therefore the difference of those present worths, shall be the present value of 1 l. Annuity for 21 Years in reversion after 7 years. 7.82383

Which multiplied by 32 (the yearly rent propounded) gives the Answer of the question 250.36256

Otherwise

Otherwise thus.

First, By the help of the said Table VI. find out how much 32 l. yearly rent for 21 years is worth in ready money, as if the 21 years were to begin presently, at the rate of 6 per centum, which ready money will be found 376.45024 l. Then by Table V. find what 376.45024 l. due at the end of 7 years to come, is worth in ready money, so will it be 250 l. 7 s. 2 d. which agrees with the Answer before found.

Quest. 4. One would bestow 630 l. to purchase a present yearly rent or Annuity of 60 l. to be paid by yearly payments, the question is to know how many Years the said Annuity must continue compound interest at 6 per centum, per annum, being allow'd on both sides. *Ans.* 17 Years, and 23 days, very near.

First, I divide 630 by 60, the quotient is 10.5, which shews that 10 Years purchase and an half are given for the Annuity; then searching for 10.5 in Table VI. in the Column of 6 per cent. I find it not exactly, but the nearest less than it, is 10.47725, standing right against 17 Years, and the next greater than 10.5 is 10.82760 which is placed against 18 Years. Whence I infer that the Annuity must continue 17 Years and more, yet less than 18 Years. Now the proportional part of a Year to be added to 17 Years, may be found out near enough for use, thus, viz. subtract the said lesser tabular number 10.47725 from the greater 10.82760, so the remainder will be found .35035: Also subtracting the said 10.47725 from 10.5 (the quotient

D d 2

tient

tient first found) the remainder will be .02275; then say by the rule of three in decimals, as .35035 the greater remainder is to .02275 the lesser; so is 1 year (the difference between 17 and 18 years) to 6649 parts of a year, or 23 days + (as will appear by the fourth *Rule* of the 26th *Chapter* of the preceding *Book*;) therefore the number of years sought by the question is 17 years, 23 days.

Quest. 5. If an Annuity of 96 *l.* payable by yearly payments for 14 Years be sold for 826 *l.* what rate of interest *per centum*, is implied in that Bargain? *Ans.* 7 *l.* 5 *s.* 7 $\frac{1}{2}$ *d.* near.

First, dividing 826 by 96, the quotient is 8.60416, which shews how many years purchase was given for the Annuity; then searching for 8.60416 in *Table VI.* in a right line passing from 14 years, equidistant to the head line of the *Table*, I find it not exactly, but the nearest less than it is 8.24423 (which stands in the Column of 8 *per cent.*) and the nearest greater is 8.74546 (which stands in the Column of 7 *per cent.*) whence I infer, that the rate of interest required is between 7 and 8 *per cent.* and the proportional part of 1 *l.* to be added to 7 *l.* may be found out near enough for practice thus, *viz.* subtract the said lesser tabular number 8.24423 from the greater 8.74546, the remainder will be .50123. Also subtract 8.60416 (the quotient first found, which falls between the said tabular numbers from the said greater tabular number 8.74546,) the remainder will be .14130; then say by the *Rule* of three in decimals, as .50123 the great remainder (or difference between the two tabular numbers) is to .14130 the lesser remainder, so is 1 *l.* (the difference between 7 *per cent.* and 8 *per cent.*) to .2819, *&c.*

or 5s. 7d. 2f. which added to 7l. gives 7l. 5s. 7d. 2f. which is near the rate of interest *per cent.* required.

Quest. 6. If a years rent, or one years purchase, be paid as a *Fine*, for renewing or adding 7 years to 14 years, yet to come of an old Lease for 21 years, and accordingly a new Lease to be taken for 21 years, to begin presently (which proportion is ordinarily observed by *Bishops, Deans and Chapters, Heads and Fellows of Colleges* in letting Leases of their Lands) what rate of interest *per centum* is implied in that Agreement? *Answ.* 11l. 11s. 8d. 1f. and somewhat more.

To solve this Question, first I search in the preceding *Table VI.* to find out two numbers so seated in some one Column of interest, that one of them may stand right against 14 years, and the other against 21 years; and so qualified that the difference between them may be exactly 1 or unity; but not finding any two numbers precisely answering those Conditions, I take those numbers that come nearest, which will be found in the Columns of 11 and 12 *per cent.* for the difference between the numbers 6.98186 and 8.07507, which stand in the Column of 11 *per cent.* right against 14 years and 21 years, is 1.09321, which exceed 1 (that is 1 years purchase) by .09321; also the difference between 6.62816 and 7.56200, which stand in the Column of 12 *per cent.* right against 14 years and 21 years, is .93384, which wants .06616 of 1: Therefore I divide 1l. (the difference between 11l. and 12l. *per cent.*) into two parts, in such proportion one to the other, as the said decimals .09321 and .06616 are one to the other; so I find the said part of 1l. to be near .5848 and .4151; or 11s. 8d. 1f. + and 8s.

3d. 2f. + the former of which being added to 11 per centum, or the latter being subtracted from 12 per cent. gives 11.5848 per cent. or 11 l. 11 s. 8 d. 1 f. +, which is very near the rate of interest required by the question.

Quest. 7. What is the present worth of 1 l. per ann. payable yearly for 10 Years, compound interest being computed at the rate of 11.5848 per cent. per ann. 5 l. 15 s. 0 d. very near, which is found out by the help of the preceding Table VI. in this manner, viz.

The tabular number for 10 Years	}	5.88923
at 11 per centum is		
The tabular number for 10 Years	}	5.65022
at 12 per centum is		
Their difference is		0.23901

Then say by the Rule of Three in decimals, as 11 (the difference between 11 and 12 per cent.) is to 5848 l. (to wit, the decimal by which the given rate in the question exceeds 11 per cent.) so is .23901 (the difference found out as above) to .13977 +, which being subtracted from 5.88923 (the greater of the two tabular numbers above-mentioned) there will remain 5.74946, or 5 l. 15 s. 0 d. which is near the present worth of one pound yearly rent to continue 10 Years, at the proposed rate of 11.5848 per centum.

After the same manner the present worth of 1 l. yearly rent payable for 21 Years at the same rate of interest, will be found to be 7.77503 l. or 7 l. 15 s. 6 d. very near, from which if you subtract 5.74946, (being the aforementioned present worth of 1 l. yearly rent for 10 years) there will remain .20557

or 2 l. 0 s. 6 d. which is near the present worth of a Lease of 1 l. rent *per annum*, for 11 years in reversion, to begin after 10 years yet to come in a Lease are expir'd: Hence it is evident, that if a Tenant to a College hath 10 years yet to come in a Lease, at 1 l. rent *per annum*, and desires to have 11 years renewed or added to those 10, and so take a new Lease for 21 years, to begin presently at the same rent, he must give 2 l. 0 s. 6 d. or two Years purchase and $\frac{1}{4}$ part of a years purchase, very near (according to the fundamental proportion before assumed in the sixth question.) The like may be done for any other term of years under 30, by the help of the said Table VI.

But yet by a Table calculated purposely for the said rate of 11.5848 l. *per centum*, (according to the fifteenth Rule of this Chapter) questions of the same kind with the two last, may be more easily answered, and therefore (for that they come often in practice) I shall here insert such a Table, as I find it ready calculated to my hand by Doctor Newton, in his Scale of Interest lately publish'd, which Table is to be used in every respect like to the preceding Table VI. and will be very ready and useful, for the proportioning of Fines, in the renewing of Leases held from Cathedral Churches and Colleges, as will be manifest by the manner of solving the two following Questions.

Concerning the
renewing of a
College Lease
of Lands.

Quest. 8. If a College Tenant hath 7 years yet to come or unspent in a Lease of Lands for 21 years, at 1*l.* yearly rent, and desires to have 14 years renewed or added to those 7 years, and so to take a new Lease for 21 years to begin presently, what must he pay for a Fine? *Ans.* 3*l.* 3*s.* 0*d.*

The rule for finding out the Answer of the Question proposed, and such like, is this, *viz.*

From 7.77507 (being the number which answers to 21 years in this Table VIII.) subtract always the tabular number which belongs to the number of years to come or unspent in the old Lease, so the remainder will shew what Fine must be paid for the years to be renewed or added, to make those unspent years in the old Lease to be 21 years compleat again, at 1*l.* yearly rent.

So to solve the question proposed.

TABLE VIII.

Shewing the present worth of one pound Annuity for any number of years under 22, at the rate of 11*l.* 11*s.* 8*d.* 11*d.* per cent. compound interest.

Years present worth

1	0.90034
2	1.69938
3	2.41922
4	3.06438
5	3.64262
6	4.16088
7	4.62540
8	5.04176
9	5.41496
10	5.74948
11	6.04934
12	6.31819
13	6.55907
14	6.77507
15	6.96868
16	7.14226
17	7.29786
18	7.43737
19	7.56243
20	7.67455
21	7.77507

From

From the present worth of 1 l. }
 yearly rent for 21 years, which is— } 7.77507
 Subtract the present worth of the }
 same rent for 7 Years, that were } 4.62540
 unspent in the old Lease. ————— }
 And there will remain the Fine }
 sought, to wit, ————— } 3.14967

That is to say, 3.14967 l. or 3 l. 3 s. 0 d. (very near)
 must be paid as a Fine, for renewing or adding 14
 years to 7 years, that were unspent in the old Lease,
 the yearly rent being 1 l. Also the said 3.14967
 shews, that such a renewal is worth 3 years pur-
 chase and near $\frac{1}{10}$ parts of a years purchase (what-
 ever the rent be.)

Quest. 9. If a Tenant that hath 17 years yet to
 come in a Lease of Lands held of a College for 21
 years, at 50 l. yearly rent, be desirous to renew 4
 years, and so make those 17 years to be 21 years
 compleat again at the same rent, what must he give
 for a Fine? *Answ.* 23 l. 17 s. 2 d. 1 f. For ac-
 cording to the Rule before given,

From the present worth of 1 l. }
 yearly rent for 21 years ————— } 7.77507
 Subtract the present worth of the }
 same rent for 17 years (that were } 7.29786
 unspent in the old Lease.) ————— }
 And there will remain, ————— } 0.47721
 Which multiplied by the rent ————— } 50

The product will be the Fine }
 sought, to wit, 23 l. 17 s. 2 d. 1 f. } 23.86050

Questions of this nature may be readily solved without the loss of one sixteenth part of a Year Purchase by the help of the following Table IX, which I have drawn from the foregoing Table VIII. for the benefit of such as understand not Decimal fractions: For example, if a College-Tenant desireth to have 10 years added to 11 years that are to come or unspent in a Lease of Lands that he may have a new Lease for the term of 21 Years to begin presently, the following Table IX, shews that he must give for a Fine 1 years Purchase, and 2 quarters of a Years Purchase, and 3 quarters of a quarter of a Years Purchase, viz. one Years rent, and half a Years rent, and three quarters of a quarter of a Years rent: Supposing then the rent to be 48 *per annum*, the Fine may be computed thus.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
One Years rent is —————	48	00	00
Half a Years rent is —————	24	00	00
Three quarters of a quarter } of a Years is —————	9	00	00
The Sum is the Fine required	81	00	00

Whence it appears that the Tenant must give 81 *l.* as a Fine, for adding of 10 Years to 11 Years that were unexpired in his old Lease, to the end he may have a new Lease for 21 Years in being.

In like manner the following Table IX shews that the Fine for renewing or adding 7 Years to 14 Years that are unspent in a Lease of Lands, to the end there may be a new Lease for 21 Years in being, is valued at 1 Years Purchase precisely, which is the fundamental proportion assumed in calculating the foregoing Table VIII, as before was said.

TABLE IX

The Fine for renewing or adding

TABLE IX.

T E A B L E IX.		Quarters of a quarter. Quarters of a Year. Years purchase..
Tears—	Tears—	
1 to 20	} is valued at	0 : 0 : 1
2 to 19		0 : 0 : 3
3 to 18		0 : 1 : 1
4 to 17		0 : 1 : 3
5 to 16		0 : 2 : 2
6 to 15	} is valued at	0 : 3 : 0
7 to 14		1 : 0 : 0
8 to 13		1 : 0 : 3
9 to 12		1 : 1 : 3
10 to 11		1 : 2 : 3
11 to 10	} is valued at	2 : 0 : 0
12 to 9		2 : 1 : 1
13 to 8		2 : 2 : 3
14 to 7		3 : 0 : 2
15 to 6		3 : 2 : 1
16 to 5	} is valued at	4 : 0 : 2
17 to 4		4 : 2 : 3
18 to 3		5 : 1 : 1
19 to 2		6 : 0 : 1
20 to 1		6 : 3 : 2

The

The Fine for renewing or adding

The like may be done for renewing any other term of years under 21, at any rent proposed.

But because it may sometimes happen, that the number of years in questions belonging to the preceding 3, 4, 5, 6. and 7 Tables may exceed 30, I shall by the five following Questions shew, how by the help of those Tables the answer to any question of that nature may be found out near the truth, when the term of years is above 30.

Quest. 10. If 340*l* be put forth at 4 per centum, compound interest, and both principal and interest be forborn until the end of 45 years, what will then be due? *Answ.* 1986*l.* very near.

To resolve this question and the like, observe this rule, viz. First make choice of such numbers of years in Table III. that if they be added together will make the number of years proposed in the question, as 17 and 28, or 15 and 30, each of which pairs make 45, then looking into Table III. in the Column belonging to 4 per centum, you will find right against 17 and 28 years these numbers, 1.94790 and 2.99870, which being multiplied one by the other will produce 5.84116+, or 5*l.* 16*s.* 10*d.* which shall be the increase of 1*l.* forborn 45 years at 4 per centum, compound interest; therefore multiplying the said 5.84116 by 340, the product will give 1985.994, &c. or 1986*l.* very near for the Answer of the Question.

The reason of the said Rule will be manifest by this Theorem, viz. If there be a rank of numbers in Geometrical proportion continued, beginning with

with 1 or unity, as 1, 2, 4, 8, 16, 32, 64, 128, &c. Also if the first term 1 be cast away, and over or under all the rest of the terms there be placed another rank of numbers, beginning at 1 and proceeding according to the natural order of numbers, as 1, 2, 3, 4, 5, 6, 7, &c. which may be called the *Indices* of those in the first rank, after the first term 1 is cast away; I say if any two of those remaining Geometrical proportionals be multiplied one by the other, the product shall be a proportional correspondent to that *Index*, which is equal to the sum of the *Indices* answering to the two proportionals that were multiplied one by the other.

Proport. 2 . 4 . 8 . 16 . 32 . 64 . 128

Indices. 1 . 2 . 3 . 4 . 5 . 6 . 7

So if 4 and 32, which are the second and fifth proportionals in the upper rank, be multiplied one by the other, the product is 128, which shall be the seventh proportional, because the sum of the *Indices* 2 and 5, which Answer to the said 4 and 32, is 7. In like manner, because the sum of the *Indices* 3 and 4 is 7, therefore if the third and fourth proportionals, to wit, 8 and 16, be multiplied one by the other, the product shall also give the seventh proportional 128. Now forasmuch as the numbers in every one of the Columns, except the first Column of years in the preceding *Table III.* are continual proportionals whose first term is 1, but 'tis excluded out of the said Columns, as appears by the Construction of that *Table*, and for that the numbers of years 1, 2, 3, 4, 5, &c. are placed

placed as *Indices* shewing the order or seat of those proportionals inserted in the Columns, therefore the rule before given for continuing that Table to any number of years is manifest.

Quest. 11. If one pound be due or payable 50 years hence, what is it worth in ready Money, by rebating at 5 per centum, per annum, compound interest? *Answ.* 08720, 9d. or 1s. 9d. + which is found out by the help of Table V. in the same manner as the Answer to the last Question; (respect being had to the second and third rules of the 26th Chapter of the preceding Book concerning the multiplication of decimal fractions.)

Quest. 12. If an Annuity of one pound payable yearly for 40 years, be all forborn untill the end of that term, what will it then amount unto, compound Interest being computed at 5 per centum per annum? *Answ.* 120l. 16s. 0d. thus found out. First, according to the second way of calculating the fourth Table in the thirteenth Section of this Chapter, find out a principal, which may have such proportion to the proposed Annuity 1l. as 100 hath to 5, saying, if 5l. interest hath 100l. for principal, what principal must 1l. interest have? *Answer,* 20l. Secondly, seek (after the manner of the preceding tenth Question) what 20l. will be augmented unto being forborn 40 years, at the rate of 5 per centum, per annum, compound interest, so you will find 140.798+, from which subtracting the said principal 20l. the remainder will be 120.798+, or 120l. 16s. which is the answer of the question.

Quest. 13. If an Annuity of one pound payable yearly for 37 years, be to be sold for present money

ney, what is it worth, compound interest being computed on both sides at 6 per centum, per annum?

Answer, 14*l.* 14*s.* 9*d.* which is found out thus:

First, according to the second way of calculating the sixth Table in the fifteenth Section of this Chap-

ter, find out a principal in such proportion to one pound (the proposed Annuity) as 100 is to 6, so

will such principal be found 16.66666+, then after the manner of the preceding eleventh question

find out the ready money which is equivalent to 16.66666, due 37 Years hence, so will such ready

money be found to be 1.92988+ (or 1*l.* 18*s.* 7*d.*) which being subtracted from the said principal

16.66666, the remainder will be 14.73678+, or 14*l.* 14*s.* 9*d.* which is the Answer of the Question

propounded.

Quest. 14. What Annuity payable by yearly payments to continue 37 Years will one pound Purchase at 6 per centum, per annum, compound interest?

Answer, 1*s.* 4*d.* near, which is found out thus: First, find out the present worth of one pound

Annuity to continue 37 Years, which present worth (by the last Question) will be found 14.73678*l.*

Then say by the Rule of Three, if 14.73678*l.* will purchase an Annuity of 1 pound, (to continue 37

Years) what Annuity to continue the same term will 1*l.* purchase? Answer, .06785+, or 1*s.* 4*d.*

which is the Answer of the Question propounded.

C H A P. VI.

A Demonstration of the Rule of Three, or Rule of Proportion.

I. **F**our numbers are said to be proportionals, when the first containeth the second so often as the third containeth the fourth; likewise when the first is such part of the second, as the third is of the fourth; so these numbers following are called proportionals, *viz.*

$$4 \times 6 . 6 :: 4 \times 9 . 9$$

$$\frac{2}{3} \times 12 . 12 :: \frac{2}{3} \times 15 . 15$$

That is to say, 4 times 6 (or 24) is said to have such proportion to 6, as 4 times 9 (or 36) hath to 9. In like manner, $\frac{2}{3}$ of 12 (or 8) hath such proportion to 12; as $\frac{2}{3}$ of 15 (or 10) hath to 15.

II. When four numbers are proportionals, the product arising from the multiplication of the two extremes is equal to the product of the two means.

Demonstration.

By the preceding Definition I these four numbers are proportionals, *viz.*

$$\left\{ \begin{array}{l} 4 \times 6 . 6 :: 4 \times 9 . 9 \\ B \times C . C :: B \times D . D \end{array} \right.$$

The

The product of the } 4 x 6 x 9
two extremes is ——— } B x C x D

The product of the } 6 x 4 x 9
two means is ——— } C x B x D

$$\text{But } \left\{ \begin{array}{l} 4 \times 6 \times 9 \\ B \times C \times D \end{array} \right\} = \left\{ \begin{array}{l} 6 \times 4 \times 9 \\ C \times B \times D \end{array} \right\}$$

Therefore the Prop. is manifest.

Likewise.

By the preceding definition these four numbers
are proportionals, viz.

$$\frac{2}{3} \times 12 . 12 :: \frac{2}{3} \times 15 . 15$$

The product of the } $\frac{2}{3} \times 12 \times 15$
two extremes is ——— }

The product of the } $12 \times \frac{2}{3} \times 15$
two means is ——— }

$$\text{But } \frac{2}{3} \times 12 \times 15 = 12 \times \frac{2}{3} \times 15$$

Wherefore the Proposition is every way pro-
ved.

III. From the last Proposition ariseth the *Rule of*
Proportion commonly called the *Rule of Three* or
Golden Rule, which teacheth by three numbers gi-
ven to find a fourth proportional number in this
manner, viz. *Multiply the second and third numbers*
actually one by the other, and divide the product by
the first number; so the quotient shall be the fourth pro-
portional number sought, in a direct proportion. This
Rule hath been fully exemplified in the 8th. Chap-
ter of the preceding Book, and the truth of the

said Rule may be thus demonstrated, viz. Let there be three numbers given to find a fourth in direct proportion, viz. If 24 gives 6, what shall 36 give? Or as 24 is in proportion to 6, so is 36 to a fourth proportional number sought, which fourth proportional (whatsoever it be) we may suppose to be Q, and then these four numbers will be proportionals, viz.

$$24 \cdot 6 :: 36 \cdot Q$$

Therefore by the second Proposition of this Chapter.

$$24 \cdot Q = 6 \cdot 36$$

And because if equal plane numbers be severally divided by one and the same number, the quotients will necessarily be equal between themselves, therefore

$$6 \cdot Q = 216$$

$$Q = \frac{216}{24}$$

Whereby it is manifest that the fourth proportional number is equal to the quotient that ariseth by dividing the product of the multiplication of the second and third proportionals by the first, which was to be proved.

Note, That every Rule of Three inverse may be made a Rule of three direct, by making the third term the first, and by proceeding forward to the other two terms; therefore one and the same demonstration serveth for both rules.

CHAP. VII.

A Demonstration of the Double Rule of Fellowship.

THE *Double Rule of Fellowship* (commonly called the *Rule of Fellowship with time*) presupposeth two things, *viz.* 1. That the particular Stocks of Merchants in Company, have continued unequal spaces of time in the common Stock. 2. That at the end of their Partnership, the total gain or loss is to be divided amongst them, in such manner, that their shares shall have such proportion between themselves, as those sums of interest money have one to another, which at any rate *per centum*, simple interest only being computed, might be gained by the particular Stocks, within the respective times of their continuance in the common Stock: Now for the effecting of such a proportional partition, the said *Double Rule of Fellowship* gives this Direction, *viz.* Divide the total gain or loss into such parts, which shall have the same proportion one to the other, as is between the products arising out of the Multiplication of each particular Stock by its correspondent time.

For Example, Suppose Two Merchants *A* and *B* to be Partners in Traffick, for a certain time first

agreed on between them, and that A doth permit his Stock of 100 l. to be implied in their joint Traffick three months, and that B forbears his Stock of 50 l. eight months; I say (according to the said Rule of Fellowship with time) whatever the total gain or loss be, that part thereof which belongs to A must have such proportion to the gain or loss of B, as 100 x 3 (or 300) hath to 50 x 8 or 400. This Rule hath been fully exemplified in the 13th. Chapter of the preceding Book, and the truth thereof, taking the two premised Suppositions for granted may be thus demonstrated.

1. Supposing 100 l. (the Stock of A) to gain in three months any certain sum of money, as two pounds; I seek how much 50 l. (the Stock of B) will gain in the same time, and at the said rate: so I find 2 x 50

— l. for,

1000

2 x 50

100 : 2 :: 50 : —

100

2. Having found what 50 l. will gain in three months, I seek how much the said 50 l. will gain in 8 months, at the same rate, and so I find —

2 x 50 x 8

100 x 3

for,

3 2 x 50 8 2 x 50 x 8

— : — :: — : —

1

100

1

100 x 3

3. Thus it appears, that if 100 l. in 3 months doth gain 2 l. then 50 l. in 8 months will gain at the

Chap

the fa

of the

A

4.

Conseq

tiplied

ducts

numbe

viz. th

5. I

assume

dent as

portion

the gai

portion

the same rate $\frac{2 \times 50 \times 8}{100 \times 3}$; so that the proportion of the gain of A to the gain of B is,

$$\text{As } 2 \text{ is to } \frac{2 \times 50 \times 8}{100 \times 3}$$

4. If both the terms (to wit, the *Antecedent* and *Consequent*) of the said proportion be severally multiplied by the said *Denominator* 100×3 , the products will be in the same proportion with the numbers of terms multiplied, (by 17 & 7. *Euclid*) viz. the gain of A will be to the gain of B,

$$\text{As } 2 \times 100 \times 3 \text{ is to } 2 \times 50 \times 8$$

5. Lastly, Because 2 (the suppositious gain first assumed) is a multiplier as well in the *Antecedent* as in the *Consequent* of the last mentioned proportion, it may be expung'd out of both, and so the gain of A will be to the gain of B in this proportion (which was to be proved) to wit,

$$\text{As } 100 \times 3 \text{ is to } 50 \times 8$$

C H A P. VIII.

A Demonstration of the Rule of Alligation alternate, and the use of the said Rule in the Composition of Medicines.

I. IN order to the Demonstration of the said Rule, I shall premise this *Lemma*, viz. if the difference of any two numbers given, be multiplied by a number assigned, the product will be equal to the difference between the products which arise from the multiplication of those two numbers severally by the number assigned.

Suppositions.

Two lines or } $AC = 10$ A B C
 numbers given. } $BC = 4$
 Their difference. $AB = 10 - 4$
 A multiplicator } $AD = 5$
 assigned. D E F

Which Suppositions, and the *Diagram* being well viewed, the truth of the said *Lemma* will be evident, viz.

$$AB \times AD = AC \times AD - BC \times BE \quad (AD)$$

$$10 - 4 \times 5 = 10 \times 5 - 4 \times 5$$

II. To add the more light to the following Demonstration of the rule of *Alligation alternate*, I shall propound a question which properly belongs to the said rule, viz. Suppose a *Vintner* having *French Wines* at 5 *d.* the quart, and at 10 *d.* the quart, would make a Mixture of them in such manner, that he might sell the mixt quantity at 7 *d.* the quart, and so make as much money of the Mixture, as if he should sell each Quantity of *Wine* at its own price; the question is to know what proportion the quantities of both sorts of *Wine* in the Mixture must bear one to the other. Here, according to the *Rule of Alligation alternate*, I take the difference between the mean price assigned for the Mixture, and the two other given prices, and place those differences alternately, viz. the difference between 7 and 10 being 3, I write 3 against 5; likewise 2 being the difference between 7 and 5, I write 2 against 10; so I conclude, that the quantity to be taken of that sort of *Wine* of 10 *d.* the quart, must have such proportion to the quantity of 5 *d.* the quart, as 2 to 3. That is to say, if 2 quarts at 10 *d.* the quart be mixed with 3 quarts at 5 *d.* the quart, the total mixture 5 quarts being sold at 7 *d.* the quart, will yield as much money as the said 3 quarts at 5 *d.* the quart, together with the said 2 quarts at 10 *d.* the quart; as is evident by the subsequent work.

$$\begin{array}{r}
 7 \left\{ \begin{array}{l} 10 \mid 2 \\ 5 \mid 3 \end{array} \right. \\
 \hline
 5
 \end{array}$$

	quarts	pence	quarts	pence
I.	I.	5	3	15
II.	I.	10	2	20
III.	15 + 20 =		7 x 5 = 35	

From the premises it appears, that when two things are given to be mixt in such manner as the Rule of *Alligation alternate* requires, the proposition to be demonstrated will be this, namely,

Three numbers A. B. C. being given in such sort that A. is less than B. but greater than C. if the difference between A. and B. be multiplied by C. and the difference between A. and B. be multiplied by B. the sum of those products will be equal to the product arising from the multiplication of A. by the sum of the said differences.

Demonstration.

mean price	extrem prices	differences alternate	
A {	B C	A - C B - A	BA - BC CB - CA
	B - C		BA - CA = B - C x A

The difference between B. and A. is B - A. which multiplied by C produceth (as is evident by the

Lemma

Chap. VIII. the Rule of Alligation 449

Lemma foregoing in the first Section of this Chapter) $CB - CA$. also the difference between A and C is $A - C$ which multiplied by B produceth $BA - BC$. Then the Sum of those two products is $BA - CA$. for $+CB$ and $-CB$ expunge one the other) which sum is manifestly the same with the product arising from the multiplication of A the mean price, by $B - C$ the sum of the aforesaid differences (to wit, the Sum of $A - C$ and $B - A$) for $+A$ and $-A$ expunge one another.

When more than two things of different prices are given to be mixt as aforesaid, the *Demonstration* will not be otherwise: For if the Sum of every two products arising from the multiplication of two alternate differences by their respective prices, be equal to the product of the mean price multiplied by the sum of the said differences; the sum of all the said products will also be equal to the products of the mean price multiplied by the sum of all the differences; as will clearly appear by view of the subsequent work.

		Products of alternate differences multiplied by their respective prices.		Products of the mean price multiplied by the several sums of alternate differences.	
	If	D +	E = F	x	G
	and	H +	K = F	x	M
Then	D + E +	H +	K = F	x	G + M.
					More

Moreover, because if equal numbers be severally divided by one and the same number, the quotients will be equal between themselves, therefore from the premises this Corollary will arise,

COROLLARY.

In the Rule of *Alligation alternate*, if the aggregate of the products arising from the multiplication of the several alternate differences by their respective prices, be divided by the sum of the said differences, the quotient will be equal to the main Price. This may be a Proof of any Example of the said Rule of *Alligation*.

OF THE COMPOSITION OF MEDICINES.

See more of this in Mr. J. Dee's Mathematical Preface, also Tom. 2. of P. Herigon and Master More's Arithmetick.

I. Medicines and Simples in respect of their Qualities are considered in some of these five ways, viz, either as they are hot or cold, moist or dry, or as they are temperate; so that such Simples or Medicines which work heat in our Bodies are said to be hot, such cold which are cause of coldness.

II. The mean or middle between the extreme Qualities of Heat and Coldness, also between Dryness and Moisture, is called Temperate or the Temperature

IV. Si
that of t

perature; from which each of the said Qualities hot, cold, moist and dry, doth differ in four degrees, so that a Medicine or Simple is said to be either temperate, or else hot, cold, moist or dry, in the first, second, third, or fourth degree.

III. If the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, be placed as you see from A to B, the differences between 5 (the middle number) and the superiour numbers 6, 7, 8, 9, will be 1, 2, 3, 4, which may represent the 4 Degrees of the Qualities hot and dry; likewise the differences between 5 and the inferiour numbers 4, 3, 2, 1, will be 1, 2, 3, 4, which may represent the 4 degrees of the qualities cold and moist, the temperature represented by 0, being the mean or middle from whence the said degrees do swerve,

Ind.		Degr.	
B	9	4	Qualities hot and dry
	8	3	
	7	2	
	6	1	
	5	0	Temperature.
	4	1	Qualities cold and moist.
	3	2	
	2	3	
A	1	4	
Ind.		Degr.	

IV. Since the Rule of Alligation alternate requires that of two things miscible, the one must exceed the mean

mean propounded and the other be less, therefore the questions of *Alligation* in this kind are to be wrought with the numbers in the aforesaid Column AB, for by them the degrees and qualities are discovered, being placed as you see in the Column adjacent to AB, and for distinction sake, those numbers in the said Column AB, may be called the *Indices* or *Exponents* of the degrees, which *Indices* are to be used in the same manner as the prices of Merchandizes in the questions of *Alligation alternate* in Chapter 14 of the preceding Book, and therefore those Examples may be compared with these.

Prop. I.

Having divers Simples whose qualities are known, to make a composition of mixture of them, in such manner that the quality of the Medicine may be some mean amongst the qualities of the Simples, and the quantity thereof any quantity assigned.

Example 1. An *Apothecary* hath four sorts of Simples, A, B, C, D, whose qualities are as followeth, viz. A is hot in the fourth degree, B is hot in the second, C is temperate, and D is cold in the third degree; the question is to know what quantities of each ought to be taken, to make a Medicine, whose quantity may be 12 ounces, and the quality in the first degree of heat? Seek in the aforesaid column AB, for the *Indices* or *Exponents* of the qualities of the Simples given, viz. for A which is hot in the fourth degree, take 9; for B which is hot in the second, take 7; for C which

which
cold
those
Merc
viz. o
the ter
degree
the m
of heat
lines o
of one
less, a
fourtee
Medic
be in
ounce
4 ounce
as will

Last
the Me
yet the
heat, as

which is temperate, take 5; and for D which is cold in the third degree, take 2; that done, rank those numbers in the same manner as the prices of Merchandises in the questions of the 14th Chapter, viz. descend from the highest degree of heat unto the temperature, and so proceed downwards to the degrees of cold, setting 6 the *Index* or *Exponent* of the mean quality propounded, which is 1 degree of heat, as common to them all; then by crooked lines or otherwise connect two such *Indices*, whereof one may be greater than the mean, and the other less, and proceeding according to the *Rule* of the fourteenth Chapter you will find that to make a Medicine of 9 ounces, and the quality resulting to be in the first degree of heat, you must take 1 ounce of A (being that Simple which was hot in 4) 4 ounces of B, 3 ounces of C, and 1 ounce of D, as will be manifest by the proof,

deg.
 $\left. \begin{array}{l} 9 \\ 7 \\ 5 \\ 2 \end{array} \right\}$

oun.
 $\left. \begin{array}{l} 1 \\ 4 \\ 3 \\ 1 \end{array} \right\}$

simpl.

The Proof.

1	A	9 x 1 = 9
4	B	7 x 4 = 28
3	C	5 x 3 = 15
1	D	2 x 1 = 2

9

9) 54 (6

Lastly, by the rule of *Proportion* you may increase the Medicine to the quantity of 12 ounces, and yet the quality to continue in the first degree of heat, according to the following operation.

Oun.

$\begin{array}{l} \text{Ounc.} \\ \text{Ounc.} \\ \text{Ounc.} \\ \text{Ounc.} \end{array} \quad \begin{array}{l} 10 \\ 9 \\ 8 \\ 7 \end{array} : \begin{array}{l} 12 \\ 12 \\ 12 \\ 12 \end{array} : \begin{array}{l} 1\frac{1}{2} \\ 5\frac{1}{2} \\ 4 \\ 1\frac{1}{2} \end{array} \quad \begin{array}{l} \text{of A} \\ \text{of B} \\ \text{of C} \\ \text{of D} \end{array}$

The Quantity assigned 12 Ounces.

By other connexions of the qualities, other quantities of each Simple would arise, but that hath been sufficiently manifested in the questions of the fourteenth Chapter.

Example 2. Suppose there are five *Simples*, A, B, C, D, E, whose *Qualities* are as followeth, viz. A is hot in 3°. B is hot in 2°. C is hot in 1°. D is cold in 1°. E is cold in 3°. and it is required to mix 4 Ounces of B, with such Quantities of the rest that the *Quality* of the *Medicine* may be temperate?

Degr.	Ounc.	Simple.	The Proof.
8	1	A	8 x 1 = 8
7	3	B	7 x 3 = 21
6	1	C	6 x 1 = 6
4	3 + 1	D	4 x 4 = 16
2	2	E	2 x 2 = 4
			11
			11)55(5

Proceed

Proceed as before, so will you find that to make a *Medicine* of 11 ounces, and the quality of the *Form* resulting to be temperate, you must take 1 ounce of A, 3 ounces of B, 1 ounce of C, 4 ounces of D, and 2 ounces of E; then since the quantity of B in the composition propounded is limited, viz. 4 ounces, find numbers which may be in such proportion to 4 (the quantity of B assigned) as the numbers 1, 1, 4, 2 (the quantities of A, C, D, E, in the aforesaid Composition of 11 ounces) are unto 3 (the quantity of B in the said Composition) in manner following:

Oun.	Oun.	Oun.	Oun.	
3	1	4	$1\frac{1}{3}$	of A.
3	1	4	$1\frac{1}{3}$	of C.
3	4	4	$5\frac{1}{3}$	of D.
3	2	4	$2\frac{2}{3}$	of E.

} to be mixed with
4 ounces of B.

Prop. II.

A *Medicine* being compounded of divers *Simples* whose qualities and quantities are known, to find the degree of the *Form* resulting, viz. the exact temperament of the *Medicine*.

Example 1. Suppose a *Medicine* to be compounded of two *Simples*, viz. 6 ounces of B hot in 4°. and three ounces of C hot in 3°. and it is required to find the temperament of the *Medicine*, viz. the degree and quality resulting from such mixture? Seek in the aforesaid Column A B for the *Indica* of

of the respective degrees and qualities of the *Simples* given, and dispose them orderly in ranks right against their respective quantities; then multiply each *Index* by its respective quantity, and divide the sum of the products by the sum of the quantities; So will the quotient be the *Index* of the degree and quality of the *Medicine*.

<i>Ind.</i>	<i>Qm.</i>	<i>Prod.</i>
9	x 6	= 54
8	x 3	= 24
—	—	—
9)	78	($8\frac{2}{3}$)

So in the said Example the Quotient will be found $8\frac{2}{3}$, which is the *Index* of $3\frac{2}{3}$ degrees of heat and therefore the said *Medicine* is hot in $3\frac{2}{3}$ degrees.

Forasmuch as any two quantities miscible according to the *Rule of Alligation alternate*, are in such proportion one to the other, as the respective alternate differences between the mean quality of the mixture and the qualities correspondent unto the said quantities, the Demonstration of the aforesaid *Rule* will be manifest by the *Corollary* foregoing in this Chapter.

Example 2. Suppose a *Medicine* to be compounded of 4 *Simples*, whose qualities and quantities are known, viz. 2 ounces of A hot in 3° . 3 ounces of B hot in 2° . 4 ounces of C temperate, and 5 ounces of D cold 4° . and let it be required to find

find the mean quality resulting from such mixture. according to the aforesaid Rule, I multiply each Index by its respective quantity, and divide the sum of the products by the sum of the quantities, so the quotient is $4\frac{1}{7}$ which is the Index of $4\frac{1}{7}$ degrees of cold (for the difference between 5 the Index of the temperature, and $4\frac{1}{7}$ the Index found, is $\frac{4}{7}$ degrees of cold) which is the quality of the said Medicine.

Ind.	Qm.	Prod.
8	x 2	= 16
7	x 3	= 21
5	x 4	= 20
1	x 5	= 5
		62

14) 62 (4 $\frac{1}{7}$

Example 3. Suppose a Medicine to be compounded of several Simples, whose qualities and quantities are as followeth, viz. 4 ounces of a Simple which is cold in 2°. and moist in 1°. 5 ounces hot in 3°. and (in respect of dryness and moisture) temperate; 3 ounces hot in 2°. and dry in 2°. 6 ounces hot in 1°. and moist in 4°. 4 ounces cold in 3°. and moist in 2°. the question is to know the temper resulting?

In the resolution of this question there must be two distinct operations, each of them like to that in the last example, viz.

F f 1. Find

1. Find in the same manner as before, the degree and quality resulting from the commixture of the qualities hot and cold; so will you find $5\frac{7}{2}$ which is the Index of $5\frac{7}{2}$ degrees of Heat (for the difference between 5 the Index of the temperature and $5\frac{7}{2}$ the Index found, is $\frac{7}{2}$ degrees of heat.

Ind.	Qm.	Prod.	Ind.	Qm.	Prod.
3	x	4=12	4	x	4=16
8	x	5=40	5	x	5=25
7	x	3=21	7	x	3=21
6	x	6=36	1	x	6=6
2	x	4=8	3	x	4=12
•22) 117 ($5\frac{7}{2}$			22) 80 ($3\frac{7}{11}$		

2. Find in the same manner, the temper resulting from the mixture of the qualities dry and moist; so will you find $3\frac{7}{11}$ which is the Index of $3\frac{7}{11}$ degree of moisture, so the quality of the said Medicine is $\frac{27}{22}$ degree of heat, and $1\frac{4}{11}$ degree of moisture, as by the operation is manifest.

Prop. III.

To augment or diminish a Medicine in quality according to any degree assigned.

Suppose a Medicine to be compounded as followeth, viz. 1 dram of a Simple hot in 4° . 2 drams hot in 3° . 2 drams hot in 2° . 1 dram hot in 1° . 1 dram cold in 1° . and 1 dram cold in 2° . Then will the quality of the said Medicine be in $1\frac{1}{2}$ degree

of heat as will be manifest by the second Proposition.) Now let it be required to augment the said Medicine in quality, viz. to add such a quantity of some one of the Ingredients (or some other simple) which may raise the quality of the Medicine $\frac{1}{2}$ degree; so that the temperament of the Medicine after it is increased in quantity, may be in 2° . of heat. Make choice of such a simple, the Index of whose quality may exceed the Index of the quality assigned, viz. make choice of that simple which is hot in 3° . whose Index is 8, then proceed according to the 1 Example of the first Proposition; so will you find that if 1 dram of the aforesaid Medicine be mixed with $\frac{1}{2}$ dram of that simple which is hot in 3° . the temper resulting from such mixture will be in 2° . of heat.

Lastly, by the Rule of Three, say, if 1 dram require $\frac{1}{2}$ dram, what shall 8 drams (the quantity of the Medicine first given) require?

Ans. 4 drams: So that if 4 drams of a simple which is hot in 3° be mixed with 8 drams of a Medicine which is hot in $1\frac{1}{2}$ degree, the temper resulting will be in 2° . of heat, as by the operation is manifest.

The Proof.

Prod.	8	=	52
Dra.	8	=	52
Ind.	8	=	32

$$8 \times 4 = 32$$

14.) 84 (7

If it be required to diminish a Medicine quality, you are to make choice of such a Simple the Index of whose quality may be less than the Index of the quality assigned, and then to proceed as before.

Here observe, that if in questions of this nature the quantities of the Simples be express'd by weight of divers denominations, they are to be reduced to that weight which is of the lowest denomination in the question, according to the sixth Rule of the seventh Chapter of the preceding Book.

The augmenting or diminishing of a Medicine respect of quantity; Also the finding of the value of any quantity of a Medicine, the prizes of Ingredients being known, will be familiar to those as understand the Rule of Proportion, and therefore I shall not insist upon them.

C H A P. IX.

A Demonstration of the common Rule of False by two Positions.

WHAT the ordinary *double Rule of False* is, and how to be used in resolving such questions which cannot readily be applied to any of the other rules of *Arithmetick*, hath been fully declared in the 15th. and 31th. Chapters of the preceding Book; it remaineth to shew what kind of operation is presupposed before the said Rule can be applied to the resolution of a question, and then to demonstrate the Truth of the Rule it self.

II. In the said *Rule of False*, look what operation the question requires to be performed with the number sought and some given number or numbers, the same kind of operation in every respect is to be made with each of the two feigned numbers commonly called *Positions*) and the said given number or numbers; which threefold process being finisht (whether it be by any one, or all of these rules, to wit, *Addition*, *Subtraction*, *Multiplication*, and *Division*) there will arise three remarkable numbers or results, to wit, one resulting from the true number sought, and two others resulting from

the two feigned numbers; then from these three Results, the Errors are collected, which are nothing else but the differences between the true Result, and each of the two false Results.

III. After the said errors or differences are discovered, the *Rule of False* will be of no force, unless this Analogy or proportionality doth arise, namely the first error must have the same proportion to the second, as the difference between the number sought and the first feigned number hath to the difference between the said number sought and the second feigned number; here therefore it may be demanded, what kind of operation will produce the said Analogy? To this I answer, when the question requires the number sought to be increased, lessened, multiplied or divided by some given number, or the number arising from such operation to be increased, lessened, multiplied or divided by some given number; in any of those cases, the aforesaid Analogy will necessarily arise, as I shall here manifest in all the said cases. First, therefore I say when unto each of three numbers (namely the number sought by the *Rule of False* and the two feigned numbers) one and the same number is added, the said Analogy will ensue, for in this case the difference between the first sum and the second will be equal to the difference between the first and second of the said three numbers; likewise the difference between the first sum and the third will be equal to the difference between the first number and the third, which may be proved in manner following.

Supposition

Let

of 2

redm

Sup

than er

Sup

ded to

three f

of 1

ed

•

The

differ

equal t

and th

the fir

rence b

of 2

of 2

The

the sec

of 2

of 2

The

the sec

of 2

The

second

of 2

Suppositions.

Let there be three numbers, to wit,

$$A \quad B \quad C$$

Suppose also that the first number A is greater than either of the numbers B and C.

Suppose also, some number as D (3) to be added to each of the said three numbers, so will the three sums be,

$$\begin{array}{r|l} A + D & 15 \\ B + D & 10 \\ C + D & 8 \end{array}$$

The Proposition to be demonstrated is, that the difference between the first sum and the second is equal to the difference between the first number and the second; also that the difference between the first sum and the third is equal to the difference between the first number and the third.

Demonstration.

The difference between the first number and the second is,

$$A - B$$

The difference between the first sum and the second is,

$$A + D - B - D$$

$$F f 4$$

But

But the latter difference is manifestly equal to the former (for $+D$ and $--D$ expunge one the other;) to wit,

$$A + D - B - D = A - B$$

Therefore the first part of the proposition is proved.

Again, the difference between the first number and the third is,

$$A - C$$

The difference between the first sum and the third is,

$$A + D - C - D$$

But the latter difference is manifestly equal to the former, for $+D$ and $--D$ expunge one the other, viz.

$$A + D - C - D = A - C$$

Wherefore the Proposition is fully proved.

The like property might be proved after the same manner, when one and the same number is subtracted from three numbers severally.

Secondly, when three numbers (namely the numbers sought by the *Rule of False* and the two feigned numbers) are severally multiplied by one and the same number; the afore-mentioned Analogy will likewise ensue, as may be thus proved.

Suppositions.

Let there be three numbers, to wit,

$$A . B . C.$$

$$3 . 5 . 8.$$

Suppose

Suppose also that the first number A is less than either of the numbers B and C .

Suppose also, each of those three numbers to be multiplied by one and the same number as D (4) and the three products to be these,

$$\begin{array}{r|l} DA & 12 \\ DB & 20 \\ DC & 32 \end{array}$$

The Proposition to be demonstrated is, that the difference between the first product and the second hath such proportion to the difference between the first product and the third, as the difference between the first number and the second hath to the difference between the first number and the third, *viz.*

$$DB-DA : DC-DA :: B-A : C-A.$$

$$8 : 20 :: 2 : 5$$

Demonstration.

Forasmuch as (by the 17th. Prop. of the seventh Book of *Euclid's Elem.*) if a number (D) multiplying two numbers ($B-A$ and $C-A$) produceth other numbers ($DB-DA$ and $DC-DA$) the numbers produced by the multiplication shall be in the same proportion as the numbers multiplied are, therefore

$$DB-DA : DC-DA :: B-A : C-A$$

which was to be demonstrated.

Likewise when 3 numbers are divided by one and the same number, the *demonstration* will not be otherwise

otherwise; and because by the second Section of this Chapter, the errors in the Rule of False are the differences between the true Result and the two False Results, therefore from the precedent demonstrations it is evident, that the aforementioned Analogy or proportionality (namely, when the first error hath such proportion to the second, as the difference between the number sought and the first feigned number hath the difference between the said number sought and the second feigned number) will succeed from such operation, as is before declared in the beginning of the third Section of this Chapter.

To know whether a question be resolvable by the Rule of False or not.

IV. Now to discern what kind of operation will not produce the said Analogy, observe this Note, viz. when a question requires some given number to be divided by the number sought or any part thereof, also when the number sought or some part thereof is to be squared, cubed; &c. likewise when some parts of the number sought are to be multiplied one by the other: I say from such operations the aforementioned Analogy will not arise, and in those cases, the ordinary Rule of False will be useless; as may partly appear by the two following examples, viz. *What number is that, by which if 360 be divided the quotient will be 24:* Here if two positions or feigned numbers be taken, and 360 be divided by each of them, the errors will not be in the same proportion with the differences between the true number sought and the 2 feigned numbers, and therefore the Rule of False will be used in vain: Yet if it be asked what number is that, which being multiplied by

by 24
latter
forme
such
alway
tive w
taken
gradua
another
mine v
place
questio

Ano

ded w
if $\frac{1}{2}$ c
same
bet, or
numbe
for the
lity w
asily

Now
parati
errours
shall
discove

V. I

cases,
noted
—, or
and the

In t
the P
errors,

by 24, the product will be 360, the ~~Answer~~ to this latter question is the same with the answer to the former, and may be found by the *Rule of False*; but such kind of interpretations and inferences are not always obvious, and therefore since the preparative work of the *Rule of False* (after the number is taken by guess for the number sought) proceeds gradually from one condition in the question to another, it will for the most part be easie to determine whether the ordinary *Rule of False* will take place or not, by comparing the conditions of a question with the Note before given.

Another Example; a certain person being demanded what number of years he had lived, answered, if $\frac{1}{5}$ of that number were multiplied by $\frac{1}{4}$ of the same number, the product would shew the number, or his Age: Here it will be in vain to search the number sought (which is 40) by the *Rule of False*; for the aforementioned Analogy or proportionality will not succeed, and the question cannot easily be resolved without *Algebra*.

Now from this supposition, that after the preparative work of the *Rule of False* is finish'd, the errors will be in such proportion as aforesaid, I shall make it manifest that the *Rule of False* will discover the number sought.

V. In the Rule of two false Positions there are 3 cases, viz. the errors are either both excesses and noted with +, or else both defects and noted with -, or lastly, one of the errors is noted with + and the other with -

In the two first cases the Rule is this, multiply the Positions or feigned numbers by the altern errors, viz. the first Position by the second error, the

the second Position by the first error, and reserve those products; then dividing the difference of the said products by the difference of the said errors, the quotient shall be the number sought by the question.

The demonstration of the said Rule here followeth.

Case I. When the errors are both excesses, and noted with +

Suppositions.

1. Let some number unknown and sought by the Rule of False be represented by } A
2. Let the first position (or feigned number) be } B
3. And the second feigned number } C
4. Suppose also that B is greater than C, and each of them greater than A.
5. Moreover, suppose the error of the first Position to be } F
6. And the error of the second Position to be } G
7. Suppose also that this Analogy will be found in the said numbers, viz.

$$B - A : C - A :: F : G$$

8. The Proposition to be demonstrated.

$$A = \frac{FC - GB}{F - G}$$

Demon.

Demonstration.

9. Forasmuch as by supposition in 7°.

$$B - A : C - A :: F : G$$

10. Therefore by comparing the rectangle of the extrems to the rectangle of the means.

$$GB - GA = FC - FA$$

11. And by equal addition of FA.

$$FA + GB - GA = FC$$

12. Again, forasmuch as by supposition in 4°.

$$B > C$$

13. And consequently out of 4° and 12°.

$$B - A > C - A$$

14. Therefore out of 9° and 13°.

$$F > G$$

15. Therefore

$$FA > GA$$

16. Therefore

$$FA - GA > 0$$

17. There

17. Therefore by equal subtraction of GB from the equation in 11°.

$$FA - GA = FC - GB$$

18. Wherefore by dividing both parts of the last equation by F-G, equal quotients will arise, viz.

$$\frac{FC - GB}{F - G} = A$$

$$A = \frac{FC - GB}{F - G}$$

which was to be demonstrated.

Case II. When the errors are both defects, and noted with—

Suppositions.

1. Let some number unknown and sought by the Rule of False be represented by } A
2. Let the first position (or feigned number) be .. } B
3. And the second feigned number ... } C
4. Suppose also that B is less than C, and each of them less than A.
5. Moreover, suppose the error of the first Position to be } F
6. And the error of the second Position .. } G
7. Suppose also that this Analogy will be found in the said numbers, viz.

$$A - B : A - C :: F : G$$

8. The Proposition to be demonstrated.

$$\frac{FC - GB}{A = \frac{FC - GB}{F - G}}$$

Demonstration.

9. Forasmuch as by supposition in 7°.

$$A - B : A - C :: F : G$$

10. Therefore by comparing the rectangle of the means to the rectangle of the extremes.

$$FA - FC = GA - GB$$

11. And by equal addition of FC.

$$FA = FC + GA - GB$$

12. Again, forasmuch as by supposition in 4°.

$$B < C$$

13. And consequently out of 4°. and 12°.

$$A - B < A - C$$

14. Therefore out of 9°. and 13°.

$$F < G$$

15. Therefore

$$FA < GA$$

16. There-

16. Therefore

$$FA - GA < 0$$

17. Therefore by equal subtraction of GA from the equation of 11°:

$$FA - GA = FC - GB$$

18. Wherefore by dividing both parts of the last equation by F - G, equal quotients will arise, viz.

$$A = \frac{FC - GB}{F - G}$$

which was to be demonstrated.

Case III. When one of the errors is an excess (to wit, noted by +) and the other a defect (noted by -)

In this third Case the Rule of False is this, viz.

Multiply the Positions by the altern errors, to wit, the first Position by the second error, also the second Position by the first error, and reserve those products; then dividing the sum of the said products by the sum of the said errors, the quotients shall be the number sought by the question.

The Demonstration of this latter Rule here followeth.

Suppositions:

1. Let some number unknown and sought } *A*
by the Rule of False be represented by }
2. Let the first Position be } *B*

3. And

3. And the second Position . . . : **C**
 4. Suppose also that B is greater than C, and also greater than A, and that C is less than A.
 5. Moreover, suppose the error of the first Position to be **F**

6. And the error of the second Position to be **G**
 7. Suppose also that this Analogy will be found in the said numbers, viz.

$$B - A : A - C :: F - G$$

8. The Proposition to be demonstrated.

$$A = \frac{GB + FC}{F + G}$$

Demonstration.

9. Forasmuch as by supposition in 7°:

$$B - A : A - C :: F : G$$

10. Therefore by comparing the rectangle of the means to the rectangle of the extremes.

$$FA - FC = GB - GA$$

11. And by equal addition of FC and GA to the last equation, this will arise,

$$FA + GA = GB + FC$$

12. Wherefore by dividing both parts of the last equation
G g
equation

equation by $F + G$, equal quotients will arise, viz.

$$A = \frac{GB + FC}{F + G}$$

which was to be demonstrated.

The learned *Herigonius* (in cap. 13. Tom. 2. of his *Curfus Mathematicus*) hath delivered another way of resolving the *Rule of False*, namely by the two following rules, viz.

When the signs of the errors are unlike.

Rule I. As the sum of the errors is to the first error, so is the difference of the supposed numbers to a fourth proportional, which being added to the first supposed number, when the said first supposition is less than the second, or subtracted from it when it exceeds the second, the sum or remainder will be the true number sought.

When the signs of the errors are unlike.

Rule II. As the difference of the errors is to the first error, so is the difference of the supposed numbers to a fourth proportional, which being added to the first supposed number when the signs are — or subtracted from it when the signs are +; the sum or remainder will be the number sought.

Both which rules the said *Herigonius* demonstrateth geometrically by lines, upon a supposition of the *Analogy* or *Proportionality* before mentioned in the third Section of this Chapter, and the same may likewise be easily demonstrated according to the precedent Method by Letters.

C H A P. X.

A Collection of pleasant and subtil Questions, to exercise all the parts of Vulgar Arithmetick. To which are also added various practical Questions about the mensuration of Superficial Figures and Solids.

Example of the Rule
of Three mixtly used
with other Rules.

Quest. 1. If a wedge of Gold weighing $17\frac{3}{7}$ lb. of Troy Weight be worth $679\frac{5}{7}$ lb. sterling, what is the value of $1\frac{1}{3}$ grain of that Gold?

Ans. 2 pence.

$$I. 1\frac{1}{3} \text{ (or } \frac{1.6}{1.3} \text{) of } \frac{1}{24} \text{ of } \frac{1}{20} \text{ of } \frac{1}{12} = \frac{1}{4680}$$

$$II. \frac{1.23}{7} \cdot 475\frac{8}{7} \div \frac{1}{4680} \cdot \frac{1}{120}$$

Quest. 2. A Man dying gave to his eldest Son $\frac{2}{3}$ of his estate, to his second Son $\frac{1}{3}$ of $\frac{1}{2}$ of his estate; and when they had counted their Portions, the one had 40 L. more than the other; the remainder of the estate was given to the Wife and younger Children. The question is, what was the Portion of the eldest Son, also of the second, and how much did belong to the Wife and younger Children?

Ans. The eldest Sons Portion 100 l. the second Sons Portion 60 l. and 440 l. for the Wife and younger Children.

The fractions being reduced, it will be manifest that the eldest Son had $\frac{2}{3}$, and the second $\frac{1}{6}$; also the

difference of the said fraction is $\frac{1}{15}$, then say,

$$\circ \cdot \frac{X1}{15} \cdot \frac{40}{1} : A \cdot \frac{H}{15} \cdot \frac{60}{1}$$

l.

The second Son's portion 60

The difference of their portions 40

The eldest Son's portion 100

$$\frac{1}{15} \cdot \frac{40}{1} :: \frac{1}{1} \cdot \frac{600}{1}$$

Lastly, $600 - 160 = 440$ for the Wife and younger Children.

Quest. 3. A young Man received $66\frac{2}{3}l.$ which was $\frac{2}{3}$ of $\frac{1}{2}$ of his eldest Brother's portion, and $3\frac{1}{2}$ times of his eldest Brother's portion, was $1\frac{1}{4}$ times of his Father's Estate, the question is, what was the Father's Estate? *Ans.* 560 l.

$$\frac{1}{3} \cdot 66\frac{2}{3} :: 1 \cdot 200$$

$$200 \times 3\frac{1}{2} = 700$$

$$1\frac{1}{4} \cdot 700 :: 1 \cdot 560$$

Quest. 4. If A can finish a Work in 20 days, and B in 30 days; in what time will the work be finished by A and B working together? *Answer* 12 days.

First find what quantity of the work will be done by each workman in one and the same time; then it will be, as the sum of those quantities is in proportion to the said time, so is 1 or the whole work to the time wherein such work will be finished by both Workmen working together.

dys

days work days work
30 1 20

add 1

sum $1\frac{2}{3}$

Hence it appears that A and B working together 20 days will finish that Work once, together with $\frac{2}{3}$ of the same Work ; therefore say again by the Rule of Three.

work days work days
 $1\frac{2}{3}$. 20 :: 1 . 12

Quest. 5.

*Aereus adsto leo, tubuli mihi lumina bina,
Osque etiam, dextri sic quoque planta pedis.
Binis dextro oculo, ternis lacus iste diebus
Impletur lava, sed pede his geminis.
Ori sufficient sex horæ. Dic simul ergo,
Quo spatio os, oculi, pesque replere valent?*

The sense is this. A brazen Lion being placed in an artificial Fountain, conveyed Water into a Cistern by two streams issuing from his Eyes, also by one from his Mouth, and by another at the bottom of his right foot. Now the Pipes through which these streams pass, are of different Capacities, in such sort, that by the right Eye set open alone, the rest of the streams being stopt, the Cistern will be filled in two days (the length of a day being supposed to be 12 hours;) by the left Eye alone in three days; by the foot alone in four days; and

Chap. X. Chapter, find how many times the Cistern will be emptied in one and the same space of time, by the Cocks of B and C running together; also how much of the Cistern will be filled by A in the same time; then will the difference shew how much of the Cistern is gained by the filling Cock in the said time: Lastly, as the Cisterns or parts gained are in proportion to the correspondent time, so is the whole Cistern, to the time wherein it will be gained or filled.

I. $\frac{2}{7} \cdot 1 :: 1 \frac{3}{7} \cdot (\frac{3}{4})$ $\left\{ \begin{array}{l} C \\ B \\ B \& C \end{array} \right\}$ in $1 \frac{3}{7}$ hou,

hou. cist. hou. cist. $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{4}$ $\frac{3}{4}$

add $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{4}$ $\frac{3}{4}$

sum $1 \frac{3}{4}$ $\frac{3}{4}$ $\frac{1}{4}$ $\frac{3}{4}$

II. $\frac{1}{4} ; 1 :: 1 \frac{3}{7} ; (2 \frac{6}{7} \text{ filled by } A)$

$1 \frac{1}{4}$ gained by A

III. $\frac{1}{4} \cdot 1 :: 1 \frac{3}{7} :: 1 \frac{3}{7} \cdot 1$

cist. hou. cist. hou.

Quest. 7. Suppose a Dog, a Wolf and a Lion were to devour a Sheep, and that the Dog could eat up the Sheep in an hour, the Wolf in $\frac{3}{4}$ hour, and the Lion in $\frac{1}{2}$ hour: Now if the Lion begin to eat $\frac{1}{4}$ hour before the other two; and afterwards all Three eat together, the question is, in what time the Sheep would be devoured? Answer, $\frac{3}{4}$ hour.

I. $\frac{1}{4} \cdot 1 :: \frac{1}{8} : \frac{1}{4}$

hou. sh. hou. sh.

If $\frac{1}{4} \cdot 1 :: \frac{1}{8} : \frac{1}{4}$

Thus it appears that $\frac{1}{4}$ of the Sheep would be eaten by the Lion, before the Dog and Wolf began to eat.

II. Proceed according to the fourth Question, so will you find the remaining $\frac{1}{4}$ to be eaten by them all in $\frac{1}{2}$ hour, which added to $\frac{1}{4}$ gives $\frac{3}{4}$ hour, in which time the Sheep would be devoured.

Quest. 8. If 120 $\frac{1}{2}$ L be to be distributed amongst three Persons, A, B, C, in such sort, that as often as A takes 5, B shall take 4, and as often as B takes 3, C shall take 2; what shall be the share of each?

Ans. A 51 $\frac{4}{7}$ L. B 41 $\frac{2}{3}$ L. C 27 $\frac{1}{3}$ L.

Find three Numbers which may express the proportions of their shares, by the *Rule of Three*, or (to avoid fractions) thus,

$$5 \dots \dots 4$$

$$3 \dots \dots 2$$

$$15 \cdot 12 \cdot 8$$

thus found

$$5 \times 3 = 15$$

$$3 \times 4 = 12$$

$$4 \times 2 = 8$$

$$\left. \begin{array}{l} 15 \cdot 51\frac{4}{7} \\ 12 \cdot 41\frac{2}{3} \\ 8 \cdot 27\frac{1}{3} \end{array} \right\}$$

Quest. 9. A Governour of a certain Garrison, being desirous to know how much Money the Port or Passage of the Garrison did amount unto in certain

tain months, made choice of a loyal servant, giving him order to receive of every Coachman passing with a Coach 4*d.* of every Horseman 2*d.* and of every Footman $\frac{1}{2}$. Now at the years end, the servant making his Account to the Governour, giveth him 94*l.* 15*s.* 10*d.* and let him know that as often as 5 passed with Coaches, 9 passed on Horseback; and as often as 6 passed on Horseback, 10 passed on Foot; the question is how many Coaches, Horsemen, and Footmen passed? *Ans.* 2500 Coaches, 4500 Horsemen, 7500 Footmen.

Find three proportional numbers after the manner of the 8 Question, which will be 5, 9, 15; then proceed as followeth,

$$\begin{array}{rcl} & d. & \\ 5 \text{ Coaches} & \dots & 20 \\ 9 \text{ Horsemen} & \dots & 18 \\ 15 \text{ Footmen} & \dots & 7\frac{1}{2} \end{array}$$

$$\text{If } 45\frac{1}{2} : 22750 :: \left\{ \begin{array}{l} 5 \cdot 2500 \\ 9 \cdot 4500 \\ 15 \cdot 7500 \end{array} \right.$$

Quest. 10. A Factor would exchange; 80*l.* Sterling for double Ducats, Dollars, and French Crowns, the Ducats at 7*s.* 6*d.* the piece, the Dollars at 4*s.* 4*d.* and the French Crowns at 6*s.* the piece, to be in such proportion, that $\frac{1}{2}$ of the number of Ducats may be equal to $\frac{1}{3}$ of the number of Dollars, and $\frac{1}{2}$ of the Dollars equal to $\frac{3}{4}$ of the Crowns, the question is, how many pieces of each coin he shall receive for his 780 pounds?

Ans. 600 Ducats, 900 Dollars, 1200 Crowns.

Find three proportional Numbers (after the manner

manner of the eight question) which will be
6, 4, 3,

$$\begin{array}{ccccccc} \frac{1}{6} & \cdot & \frac{1}{4} & \cdot & \frac{1}{3} & \cdot & \frac{1}{2} \\ \hline \frac{1}{6} & \cdot & \frac{1}{4} & \cdot & \frac{1}{3} & \cdot & \frac{1}{2} \\ \hline 6 & \cdot & 4 & \cdot & 3 & \cdot & 2 \end{array}$$

Thus it appears that six times the number of Ducats must be equal to four times the number of Dollars, also equal unto three times the number of Crowns. Then make choice of three numbers to answer those proportions, such as are these 2, 3, 4, (for $6 \times 2 = 4 \times 3 = 3 \times 4$) with which numbers proceed as followeth,

$$\begin{array}{l} 2 \text{ ducats} \dots \frac{3}{4} \\ 3 \text{ dollars} \dots \frac{1}{2} \\ 4 \text{ crowns} \dots 1\frac{1}{3} \end{array}$$

l. l. l.

$$\text{say if } \dots 2\frac{3}{8} \cdot 780 :: \left\{ \begin{array}{l} \frac{3}{4} \cdot 225 \\ \frac{1}{2} \cdot 195 \\ 1\frac{1}{3} \cdot 360 \end{array} \right.$$

$$\begin{array}{l} \frac{1}{2} \cdot 1 :: 225 \cdot 600 \text{ ducats.} \\ \frac{1}{2} \cdot 1 :: 195 \cdot 900 \text{ dollars.} \\ \frac{3}{10} \cdot 1 :: 360 \cdot 1200 \text{ crowns.} \end{array}$$

Quest. 11. Twenty Knights, 30 Merchants, 8 Lawyers, and 24 Citizens, spent at a Dinner of a pound, which was divided amongst them in such manner, that 4 Knights paid as much as 5 Merchants, 10 Merchants as much as 16 Lawyers; and

8 Lawyers as much as 12 Citizens; the Question is, to know the sum of money paid by all the Knights, also by the Merchants, Lawyers and Citizens.

Answer, The 20 Knights paid 20 pounds, the 30 Merchants 24 pounds, the 24 Lawyers 12 pounds, and the 24 Citizens 8 pounds.

Find four numbers to express the proportions of their payments by the *Rule of Three*, or (to avoid fractions) in manner following, so will the proportional numbers be 4, 5, 8, 12, viz. 4 Knights paid as much as 5 Merchants, or 8 Lawyers, or 12 Citizens.

4		5
10		16
8		12

320 . 400 . 640 . 960

4 5 8 12

~~~~~

*thus found,*

$$4 \times 10 \times 8 = 320$$

$$10 \times 8 \times 5 = 400$$

$$8 \times 5 \times 16 = 640$$

$$5 \times 16 \times 12 = 960$$

Then presupposing that a Knight is to pay 4s proceed as followeth, viz.

20 Knights

20 Knights . . . 4

30 Merchants . . .  $4\frac{1}{2}$ 24 Lawyers . . .  $2\frac{1}{2}$ 24 Citizens . . .  $1\frac{1}{2}$ 

say, if 12 : 64 ::

$$\left. \begin{array}{l} 4 \cdot 20 \\ 4\frac{1}{2} \cdot 24 \\ 2\frac{1}{2} \cdot 12 \\ 1\frac{1}{2} \cdot 8 \end{array} \right\}$$

64

**Quest. 12.** A certain Man with his Wife did usually drink out a Vessel of Beer in 12 days, and the Husband found by often experience, that his Wife being absent, he drank it out in 20 days; the question is, in how many days the Wife alone could drink it out? *Ans.* 30 days.

*Note,* It is to be supposed that the Husband in 8 of the 20 days where he drank alone, did drink as much as in the 12 days wherein he drank with his Wife; hence it followeth, that in the remaining 8 of the said 20 days, he drank as much as his Wife did in 12 days. Therefore by the Rule of Three say, If 8 give 12, what 20? *Ans.* 30. View the following form of the Work.

From 20

Subtract 12

Then if 8 . 12 :: 20 . 30

**Quest. 13.** If a House be to be built by Three Carpenters, A, B, C, working in such sort, that A alone will finish it in 30 days, B in 40 days,

Cha  
and  
cou  
I  
in wh  
the h

II. S  
med by  
by D  
15 day  
stion) i  
120 day

The I  
Fourth c  
**Quest.**  
Journey  
viz. A tr  
led 8 day  
B sets for



and A, B, C, together in 15 days, in what time could C alone build the House? *Ans.* 120 days.

I. After the manner of the fourth question, (find in what time A and B working together will finish the house;) *Ans.*  $17\frac{1}{7}$  days.

days work days work

40 . 1 . 30 .  $\frac{3}{4}$   
add 1

sum  $1\frac{3}{4}$

work days work days

$1\frac{3}{4}$  . 30 :: 1 .  $17\frac{1}{7}$

II. Supposing the work of A and B to be performed by one Person, as D, the House will be built by D in  $17\frac{1}{7}$  days, but by D and C together in 15 days; Then find (according to the 12th. Question) in what time C will build the same. *Ans.* 120 days.

From  $17\frac{1}{7}$   
Subtract 15

Then if  $2\frac{1}{7} . 15 :: 17\frac{1}{7} . 120$

The Proof may be wrought according to the Fourth or Fifth Questions.

*Quest.* 14. Two Travellers, A and B perform a Journey to one and the same place in this manner, viz. A travels 14 miles every day, and had travelled 8 days before B began; upon the ninth day B sets forward, and travels 22 miles every day; the

the question is, to find in what times B shall overtake A? *Answ.* at the end of 14 days.

I. Find how many miles A had travelled before B set forward? *Answ.* 112 miles; For

day miles days miles

I . 14 :: 8 . 112

II. Find how many miles B gains of A in a day; *Answ.* 8 miles; For,

$$22 - 14 = 8$$

III. If miles day miles days  
8 . 1 :: 112 . 14

*Quest.* 15. There is an Island which is 36 miles in compass. Now if at the same time, and from the same place, two Footmen A and B set forward to travel round about the said Island, and follow one another in such manner that A travel-eth every day 9 miles, and B 7 miles; the question is to find in what space of time they will again meet, also how many miles, and how many times about the Island each Footman will then have travelled?

*Answ.* They will meet at the end of 18 days from their first parting; and then A will have travelled 162 miles (or  $4\frac{1}{2}$  times the compass of the Island) and B will have travelled 126 miles (or  $3\frac{1}{2}$  the compass of the Island.)

miles

|          |     |                  |       |      |                  |
|----------|-----|------------------|-------|------|------------------|
|          |     | miles            |       |      |                  |
| From     | 9   |                  |       |      |                  |
| Subtract | 7   |                  |       |      |                  |
| <hr/>    |     | day              | miles | days |                  |
| 2.1      |     | ::               | 36    | :    | 18               |
| mult     | 18  |                  | mult. | 18   |                  |
| by       | 9   |                  | by    | 7    |                  |
| <hr/>    |     |                  |       |      |                  |
| 36)      | 162 | (4 $\frac{1}{2}$ | 36)   | 126  | (3 $\frac{1}{2}$ |

**Quest. 16.** Two Footmen A and B depart at the same time from *London* towards *Tork*, travelling at this rate, viz. A goeth 8 miles every day, B goeth 1 mile the first day, 2 miles the second day, 3 miles the third day, and in that progression he goeth forward, travelling in every following day one mile more than in the preceding day; the question is to know in how many days B will overtake A?

**Ans.** 15 days.

To resolve this and such like questions, double 8 (the number of miles which A travelles daily) which make 16, from which subtract 1, the remainder is 15, the number of days sought.

**Quest. 17.** If *Exeter* be distant from *London* 140 miles, and that at the same time one Footman A departed from *London* towards *Exeter*, travelling every day 8 miles, and another B from *Exeter* towards *London*, travelling every day 6 miles, the question is in how many days they will meet one another, and how many miles each Footman will have then travelled?

**Answer,**



*Answer*, They will meet at the end of 10 days, and then A will have travelled 80 miles, and B 60 miles.

add  $\left\{ \begin{array}{l} 8 \text{ miles travelled daily by A.} \\ 6 \text{ miles travelled daily by B.} \end{array} \right.$

sum 14 miles which A and B together did travel daily.

*m. da. miles da.*

14. 1 :: 140. 10 in which time A and B will meet each other.

$10 \times 8 = 80$  miles travelled by A.

$10 \times 6 = 60$  miles travelled by B.

*Quest.* 18. A certain Footman A departed from London towards Lincoln, and at the same time another Footman B departed from Lincoln towards London; also A travelleth every day  $2\frac{1}{2}$  miles more than B. Now supposing those two Cities to be 100 miles distant one from the other, and that those two Footmen do meet one another at the end of 8 days after the beginning of their Journies; the question is, how many miles each will have then travelled, as also how many miles each travelled daily?

*Ans.* A 60 miles, B 40 miles. Also A travelled  $7\frac{1}{2}$  miles every day, and B 5 miles.

*day miles days miles*

1 .  $2\frac{1}{2}$  :: 8 . 20

Hence it appears that at the time of their meeting A had travelled 20 miles more than B, which

10 Miles being subtracted from 100 miles leave 90 miles, whereof the half is 45 miles which B had travelled, therefore A had travelled 60 miles.

Now to find how many miles each travelled daily, say,

|      |       |     |       |
|------|-------|-----|-------|
| days | miles | day | miles |
| 8    | 40    | 1   | 5     |

miles

Therefore { B } travelled {  $7\frac{1}{5}$  } daily.

**Quest. 19.** There is an Island which is 134 miles in compass; now at the same time, and from the same place, two Footmen A and B begin a Journey round about the said Island, but they travel towards contrary parts, at this rate, viz. A travelleth 11 miles in every 2 days, and B 17 miles in 3 days, the question is to find in what space of time A and B will meet one another; and how many miles each will then have travelled?

**Answer.** They will meet at the end of 12 days, and then A will have travelled 66 miles, and B 68 miles.

After the manner of the fourth Question of this Chapter, the time sought will be found 12 days.

|      |       |      |       |
|------|-------|------|-------|
| days | miles | days | miles |
|------|-------|------|-------|

|   |   |    |    |   |   |                  |
|---|---|----|----|---|---|------------------|
| 2 | . | 11 | :: | 3 | . | 16 $\frac{1}{3}$ |
|---|---|----|----|---|---|------------------|

add 17

|       |  |      |       |      |
|-------|--|------|-------|------|
| <hr/> |  | days | miles | days |
|-------|--|------|-------|------|

|                  |   |   |   |   |     |   |    |
|------------------|---|---|---|---|-----|---|----|
| 33 $\frac{1}{3}$ | . | 3 | : | : | 134 | . | 12 |
|------------------|---|---|---|---|-----|---|----|

H h

The

The miles travelled by each will be found in this manner.

| days | miles | days | miles                   |
|------|-------|------|-------------------------|
| 2    | 11    | 12   | 66 miles travelled by A |
| 3    | 17    | 12   | 68 miles travelled by B |

**Quest. 20.** If a Clock hath two Indices (or Hands) one of which (to wit *A*) is carried twice round the whole Circumference of the Dial in one day and the other (*B*) once in 30 days, and that both at once shewing the same point begin to be moved; the Question is, in what time they will be again conjoined?

**Answer,**  $\frac{1}{2}$  day or  $\frac{1}{2}$  hours.

| day | circum. | days | circum. |
|-----|---------|------|---------|
|-----|---------|------|---------|

|   |   |    |    |
|---|---|----|----|
| 1 | 2 | 30 | 60 |
|---|---|----|----|

subtrah

59

Hence it appears, that in 30 days *A* will have run through 60 Circumferences, and *B*, one Circumference only in the same time; therefore *A* gains of *B* 59 Circumferences in 30 days, therefore say,

| circum. | days | circum. | day. |
|---------|------|---------|------|
|---------|------|---------|------|

|    |    |   |                |
|----|----|---|----------------|
| 59 | 30 | 1 | $\frac{1}{59}$ |
|----|----|---|----------------|

**Quest. 21.** If 6 lb. of Sugar be equal in value to 7 lb. of Raisins; 5 lb. of Raisins to 2 lb. of Almonds; 3 lb. of Almonds to 5 lb. of Currants; 2 lb. of Currants to 18 d. how many pence are the value of 3 lb. of Sugar? **Answer.** 21 d.



$$\left. \begin{array}{l} 6S. = 7R. \\ 5R. = 2A. \\ 3A. = 5C. \\ 2C. = 18d. \\ ?d. = 3S. \end{array} \right\} \begin{array}{l} \text{mult. these contin.} \\ \text{mult. these contin.} \end{array}$$

$$180. \quad 3780 \quad (21)$$

**Quest. 22.** If 3 dozen pair of Gloves be equal in value to 2 pieces of Ribbon; 3 pieces of Ribbon to 7 dozen of Points; 6 dozen of Points to 2 yards of Flanders-lace; and 3 yards of Flanders-lace to 81 shilling; how many dozen pair of Gloves may be bought for 28 shillings?

**Ans.** 2 dozen pair of Gloves.

$$\left. \begin{array}{l} 3G. = 2R. \\ 3R. = 7P. \\ 6P. = 2L. \\ 3L. = 81S. \\ 28S. = ?G. \end{array} \right\} \begin{array}{l} \text{mult. these contin.} \\ \text{mult. these contin.} \end{array}$$

$$4536 \quad 2268$$

**Quest. 23.** Suppose a Greyhound to be courting a Hare, in such sort that the Hare takes five leaps for every four leaps of the Greyhound, and that the Hare is one hundred of her own leaps distant from the Greyhound; now if three of the Greyhounds leaps be equal to four leaps of the Hares: The question is to know how many leaps the Greyhound must take before he obtain his prey?

**Ans.** 1200 leaps.

I. If  $3 : 4 :: 4 : 5\frac{1}{3}$ .

Thus it appears, that 4 of the *Greyhounds* leaps are equal to  $5\frac{1}{3}$  of the *Hares* leaps; and because by the question the *Greyhound* takes 4 leaps for every 5 of the *Hares*, therefore the *Greyhound* in every four of his leaps gains  $\frac{1}{3}$  of one of the *Hare's* leaps; therefore say by the *Rule of Three*,

II. If  $\frac{1}{3} : 4 :: 100 : 1200$

*Quest. 24.* There is a certain Room whose Basis is a long square, which is in circuit  $50\frac{1}{2}$  feet, and the height of the walls or sides of the Room is  $8\frac{1}{4}$  feet; all which walls of the Room except a space taken out for a Window in the form of a long square, whose height is five feet, and breadth four feet, are to be furnished with Hangings of all-broad stuff at 3 s. 4 d. the yard, the question is to know how much money the stuff will cost.

*Ans.* 5 l. 17 s.  $6\frac{2}{3}$  d.

$$50\frac{1}{2} \times 8\frac{1}{4} = 416\frac{1}{8} \text{ Square feet.}$$

$$5 \times 4 = 20 \text{ subtract.}$$

---


$$396\frac{5}{8}$$

$$3\frac{3}{4} \times 3 = 11\frac{1}{4} \text{ square feet in one yard of stuff}$$

|                      |         |                    |                   |
|----------------------|---------|--------------------|-------------------|
| feet                 | d       | feet               | d                 |
| If $11\frac{1}{4} :$ | $40 ::$ | $396\frac{5}{8} .$ | $1410\frac{3}{4}$ |

*Quest. 25.* There is a certain Walk which is a long

long square, whose length is 40 yards, and breadth 7 yards, to be paved with Stones, each of which being in form of a long square is 28 inches in length, and 24 inches in breadth; the Question is to know how many such Stones will be requisite to pave the said Walk?

Ans. 540.

$$\begin{array}{rcl} \text{Inches} & & \text{Inches} \\ 1440 & \times & 253 = 362880 \text{ Square Inches.} \end{array}$$

$$28 \times 4 = 672 \text{ Square Inches.}$$

$$672 \cdot 1 :: 362880 \cdot 540 \text{ Stones.}$$

Quest. 26. Suppose a piece of Tapestry to be  $5\frac{1}{4}$  yards English in length, and  $3\frac{7}{8}$  yards in breadth, the Question is, how many square ells Flemish are contained in that piece of Tapestry, when the length of 1 ell Flemish is equal  $\frac{1}{4}$  of a yard English?

Ans.  $37\frac{1}{8}$  square ells Flemish.

$$5\frac{1}{4} \times 3\frac{7}{8} = 13\frac{3}{4} \text{ square yards.}$$

Then because  $\frac{2}{16}$  of a square yard is equal to ell square of Flemish measure (for  $\frac{3}{4} \times \frac{1}{4} = \frac{2}{16}$ ) say,

$$\text{If } \frac{2}{16} \cdot 1 :: 13\frac{3}{4} \cdot 37\frac{1}{8}$$

Quest. 27. A Workman hath performed a piece of Tiling bearing the form of a long square, whose length is 273 feet, 7 inches; and breadth 21 feet 5 inches; now when the Tiles are sold at the rate of 11s. 10 $\frac{3}{4}$ d. 1000, and every square of Tiling consisting of 10 feet as well in length as in breadth doth take up 1000 Tiles, what doth the said piece of Tiling amount unto?

H h 3

Ans.



*Ans.* 34 l. 17 s. 0  $\frac{400}{1000}$  d.

I.  $273\frac{7}{8} \times 21\frac{1}{2} = \frac{34173}{1000}$  Square feet.

II.  $100 \cdot 142\frac{3}{4} :: \frac{84373}{1000} \cdot 8364\frac{400}{1000}$

*Quest.* 28. A Merchant would bestow 220 l. in Clove, Mace, and Nutmegs, the Cloves being at 5 s. the pound; the Mace at 11 s. the pound, and the Nutmegs at 6 s. the pound; now he would have of each sort an equal quantity, the question is how many pounds he may have of each sort?

*Ans.* 200 lb.

5  
5  
11  
6  
—

220 : 1 :: 4400 : 200

### The Proof.

| lb.                    | s.  | l.        |
|------------------------|-----|-----------|
| 200 at 5 amounts unto  | 50  | 50        |
| 200 at 11 amounts unto | 110 | 110       |
| 200 at 6 amounts unto  | 60  | 60        |
|                        |     | <hr/> 220 |

*Quest.* 29. A Factor is to receive a sum of money, and is offered Dollars at 4 s. 4 d. which are worth but 4 s. 3 d. or French Crowns at 6 s. 1  $\frac{1}{2}$  d. which

are worth but 6 s. the question is by which Coin he shall sustain the least loss?

*Ans.* the Dollars.

$2 d. \text{ in } 1 d. \quad 1 d. \text{ in } 1 d. \quad 1 d. \text{ in } 1 d. \quad 1 d. \text{ in } 1 d.$   
 $52 : 1 :: 73 \frac{1}{2} : 1 \frac{43}{54}$

That is, in receiving the Dollars every 6 s.  $1 \frac{1}{2} d.$  loseth  $1 \frac{43}{54} d.$  but in receiving the Crowns 6 s.  $1 \frac{1}{2} d.$  loseth  $1 \frac{1}{2} d.$  which is a greater loss than  $1 \frac{43}{54} d.$

*Quest.* 30. A Butcher agrees with a Grasier, for the feeding of 20 Oxen, during the space of 12 equal months, but at 2 months end, the Butcher adds 5 Oxen more, and  $6 \frac{1}{2}$  months after that, he added 10 Oxen more, and then it is agreed between them that the Grasier shall feed them all, so long time as will be equivalent to the keeping of the first twenty during 12 months; the question is how long time he shall feed them all, after the putting in of the last 10?

*Ans.* 1 month.

Consider, that as he receives more Oxen to feed he ought to keep them all the less time; therefore work as the question imports by the *Rule of Three Inverse*.

|         | Mon.  | Oxen |                 | Mon. | Oxen |
|---------|-------|------|-----------------|------|------|
|         | 12    | 20   |                 |      |      |
| Oxen    | 2     | 5    |                 |      |      |
| <hr/>   |       |      |                 |      |      |
| If 20 : | 10 :: | 25 . | (8              | 25   |      |
|         |       |      | $6 \frac{1}{2}$ | 10   |      |
| <hr/>   |       |      |                 |      |      |

If 25 . . . .  $1 \frac{1}{2}$  .. 35 (1 Mon.

H h 4

*Quest.*

Examples of  
the Rule of  
Fellowship.

**Quest. 31.** Two Merchants, viz. A and B, have entred Company; A puts in 500*l.* and at 4 months end takes out a certain sum, leaving the remainder to continue 8 months longer, B puts in 250*l.* and at five months end puts in three hundred pounds more, and then his whole sum continues seven months longer. Now at the making of their Account A findeth that he hath gained 106 $\frac{2}{3}$  pounds, and B gained 133 $\frac{1}{3}$  pounds; the question is to know how much A took out of the Bank at 4 months end?

**Ans.** 240*l.*

$$\begin{array}{r} 250 \times 5 = 1250 \\ \text{add } 300 \\ \hline \end{array}$$

$$550 \times 7 = 3850$$

$$5100$$

$$133\frac{1}{3} \cdot 5100 :: 106\frac{2}{3} \cdot 4080$$

$$500 \times 4 = 2000 \text{ (Subtra)} \quad \underline{\hspace{2cm}}$$

$$8) 2080 \text{ (260)}$$

Lastly, 500—260 = 240 taken out by A.

*The Proof.*

|            |            |
|------------|------------|
| l.         | mon.       |
| 500        | x 4 = 2000 |
| Subtra 240 |            |
| <hr/>      |            |
| 260        | x 8 = 2080 |
| <hr/>      |            |
|            | 4080       |

*Quest.*

Chap. V

*Quest.*  
E, have  
fort that  
of the sh  
tion is,

*Ans.*  
Divid  
which r  
quired,  
operatio

A 2  
B 4  
C 5  
D 6  
E 8

If 25

*Quest.*  
pany, t  
of A co  
ney of l  
divide e  
each Me

*Ans.*  
Divid  
in prop  
be the S  
which S  
times, t



*Quest.* 32. Five Merchants, viz. A, B, C, D, and E, have gained 2025 l. which they divide in such sort that  $\frac{1}{2}$  of the share of A is equal severally to  $\frac{1}{4}$  of the share of B,  $\frac{1}{3}$  of C,  $\frac{1}{5}$  of D,  $\frac{1}{8}$  of E. The question is, what was the share of each Merchant?

*Ans.* A 162 l. B 324 l. C 405 l. D 486 l. E 648 l.

Divide a number at pleasure into such parts which may be in such proportion as the shares required, and proceed according to the subsequent operation.

A 2  
B 4  
C 5  
D 6  
E 8

If 25 . 2025 ::

l.

|   |                                           |
|---|-------------------------------------------|
| { | 2 (162 for A, whereof $\frac{1}{2}$ is 81 |
| { | 4 (324 for B, whereof $\frac{1}{4}$ is 81 |
| { | 5 (405 for C, whereof $\frac{1}{5}$ is 81 |
| { | 6 (486 for D, whereof $\frac{1}{6}$ is 81 |
| { | 8 (648 for E, whereof $\frac{1}{8}$ is 81 |

2025

*Quest.* 33. Two Merchants A and B are in Company, the sum of their Stocks in 300 l. the money of A continuing in Company 9 months, the money of B 11 months, they gain 200 l. which they divide equally, the question is to know how much each Merchant did put in?

*Ans.* A 165 l. B 135 l.

Divide 300 into two such parts which may be in proportion as 11 to 9, so will the greater part be the Stock of A, and the lesser the Stock of B, which Stocks being multiplied by their respective times, the products will be equal.

$$\begin{array}{r}
 11 \\
 9 \\
 \hline
 20 \cdot 300 ::
 \end{array}
 \left. \begin{array}{l}
 \\
 \\
 \end{array} \right\}
 \begin{array}{l}
 11 \cdot 165 \text{ for } A \\
 9 \cdot 135 \text{ for } B
 \end{array}$$

**Quest. 34.** Two Merchants, viz. *A* and *B*, are in Company, *A* did put in 325 *l.* more than *B*, and the stock of *A* continued in Company  $7\frac{1}{2}$  months *B* put in a certain sum which is unknown, and continued in Company  $10\frac{3}{4}$  months: After a certain time they divided the gain equally; the question is, what each Merchant did put in?

*Ans.* *B* 750 *l.* and *A* 1075 *l.*

Divide the product of the difference of their Stocks multiplied by the time of *A*, by the difference of their times, so will the quotient be the Stock of *B*, which added to 325 *l.* gives the Stock of *A*.

$$\begin{array}{r}
 325 \times 7\frac{1}{2} = 2437\frac{1}{2} \\
 3\frac{1}{4}) 2437\frac{1}{2} \quad (750 \text{ stock of } B \\
 \text{add } 325 \\
 \hline
 1075 \text{ stock of } A
 \end{array}$$

**Quest. 35.** A Goldsmith hath some Gold of 22 Carets, others of 22 Carets, and another sort of 18 Carets fine; he would so mix these together that the mass mixed might be 60 *lb.* and that the whole Mixture might be 20 Carets fine. How much of each sort must he take?

*Examples of the Rule of Alligation, How the fineness of gold and silver is estimated, v. p. 171.*

lb.

Ans<sup>w</sup>.  $\left\{ \begin{array}{l} 12 \text{ of } 24 \text{ Carets.} \\ 12 \text{ of } 22 \text{ Carets.} \\ 36 \text{ of } 18 \text{ Carets.} \end{array} \right.$

$$\begin{array}{r|l} \begin{array}{l} 24 \\ 22 \\ 18 \end{array} & \begin{array}{l} 2 \\ 2 \\ 4+2 \end{array} \end{array} \quad \begin{array}{l} 2 \\ 2 \\ 6 \end{array}$$

$$10 \cdot 60 :: \left\{ \begin{array}{l} 2 \cdot 12 \\ 2 \cdot 12 \\ 6 \cdot 36 \end{array} \right.$$

*Note*, Some may think that Questions of *Alligation* are capable only of so many several Answers as there are different ways to connect the mean Rate or Price with the extream Rates or Prices; yet it is most certain, that any ordinary Question of *Alligation*, where three or more things are propounded to be mixt in such manner as that Rule requires, is capable of infinite Answers, if Fractions be admitted, and sometimes of many Answers in whole numbers, which are not discoverable by the common Rule of *Alligation*: So albeit to the last mentioned question, the said Rule of *Alligation* can find but one Answer only, which is before given; yet there are eight other Answers in whole Numbers, which are these that follow (the Invention whereof I have shewn in the 19th. Question of the thirteenth Chapter of my second Book of *The Elements of Algebra*.)

Of



|              |    |    |    |    |
|--------------|----|----|----|----|
| Of 24 Carets | 18 | 16 | 14 | 10 |
| Of 22 Carets | 3  | 6  | 9  | 15 |
| Of 18 Carets | 39 | 38 | 37 | 35 |

|              |    |    |    |    |
|--------------|----|----|----|----|
| Of 24 Carets | 8  | 6  | 4  | 2  |
| Of 22 Carets | 18 | 21 | 24 | 27 |
| Of 18 Carets | 34 | 33 | 32 | 31 |

See Chap. 8. of  
this Appendix.

*Quest. 36.* An Apothecary hath several Simples, viz. A hot in  $3^{\circ}$ . B hot  $2^{\circ}$ . C temperate, D cold in  $2^{\circ}$ . and E cold in  $4^{\circ}$ . Now he desires to make a Medicine of those Simples in such sort that the temper thereof in respect of quality may be in  $1^{\circ}$ . of heat, and the quantity  $8\frac{1}{2}$  Drams; the Demand is what quantity of each Simple he must take?

*Ans.*  $4\frac{1}{2}$  Drams of A,  $\frac{1}{2}$  Dram of B,  $\frac{1}{2}$  Dram of C, 1 Dram of D, and 1 Dram of E.

| Indices |  | Drams   |   |
|---------|--|---------|---|
| 8       |  | 1, 3, 5 | 9 |
| 7       |  | 1       | 1 |
| 5       |  | 2, 1    | 3 |
| 3       |  | 2       | 2 |
| 1       |  | 2       | 2 |

|                            |   |                    |    |
|----------------------------|---|--------------------|----|
|                            |   | 17                 |    |
|                            |   | Drams.             |    |
| $17 \cdot 8\frac{1}{2} ::$ | { | 9 . $4\frac{1}{2}$ | A. |
|                            |   | 1 . $0\frac{1}{2}$ | B. |
|                            |   | 3 . $1\frac{1}{2}$ | C. |
|                            |   | 2 . 1              | D. |
|                            |   | 2 . 1              | E. |

$8\frac{1}{2}$

*Quest.*

*Quest.* 37. A Merchant buyeth 2 sorts of Cloaths, viz. of blacks and of whites for 68*l*. 2*s*. after the rate of 21*s*. the yard for the black, and 12*s*. the yard for the white, and he taketh so much of each sort, that  $\frac{3}{4}$  of the number of yards of the black, are equal to  $\frac{7}{4}$  of the white; the demand is how many yards be bought of each sort?

*Examples of the Rule of False Position.*

*Ans.* 42 yards of black, and 40 yards of white.

*Quest.* 38. A certain Person A payeth unto the use of B for ever 2500*l*. in present money, upon this Condition, that B shall pay unto A an Annuity of yearly rent to be continued four years; the equality of their agreement being thus grounded, viz. the said 2500*l*. is supposed to be put forth at interest for a year (to commence from the time of their agreement) at the rate of 8 per centum, per annum. Then from the sum of that principal and interest (arising due at the years end) the first payment of the Annuity being subtracted, the remainder is likewise supposed to be put forth at the same rate of interest for the second year; then from the composed of this principal and interest (due at the second years end) the second payment of the Annuity being subtracted, the remainder is likewise supposed to be put forth at the same rate of interest for the third year; then from this principal and interest the third payment of the Annuity being subtracted, the remainder is in like manner supposed to be put forth at the same rate of interest for the fourth Year: Lastly, from this principal and interest the fourth and last payment of the Annuity being subtracted, there must be nothing left: The question is, what sum of money must be yearly

yearly paid to satisfy those Conditions?

*Ans.* 754  $\frac{14117}{17602}$  l. as will be manifest by the subsequent proof.

I. 100 . 108 :: 2500 . 2700

Subtract the first payment 754  $\frac{14117}{17602}$

1945  $\frac{1485}{17602}$

II. 100 . 108 :: 1945  $\frac{1485}{17602}$  . 2100  $\frac{14315}{17602}$

Subtract the second payment 754  $\frac{14117}{17602}$

1346  $\frac{108}{17602}$

III. 100 . 108 :: 1346  $\frac{108}{17602}$  . 1453  $\frac{12194}{17602}$

Subtract the third payment 754  $\frac{14117}{17602}$

698  $\frac{15679}{17602}$

IV. 100 . 108 :: 698  $\frac{15679}{17602}$  . 754  $\frac{14117}{17602}$

Subtract the last payment . . . 754  $\frac{14117}{17602}$

000

Quest. 39.

*Mula, Asinaque duos imponit servulus utres*

*Impletos vino; segnemque ut vidit Asellam*

*Pondere defessam vestigia figere tarda,*

*Mula rogat; quid chara parens cunctare, gemisque?*

*Unam ex utro tuo mensuram si mihi reddas,*

*Duplum oneris tunc ipsa feram; sed si tibi tradam*

*Unam mensuram, fient aequalia utrique*

*Pondera, mensuras dic docte Geometer istas?*

The sense is this. A Mule and an As carried two unequal quantities of Wine, each consisting of a certain



certain number of measures, in such sort, that if the *Ass* imparted one of her measures to the *Mule*, then the *Mules* number of measures so increased would be the double of those which the *Ass* had remaining; but if the *Mule* gave one measure to the *Ass*, then the *Asses* measures with that increase would be equal to the *Mules* remaining measures. The Question is, how many Measures each carried?

*Ans.* The *Mule* 7, and the *Ass* 5.

## Quest. 40.

*Æs, ferrum, stannum miscens, aurique metallum,  
Sexaginta minas pensantem finge coronam.*

*Æs aurumque duos simul efficiunto trientes.*

*Ternos quadrantes stanno mixtum impleat aurum.*

*At totidem quintas auri vis addita ferro.*

*Ergo age dic fulvi quantum tibi conjicis auri*

*Miscendum: dic quantum æris stannique requiras:*

*Dic quoque sufficiant auri quot pondera ferri:*

*Præscriptam ut valeas rite efformare coronam.*

The sense is this, Suppose a Crown that shall weigh 60 l. is to be made of Gold, Brass, Iron, and Tin, mixed together in such proportion, that the weight of the Gold and of the Brass together may be 40 l. the joint weight of the Gold and of the Tin 45 lb. and the joint weight of the Gold and of the Iron 36 lb. The Question is how much of every one of those four Metals must be taken?

*Answer,*  $\left\{ \begin{array}{l} 30\frac{1}{2} \text{ of Gold,} \\ 9\frac{1}{2} \text{ of Brass,} \\ 5\frac{1}{2} \text{ of Iron,} \\ 14\frac{1}{2} \text{ of Tin.} \end{array} \right.$

*Quest.*

*Quest.* 41. One being demanded what was the present hour of the day, answered, That the time then past from noon was equal to  $\frac{1}{2}$  of  $\frac{3}{4}$  of the time remaining until midnight. The question is, what a Clock it was? (supposing the time between noon and midnight to be divided into twelve equal parts or hours)

*Ans.*  $\frac{3}{4}$  hour after Noon.

*Quest.* 42. A Factor delivers 6 French Crowns and 2 Dollars for 45 shillings sterling; also at another time he delivers 9 French Crowns and 5 Dollars (at the same rate with the former) for 76 shillings. The question is to know the value of a French Crown, also of a Dollar?

*Ans.* A Crown was valued at 6 s. 1 d. and a Dollar at 4 s. 3 d.

*Quest.* 43. A certain Ufurer received 36 Dollars for the simple interest of 186 l. lent for a certain time unknown; also he received 90 Dollars for the gain of 360 l. at the same rate of interest for a certain time unknown; now the sum of the months wherein both the said numbers of Dollars were gained was twenty months. The question is to know in what time as well the 36 Dollars as the 90 Dollars were gained?

*Ans.* The 36 Dollars were gained in  $8\frac{1}{11}$  months, and the 90 Dollars in  $11\frac{1}{11}$  months, as may be proved by the *Double Rule of Three*.

Which answer may be discovered by the following Canon found out by the *Algebraick Art*.

Multiply the Dollars first gained, the latter Principal, and the given time, according to the rule of continual Multiplication, for a dividend; then multiply the first principal by the Dollars last gained;

also

also multiply the latter principal by the Dollars first gained, and reserve the sum of these two last products for a Divisor; Lastly, divide the Dividend first found by the said Divisor, so shall the quotient be the time wherein the first number of Dollars was gained, which subtracted from the time given in the question, discovers the time wherein the latter number of Dollars was gained.

$$36 \times 360 \times 20 = 259200.$$

$$186 \times 90, + 300 \times 36, = 29700$$

$$\text{And consequently } \dots 20 - 8\frac{2}{11} = 11\frac{3}{11}.$$

*Quest. 44.* 3481 Soldiers are to be placed in a square Battel, how many are to be set in Rank or in File? *Examples of the Extraction of Roots.*

*Ans.* 59. (for the square root of 3481 is 59.)

*Quest. 45.* If 4050 Soldiers are to be set in Battel in a figure, which beareth the form of a long square, in such manner, that the number in File may be to the number in Rank, as 1 to 2, how many Soldiers are to be placed in Rank, and how many in File?

*Ans.* 90 in Rank and 45 in File (found by this Canon or general Rule) viz.

As the greater term of the proportion given is the lesser, so is the number of Men to be placed in Battel to a fourth proportional, whose square root the lesser number sought (whether it be for the Rank or File:) Also the lesser term of the given proportion is to the greater; so is the number of Men to be set in Battel to a fourth proportional;

11

whose



whose Square root is the greater number sought (whether it be for the Rank or File.)

|      |             |                         |
|------|-------------|-------------------------|
| I.   | 2.          | I :: 4050 . 2025        |
| II.  | $\sqrt{q.}$ | 2025 = 45 (Men in File) |
| III. | I.          | 2 :: 4050 . 8100        |
| IV.  | $\sqrt{q.}$ | 8100 = 90 (Men in Rank) |

### The Proof.

$$45 \times 90 = 4050$$

Also  $45 \cdot 90 :: 1 \cdot 2$

Or when one of the numbers sought (whether it be for the Rank or File) is found, the other may be discovered by *Division*, viz.

$$\begin{array}{r} 45 \overline{) 4050} \quad (90 \\ 90 \overline{) 4050} \quad (45 \end{array}$$

**Quest. 46.** Suppose the Wall of a Garrison to be in height 21 feet, and the breadth of the Moat surrounding the said Wall to be 28 feet; the Question is, what length must a scaling Ladder have to reach from the outermost side of the Moat to the top of the Wall?

**Ans. 35.** (to wit, the square root of the sum of the squares of 21 and 28.)

$$\begin{array}{r} 21 \times 21 = 441 \\ 28 \times 28 = 784 \end{array}$$

$$\sqrt{q.} \quad 1225 \quad 35$$

*Quest.* 47. If 100*l.* being put forth for interest at a certain rate, will at the end of two years be augmented unto 112  $\frac{36}{100}$  *l.* (compound interest, or interest upon interest being computed) what principal and interest will be due at the first years end?

*Ans.* 106*l.* composed of 100*l.* principal and 6*l.* interest (which 106 is a mean Geometrically proportional between 100 and 112. 36 (and many be found by the eighteenth rule of the fifth Chapter of this Appendix.)

$100 \times 112.36 = 11236$  (106

*Quest.* 48. If 100*l.* being put forth for interest at a certain rate, will at the end of 3 years be augmented unto 115.7625*l.* (compound interest being computed) what principal and interest will be due at the first years end?

*Ans.* 105*l.* composed of 100*l.* Principal, and 5*l.* interest) which 105 is the first of two mean proportional numbers between 100 and 115.7625*l.* (See the nineteenth rule of the fifth Chapter of this Appendix.)

*Various Practical Questions to exercise Decimal Arithmetick, in the mensuration of Superficial Figures and Solids.*

*Quest.* 49. If the side of a square Superficies be 3 feet, what is the Area or Content of that Superficies? Or (which is the same thing) how many Squares, each of which is a Foot square, are contained in that Superficies?

*See the second Section of the 23d. Chapter of the preceding Book.*

*Ans.* 9 square feet, which content is found out by multiplying the given side 3 by it self, viz. 3 multiplied by 3 produceth 9.

In like manner, if the side of a square Pavement of stone be 15.7 feet, the superficial content of that Pavement will be 246.49 feet, that is 246 feet and an half very near, (for 15.7 multiplied by it self produceth 246.49.)

Likewise, a square piece of Wainscot whose side is 3.24 yards, will be found to contain 10.49 + yards, or 10 yards and an half almost; for 3.24 multiplied by it self, to wit, by 3.24, will produce 10.49 +.

Also if the side of a square piece of Land be 37.25 perches, the content in square perches (neglecting the fraction in the product) will be found 1387, which being reduced (according to the seventh Table in Rule 4, Chapter 7 of the preceding Book) will give 8 Acres, 2 Roods, and 27 perches for the content of that square piece of Land.

*Quest.* 50. If a long square be 8 feet in length and 5 feet in breadth, what is the superficial content?

*Ans.* 40 feet; which content is found out by multiplying the length by the breadth, viz. 8 multiplied by 5 produceth 40. So if one of the Lights of a Glass window supposed to be in the form of a long square, hath for its length 3.06 feet, and breadth 1.47 feet, the content of that Glass will be 4.4982 feet, or 4 feet, and an half almost, (for 3.06 multiplied by 1.47 produceth 4.4982.)

In like manner if there be a piece of Wainscot, Plastring, or any other superficies in the form of



a long square, which is in length 6.325 yards, and in breadth 3.214 yards; the superficial content will be found 20 32 + yards, that is 20 yards, one quarter of a yard, and somewhat more, for, 6.325 multiplied by 3.214 produceth 20 32 +.

Likewise a piece of Tiling in the form of a long square whose length is 18.5 feet, and breadth 11.7 feet will be found to contain 216.45 square feet, which will be reduced to 2.1645 squares of Tiling by allowing (according to Custom) 100 square feet to one square of Tiling.

Also if a piece of Land in the form of a long square be 48.75 perches in length, and 36.25 in breadth, the Area or Content in Perches, will be found 1767.18 +, which 1767 perches being reduced will give 11 acres and 7 perches for the content of that piece of Ground.

*Quest.* 51. If it be required to set forth in a Meadow one Acre of Grass to lie in the fashion of a long square, and that the length thereof be limited or agreed to be 20 perches, what must the breadth be?

*Ans.* 8 perches, which breadth is found out by dividing 160 (the number of square perches contained in an Acre) by the given length 20. If two Acres were required, then 320 (to wit, twice 160) must be divided by the given side, whether it be the length or breadth; so if 7.25 perches be prescribed for the breadth of two Acres, the length must be 44. 13 + perches.

In like manner, if the breadth of a Board be 1. 32 foot, and it be demanded how far one ought to measure along the side thereof to have a superficial foot, or a foot square of that Board; divide

1 by the given breadth, so you will find in the quotient this decimal fraction  $.757+$ , which represents three quarters of a foot or nine inches and somewhat more, and so much in length ought to be measured along the side of that Board to make a superficial Foot. Likewise if the breadth of a board be given in inches, then 144 (the number of square inches contained in a superficial foot square) being divided by the given breadth, the quotient will shew how many inches ought to be measured along the side of that board to make a superficial foot; so the breadth of a board being 9 inches, the length forward to make a superficial foot will be found 16 inches.

*Quest.* 52. If the three sides of a piece of Land that lies in the form of a triangle be 15 perches, 14 perches, and 13 perches, what is the Area or number of square perches contained in that triangle?

*Ans.* 84 Perches, or half an Acre and four Perches, which Content is found out by this Rule, *viz.*

From half the sum of the three sides of any plain triangle, subtract each of the three sides severally, and note the three remainders; then multiply the said half sum and those three remainders one into the other (according to the rule of continual Multiplication;) that done, extract the square root of the last product, so shall such square root be the Area or Content of the triangle.

The

Perches

The 3 sides of a triangle

$$\left. \begin{array}{r} 15 \\ 14 \\ 13 \end{array} \right\}$$

The sum of the three sides

42

The half of that sum

21

The 3 remainders found out by subtracting each side from the half sum

$$\left. \begin{array}{r} 6 \\ 7 \\ 8 \end{array} \right\}$$

The product arising from the continual multiplication of the four last numbers

7056

The square root of which product is the content required, to wit,

84

## Another Example.

Perches

The 3 sides of a triangle

$$\left. \begin{array}{r} 120.3 \\ 112.6 \\ 90.3 \end{array} \right\}$$

The sum of the 3 sides

323.4

The half of that sum

161.7

The 3 remainders found by subtracting each side from the half sum

$$\left. \begin{array}{r} 41.2 \\ 49.1 \\ 71.4 \end{array} \right\}$$

The product arising from the continual multiplication of the four last numbers

23355380.1096

The square root of that product—4832.7+

114

Wherefore



Wherefore I conclude that the content of a plain triangle, whose three sides are 120.5 perches 112.6 perches, and 90.3 perches, is 4832.7 + perches, which reduced give 30 Acres and 32 perches (the fraction of a perch being neglected.)

Now forasmuch as every irregular piece of Ground may be divided into triangles, for a four-sided Field will be divided into two triangles by one imaginary straight Line leading overthwart from corner to corner called a *Diagonal Line*; a five-sided Field into three triangles by two *Diagonals*; a six-sided Ground into four triangles by three *Diagonals*, &c. the Rule before given will be of excellent Use to find out the Contents of large Fields, especially if the Land be of a dear value, as also when any controversy ariseth by the reason of the different admeasurements of Surveyors of Land: For if the sides of those triangles be measured in the Fields, and their lengths be agreed on, all Artists to whom the reason of the rule before given is known, will agree in one and the same content. But yet this way of measuring presupposeth that there is no Obstacle, as Water, Wood, or other Impediment, to hinder the measuring of the sides of those Triangles into which the Field is divided as aforesaid.

*Quest.* 53. If the Diameter of a Circle be 28.25, what is the Circumference?

*Ans.* 88.749.4: For as 113 is in proportion to 355; or as 1 is to 3.14159 so is the Diameter to the Circumference: Therefore multiplying always the Diameter given by the said 3.14159 the product shall be the Circumference required.

*Quest.*

*Quest. 54.* If the Diameter of a Circle be 28.25, what is the superficial Content of that Circle?

*Ans.* 626.79 + ; For as 1 is in proportion to .78539, so is the square of the Diameter to the superficial content. Therefore multiplying always the said decimal Fraction .78539 by the square of the given Diameter (which square is the product of the multiplication of the Diameter by it self) the product shall be the superficial content required.

*Quest. 55.* If the Diameter of a Circle be 28.25. what is the side of a square which may be inscribed within the same Circle?

*Ans.* 19.975 + ; For the square root of half the square of the Diameter, or the square root of the double of the square of the Semi-diameter, shall be the side of the inscribed square sought. Otherwise, as 1 is to .707106, so is the Diameter to the side required. Therefore if you multiply (always) the said .707106, by the Diameter given, the product will be the side of the inscribed square required.

*Quest. 56.* If the Circumference of a Circle be 88.75 what is the Diameter?

*Ans.* 28.249 + ; For as 355 is to 113, or as 1 is to .318309, so is the Circumference to the Diameter. Therefore if .318309 be multiplied always by the given circumference, the product shall be the Diameter required.

*Quest. 57.* If the circumference of a Circle be 88.75, what is the superficial content of that Circle?

*Ans.* 626.801 + ; For as 1 is to .079578, so is the square of the circumference to the superficial content. Therefore if .079578 be always multiplied by the square of the given circumference, the product shall be the superficial content sought.

*Quest.*

*Quest.* 58. If the circumference of a Circle be 88.75. what is the side of a square that may be inscribed within the same Circle?

*Ans.* 19.975 + ; for as 1 is to .225078, so is the Circumference to the side required; Therefore if .225078 be always multiplied by the Circumference given, the product will be the side of the inscribed square sought.

*Quest.* 59. If the superficial Content of a Circle be 626.8, what is the diameter?

*Ans.* 28.25 + ; for as 1 is to 1.27224, so is the content to the square of the diameter. Therefore multiplying always 1.27224 by the given content, the square root of that product shall be the diameter required.

*Quest.* 60. If the superficial content of a Circle be 626.8, what is the circumference?

*Ans.* 88.75 + ; for as 1 is to 12.5664, so is the content to the square of the circumference. Therefore if 12.5664 be always multiplied by the given content, the square root of the product shall be the circumference required.

*Quest.* 61. If the superficial content of a Circle be 626.8, what is the side of a square equal to the same Circle?

*Ans.* 25.025 + ; for the square root of the given content is the side of the square required.

*Quest.* 62. If the side of a Cube be 12 inches, how many cubical inches are contained in that Cube?

*Ans.* 1728 ; what a Cube is may be well represented by a Dye, which is a little Cube it self being a rectangular or square solid, that hath an equal length, breadth and depth, and is comprehended

Chap.  
hended  
one of  
of the  
of that  
ording  
died by  
he dept  
nd luc  
ide is 1  
stone in  
ndersto  
Dye-squ  
olid con  
foot so  
In lik  
53 fee  
ound 1  
half prob  
uct bein  
6.194-  
Also i  
6 inches  
16 cub  
for 6 m  
s. multi  
t may  
wood, e  
square, a  
for 8 tim  
ber of cu  
wise 8 ti  
foot soli  
*Quest.*  
umber,



ended under six equal squares; now if the side of one of those equal squares (which is also the side of the Cube) be 12 inches, the superficial content of that square will be 144 square inches (for according to the preceding 49th question, 12 multiplied by 12 produceth 144) which multiplied by the depth 12 inches, produceth 1728 cubical inches; and such is the solid content of that Cube whose side is 12 inches: So that by one foot of timber or stone in whatsoever kind of solid it be found, is understood a Cube, containing 1728 cubical or square inches, and consequently half a foot solid contains 864 cubick inches, and a quarter of a foot solid contains 432 cubick inches.

In like manner, if a side of a Cube of stone be 253 feet, the solid content of that Cube will be found 16.194 + feet, for 253 being multiplied by it self produceth 64009 superficial feet, which product being multiplied by the said 253 will produce 16.194 + solid feet.

Also if the side of a Cube of stone or wood be 6 inches, or .5 foot, the solid content will be found 216 cubick inches, or .125 parts of a foot solid for 6 multiplied cubically produceth 216, likewise .5 multiplied cubically produceth .125; whence it may be infer'd, that 8 little cubes of stone or wood, each of which is half a foot or 6 inches square, are contained in a foot of stone or timber; for 8 times 216 produceth 1728 (being the number of cubick inches contained in a foot solid (likewise 8 times .125 produceth 1 (to wit, one entire foot solid.)

Quest. 63. If the breadth of a squared piece of timber, supposed to be streight and terminated at both

both ends by two equal squares, be 1.55 foot, the depth also 1.55 foot, and the length 17.33 feet, how many cubick feet are contained in that piece of Timber?

*Ans.* 41.635 feet, that is, 41 feet and an half, and about half a quarter of a Foot. Which solid content is found out by this rule, viz. multiply the breadth 1.55 by the depth 1.55, the product will be 2.4025 superficial Feet, which is the content of the Base (that is, the Area of either of the two equal squares at the ends of the piece.) Lastly multiplying the said Base 2.4025 by the length 17.33 the product will be 41.635 +, which is the solid content required.

In like manner if the breadth of a squared piece of Timber, supposed to be straight and terminated at both ends by two equal long squares (which are called the Bases) be 2.34 feet, the depth 11.61 foot, and the length 17.58 feet, the solid content will be 66.23 + feet; for (as before) multiplying the breadth by the depth, and that product by the length, the last product shall be the solid content required.

*Quest.* 64: If the breadth, as also the depth of a squared piece of Timber having equal square Bases, be 1.55 foot, how far ought one to measure along the length of that piece of Timber to make a foot solid?

*Ans.* 416 parts of a foot, or 5 inches very near; which decimal is thus found, viz. First find the superficial content of the Base, which will be 2.4025 (for 1.55 multiplied by 1.55 produceth 2.5025; Then dividing 1 (to wit 1 solid foot) by the Base 2.4025 the quotient will be .416 +

or  $\frac{4}{10}$  parts of a foot, or five inches almost, and so far ought to be measured along the length of the piece to make a foot solid. In like manner, if the breadth be 2.34 feet, and the depth 1.61 feet, the length forward along the piece to make one solid foot will be found .265 parts of a foot, or three inches and almost  $\frac{1}{4}$  part of an inch.

*Quest. 65.* If a freight squared piece of timber be terminated by unequal Bases, whereof one contains 1.92 superficial foot, the other .85 foot, and the length of that piece of Timber be 17.4 feet; what is the solid content; or how many cubical Feet are contained in that piece of Timber?

*Ans.* 23.474 + feet (found out by one of Mr. Oughtred's Rules for measuring a segment of a Pyramid in Problem 21. Chapter 19. of his *Clavis Mathematicæ*.) The Rule is this.

Multiply the greater Base by the less, and extract the square root of that product, then multiply the sum of the two Bases and that square root by one third part of the length of the solid propounded, so shall the last product be the solid content required.

*Example*



## Example.

The greater Base \_\_\_\_\_ 10 : 92  
 The lesser Base \_\_\_\_\_ 6 : 85  
 The product of the multiplication }  
 of those two Bases \_\_\_\_\_ } 1 : 6320  
 The square root of that product \_\_\_\_\_ 1 : 2774  
 The sum of that square root and }  
 the two Bases \_\_\_\_\_ } 4 : 0474  
 One third part of the length is \_\_\_\_\_ 5 : 8  
 The product of the multiplication }  
 of the two last numbers is the solid } 23 : 474+  
 content required \_\_\_\_\_ }

**Quest. 66.** A Pyramid is a solid comprehended under plain surfaces, and from a triangular, quadrangular, or any multangular Base, diminisheth equally less and less till it finish in a point at the top; now if the superficial content of the Base of a Pyramid be 5.756 Feet, and the height thereof 14.25 feet (which height is the length of the perpendicular line that falleth from the top of the Pyramid to the Base) what is the solid content of that Pyramid?

**Ans.** 27.341 + feet; for if the Area of the Base of a Pyramid, be multiplied by one third part of the height thereof, the product shall be the solid content of the Pyramid; therefore  $5.756 \times 4.75 = 27.341$  feet =, the solidity of the Pyramid propounded.

**Note,** If a Pyramid be cut into two segments by a Plane parallel to the Base, one of those segments will be a Pyramid, and the other will have two unequal Bases, for the measuring of which latter segment;

ment; a rule hath been already given in the sixty-fifth question, the Area of each Base being known.

*Quest. 67.* A Cone is a solid, which hath a Circle for its Base, from whence it grows equally less and less (like a round Steeple of a Church) till it finish in a point at the top; now if the Area of the Base of a Cone be 5.756 feet, and the height thereof be 14.25 feet, what is the solid content of that Cone?

*Ans.* 27.341 feet; for if the Area of the Base of a Cone be multiplied by one third part of the height thereof, the product shall be the solid content of the Cone.

*Notes.* If a Cone be cut into two segments by a Plane parallel to the Base, one of those segments will be a Cone, and the other segment will have two unequal Bases which are Circles; the solidity of which latter segment may be found out by the rule before given in the 65. question, the Area of each Base (or Circle) being known.

*Quest. 68.* A Cylinder is a solid which may be well represented by a Stone-roll, such as are used in Gardens for the rolling of Walks. Now if the circumference of a Cylinder be 4.57 feet, and the length 3.25 feet, what is the solid content of that Cylinder?

*Ans.* 5.41 Feet, thus found out: First by the help of the given circumference 4.57, find out the superficial content of that Circle (being the Base of the Cylinder) which content (by the preceding 57th question) will be found 1.6619 + feet, then multiplying the said 1.6619 by the given length 3.25, the product will be 5.4008, which is the solid content required.

*Quest.*

*Quest.* 69. If the Base of a Cylinder be 1.6619 foot, how much in length of that Cylinder will make a foot solid?

*Ans.* .601 parts of a foot; for 1 (to wit, 1 solid foot) being divided by the Base 1.6619, gives in the quotient the decimal .601 + for the length required.

*Quest.* 70. A Globe is a perfect round Body contained under one Plane; in the middle of the Globe there is a point called the Center, from whence all straight Lines drawn to the outside are of equal length, and called Semi-diameters, the double of any one of which is equal to the Diameter of the Globe; now if the Diameter of a Globe of Stone be 1.75 feet, how many Feet solid are contained in that Globe?

*Ans.* 2.807 + feet, for as 21 is in proportion to 11, or as 1 is to .5238, so is the Cube of the Diameter to the solid content of the Globe: Therefore, multiplying always the Cube of the Diameter by the said decimal .5238, the product shall be the solid content required: So the Diameter 1.75 being first multiplied by it self, the product will be 3.0625, which multiplied by the said 1.75, gives in the product 5.359375, to wit, the Cube of the diameter, which being multiplied by .5238, the product thence arising will be 2.807. +, which is the solidity of the Globe propounded.

*Quest.* 71. What is the Diameter of a Globe of Stone, which contains 4 cubical or solid Feet?

*Ans.* 1.96 + foot, for as 11 is in proportion to 21, or as 1 is to 1.9090909 so is 4 (the solid content given) to a fourth proportional, to wit, 7.636363 + whose cubick root is 1.96 + the Diameter required.

Concern



## Concerning the gauging of Vessels.

The easiest and aptest way for practice in gauging, are those which are performed by the help of Tables, or Gauging-rods purposely compos'd: Nevertheless, to give the Reader of this Treatise some light in this matter, I shall here insert one Rule to find out the number of Gallons contained in a full Tun, Pipe, Hoghead, Barrel, or such like Vessel, according to Mr. Wingate's way of reducing a Vessel to a Cylinder.

*The Rule is this;*

Having found the difference of the two diameters at the Bung and Head of the Vessel, take  $\frac{1}{2}$  of that difference and add it to the lesser diameter; then square that sum and reserve the product; that done, if the content be required in Wine Gallons, multiply the product reserved, this decimal fraction .0034, and the length of the Vessel, one into the other (according to the Rule of continual Multiplication) so shall the last product be the number of Wine Gallons required: But if the content be required in Ale Gallons, multiply the product before reserved, this decimal fraction .0027, and the length of the Vessel, one into the other continually, so shall the last product be the content in Ale Gallons: This Rule I shall first explain by two questions, and then shew how it is rais'd.

*Quest. 72.* If the diameter at the Bung of a Vessel be 32 inches, the diameter at the Head 28 2 inches, and the length 39 inches (which dimensions

are said to agree very near with those of an English Vessel, called a Pipe) what is the content of that Vessel in Wine Gallons?

*Ans.* 126.278 Wine Gallons, that is 126 Wine Gallons and about a quart more (found out by the Rule above given, as will be manifest by the following Operation.

*Explication*

|                                                                                |         |      |
|--------------------------------------------------------------------------------|---------|------|
| The diameter at the Bung                                                       | — 32    | 1. 0 |
| The diameter at the Head                                                       | — 28    | 7. 2 |
| The difference                                                                 | — 3     | 8    |
| Which multiplied by 4, that is                                                 | — 0     | 7    |
| The product will be                                                            | — 2     | 66   |
| Which added to the lesser diameter gives the mean diameter                     | 30      | 86   |
| Which mean diameter, being squared (that is, multiplied by it self), produceth | 952     | 3396 |
| Which product multiplied by                                                    | 0       | 0034 |
| The product thence arising will be                                             | 3       | 2379 |
| Which multiplied by the length of the Vessel                                   | 39      | 0    |
| The product is the number of Wine Gallons sought, viz.                         | 126.278 | +    |

*Quest. 73.* If the Diameter at the Bung of a Barrel be 23 inches, the Diameter at the Head 19 inches, and the length 27.4 inches; what is the content of that Barrel in Ale Gallons?

*Ans.* 36.031 Ale Gallons, that is 36 Gallons and about a quarter of a Pint more (found out by the preceding Rule.)

*Explication*

*Explication.*

The diameter at the Bung ————— 23 . 0  
 The diameter at the Head ————— 19 . 9  
 Their difference ————— 3 . 1  
 Which multiplied by  $\frac{7}{10}$ , that is ————— 0 . 7  
 The product will be ————— 2 . 17  
 Which added to the lesser diameter }  
 gives the mean diameter ————— 22 . 07  
 Which mean diameter being Squa- }  
 red (that is, multiplied by it self) } 487 . 0849  
 produceth —————  
 Which product multiplied by ————— 0 . 0027  
 The product thence arising is ————— 1 . 315 +  
 Which multiplied by the length of }  
 the Vessel ————— } 27 . 4  
 The product is the number of Ale }  
 Gallons sought, to wit ————— } 36 . 031 +

*The Reason of the Rule.*

Two Things are taken for granted in the said Rule, viz. First it is supposed that if  $\frac{7}{10}$  of the difference of the two diameters at the Bung and Head, be added to the lesser diameter, the sum shall be an equated or mean diameter (near enough for practical use though it be not exact,) viz. If there be a Cylinder whose diameter is equal to that mean diameter, and whose length is equal to the length of the Vessel, that Cylinder shall be equal to the capacity of the Vessel very near. Secondly,



the said Rule presupposeth that 231 cubick inches are equal to a Wine Gallon, and 282 equal to an Ale Gallon; concerning which Equalities (especially the latter) Artifts differ somewhat in their experiments; but according to any Equality which in that particular shall be agreed on, from this that follows a Rule may be framed, and Tables thence calculated for gauging a full Vessel without considerable Error.

Taking then those two things above mentioned for granted, we may rightly infer that if a Cylinder hath for its Base a Circle whose superficial content is 231 inches, every inch in length of that Cylinder will contain 231 cubick inches, or one intire Wine Gallon; now forasmuch as all Circles are in such proportion one to the other as the squares of their diameters, it shall be as 294.11844 (to wit, the square of the diameter of that Circle whose superficial content is 231) is to 1 (to wit, the superficial content 231 considered as the Base of one Wine Gallon;) or as 1 is to .0034: So is the square of the equated (or any other) diameter, to the superficial content of that Circle in Wine Gallons and parts of a Gallon, which content multiplied by the length of the Vessel will produce its solidity or capacity in Wine Gallons. Therefore the first part of the preceding Rule for finding of the number of Wine Gallons contained in a full Vessel is manifest: And after the said manner, supposing as before 282 cubick inches are equal to an Ale Gallon, the decimal .0027 prescribed in the said Rule will be found out.

Upon those Grounds Mr. *Wingate* compos'd his Gauging-Rod; Mr. *Oughtred* also in his Circles of Proportion.

Proportion hath delivered another Rule for Gauging, from whence his Gauging-rod is deduced; but the particular Constructions of those Rods, and likewise the making of Tables for the same purpose, being handled by several Artists, I shall not insist upon them.

Now if the industrious and more curious Arithmetician, after he is well exercis'd in vulgar Arithmetick, desires further Knowledge in finding out the Answer of subtil Questions about numbers, his best Guide will be the admirable *Algebraical* Art, which discovers Rules for the solving of *Problems*, as well Arithmetical as Geometrical, that are above the reach of any of the Rules of common Arithmetick, or practical Geometry, as may partly appear by the two Rules in the foregoing 52 and 65 Questions, as also by the two following Questions, with which I shall conclude this Chapter.

*Quest* 74. To find two numbers in a given proportion, suppose the lesser to the greater as 2 to 3 and such, that if the lesser number be added to the square of the greater, also if the greater number be added to the square of the lesser, the two sums shall be square numbers whose roots are expressible by rational or true numbers (fractions being admitted for numbers.)

*Ans.*  $\frac{1}{10}$  and  $\frac{3}{10}$ .

The square of  $\frac{1}{2}$  (the greater number) is  $\frac{1}{4}$ .

To which adding the lesser number  $\frac{1}{3}$  —

The sum in its least terms will be  $\frac{5}{12}$ .

Which is a square number, whose root is  $\frac{5}{12}$ .

Again, the square of  $\frac{1}{3}$  (the lesser number) is  $\frac{1}{9}$ .

To which adding the greater number  $\frac{1}{2}$  —

The sum in its least terms will be  $\frac{7}{6}$ .

Which is a square number whose root is  $\frac{7}{6}$ .

Also the said numbers  $\frac{1}{2}$  and  $\frac{1}{3}$  are one to the other as 2 to 3, wherefore the question is solved, which numbers  $\frac{1}{2}$  and  $\frac{1}{3}$  are found out by this following

### Theorem.

If the fraction  $\frac{1}{2}$  be divided into any two parts; either of those parts being increased with the square of the other part shall give a fraction having a rational square root.

Wherefore by dividing  $\frac{1}{2}$  into the two fractions  $\frac{1}{3}$  and  $\frac{1}{6}$ , which are in the prescribed proportions of 2 to 3, those fractions will satisfy the conditions in the question propounded.

Likewise these two fractions  $\frac{7}{12}$  and  $\frac{1}{12}$  will answer the question, and are found out without extracting any root; but the manner of finding out the said Theorem and last mentioned fractions, I have shewn in the 24th. question of my Third Book of the Elements of Algebra.

Quest.



*Quest. 75.* To find 3 numbers, such that the square of any one of them being added to the other two numbers, the sum of such addition shall be a square number, whose root is a rational number.

*Ans.* 1,  $\frac{8}{3}$ , and  $1\frac{6}{5}$

*The Proof.*

First, the square of the first number }  
1 is \_\_\_\_\_ } 1

To which adding the second and }  
third numbers  $\frac{8}{3}$  and  $1\frac{6}{5}$ , the sum will be } 9

Which is a square number whose }  
root is \_\_\_\_\_ } 3

Secondly, the square of the second }  
number  $\frac{8}{3}$  is \_\_\_\_\_ }  $\frac{64}{9}$

To which adding the first and third }  
numbers 1 and  $1\frac{6}{5}$ , the sum in its least }  $1\frac{1}{5}$   
terms will be \_\_\_\_\_ }  $\frac{6}{5}$

Which is a square number whose }  
root is \_\_\_\_\_ }  $1\frac{1}{5}$

Thirdly, the square of the third }  
number  $1\frac{6}{5}$  is \_\_\_\_\_ }  $3\frac{6}{5}$

To which adding the first and second }  
numbers 1 and  $\frac{8}{3}$ , the sum in its least }  $2\frac{2}{3}$   
terms will be \_\_\_\_\_ }  $\frac{8}{3}$

Which is a square number whose }  
root is \_\_\_\_\_ }  $1\frac{1}{3}$

Wherefore is it manifest that the three numbers 1,  $\frac{8}{3}$  and  $1\frac{6}{5}$  will satisfie the conditions in the question, which may be solved also by other numbers, but the manner of finding them out I have shewn in the 32d. Question of my 3d. Book of the Elements of Algebra.

## C H A P. XI.

## Of Sports and Pastimes.

## Probl. I.

*To discover a number which any one shall have in his mind, without requiring him to reveal any part of that or any number whatsoever.*

**A**FTER any one hath thought upon a number at pleasure, bid him double it, and to that double bid him add any such even number which you please to assign, then from the sum of that addition let him reject one half, and reserve the other half: Lastly, from this half bid him to subtract the number which he first thought upon; then may you boldly tell him what number remaineth in his mind after that subtraction is made, for it will always be half the number which you assigned him to add.

For Example; suppose he thought upon 6, the double thereof is 12, to which bid him add some even number at your pleasure, suppose 4, so will the sum be 16, whereof the half is 8, from which if he subtract 6 (the number first thought on) the remainder is 2 (to wit, half the number 4, which was by you assigned to be added;) which remainder you discover, notwithstanding all the operation was performed in his mind, without his making known of any number whatsoever. Note, that the adding of an even number as aforesaid is not of necessity, but only to avoid a fraction which will arise by taking the half of an odd number.

The

If to  
for dis  
be add  
sift of  
therefo  
be sub  
half of

Two n  
prop  
(out  
bers  
shall

Sup  
John t  
odd, a  
to chu  
Now t  
have c  
even a  
multip  
by 2;  
which  
them a  
make  
them v  
any ot  
it, by t

*The Reason of the Rule.*

If to the double of any number (which number for distinction sake I call the first) a second number be added, the half of the sum must necessarily consist of the said first number, and half the second; therefore if from the said half sum the first number be subtracted, the remainder must of necessity be half of the second number which was added.

*Probl. II.*

*Two numbers, the one even and the other odd, being propounded unto two Persons, to the end they may (out of your sight) severally chuse one of those numbers; to discover which of these numbers each person shall have chosen.*

Suppose you have propounded unto *Peter* and *John* two numbers, the one even and the other odd, as 10 and 9, and that each of those Persons is to chuse one of the said numbers unknown to you. Now to discover which number each person shall have chosen, you must take two numbers, the one even and the other odd, as 2 and 3; then bid *Peter* multiply that number which he shall have chosen by 2; and cause *John* to multiply that number which he shall have chosen by 3; that done, bid them add the two products together, and let them make known the sum to you, or else demand of them whether the said sum be even or odd, or by any other way more secret endeavour to discover it, by bidding them to take the half of the said sum,  
for



for by knowing whether the said sum be even or odd, you do obtain the principal end to be aimed at, because if the said sum be an even number, then infallibly he that multiplied his number by your odd number (to wit, by 3) did chuse the even number (to wit, 10;) but if the said sum happen to be an odd number, then he whom you caused to multiply his number by your odd number (to wit, by 3) did infallibly chuse the odd number (to wit, 9.)

For Example, if *Peter* had made choice of 10, and *John* 9, suppose you willed *Peter* to multiply his number 10 by 2, and *John*, to multiply his number 9 by 3; the Products will be 20 and 27, whereof the sum is 47, which being an odd number, you may thence conclude that *John* whom you caused to multiply his number by 3, did chuse the odd number 9, and therefore *Peter* did chuse 10. But if you had willed *John* to have multiplied his number 9 by 2, and *Peter* to have multiplied his number 10 by 3, the products would have been 18 and 30, whereof the sum is 48, which is an even number, from whence you may infer, that he that multiplied his number by 3 did chuse the even number, and therefore *Peter* chose 10, and *John* 9.

#### Demonstration.

The reason of the said Rule is very easie, and dependeth principally upon the 28th. and 29th. Propositions of the 9th. Book of *Euclid*; for one may infer from the 21st. of the same Book, that an even number multiplied by any number whatsoever produceth an even number, but an odd number is of a different nature, for if it be multiplied by an even number,

number, the Product is an even number (by the said 28 Proposition;) and if it be multiplied by an odd number, the product is odd (by the said 29th. Proposition.) Therefore if in making this Sport it happeneth that the even number be multiplied by your odd number, both the products shall be even, and consequently the sum shall be infallibly an even number (by the said 21 proposition.) But if it happen that you cause the odd number to be multiplied by your odd number, that product will be odd, and the other product even, therefore the sum of these two products shall be an odd number (as *Clavius* hath demonstrated upon the 23d. of the 9th. of *Euclid*.)

## Probl. 3.

A certain number of distinct Things being propounded, to dispose them in such an order, that casting away always the ninth, or the tenth, or any other that shall be assigned, unto a certain number, those remaining may be such as were first intended to be left.

This Problem is usually propounded in this manner, viz. Fifteen *Christians* and Fifteen *Turks* being at Sea in one and the same Ship in a terrible Storm, and the Pilot declaring a necessity of casting the one half of those Persons into the Sea, that the rest might be saved; they all agreed that the Persons to be cast away should be set out by Lot after this manner, viz. the thirty Persons should be placed in a round form like a Ring, and then beginning to count at one of the Passengers, and proceeding circularly, every Ninth Person should be cast into the Sea, until of the thirty Persons there

there remained only fifteen. The question is, how those thirty Persons ought to be placed, that the Lot might infallibly fall upon the fifteen *Turks* and not upon any of the fifteen *Christians*? For the more easie remembring of the Rule to resolve this question, I shall presuppose the five Vowels *a, e, i, o, u*, to signifie five numbers, to wit, (*a*) one, (*e*) two, (*i*) three, (*o*) four, and (*u*) five; then will the Rule it self be briefly comprehended in these two following verses.

*From numbers, aid, and art  
Never will Fame depart.*

In which Verses you are principally to observe the vowels, with their correspondent numbers before assigned, and then beginning with the *Christians*, the vowel *o* (in *from*) signifieth that four *Christians* are to be placed together; next unto them the vowel *u* (in *num*) signifieth that five *Turks* are to be placed; in like manner *e* (in *hers*) denoteth 2 *Christians*; *a* (in *aid*) 1 *Turk*; *i* (in *aid*) 3 *Christians*; *a* (in *and*) 1 *Turk*; *a* (in *art*) 1 *Christian*; *e* (in *ne*) 2 *Turks*; *e* (in *ver*) 2 *Christians*; *i* (in *will*) 3 *Turks*; *a* (in *fame*) 1 *Christian*; *e* (in *fame*) 2 *Turks*; *e* (in *de*) 2 *Christians*; *a* (in *part*) 1 *Turk*.

The Invention of the said Rule, and such like dependeth upon the subsequent demonstration, viz. if the number of Persons be thirty, let thirty Figures or Cyphers be placed circularly, or else in a right Line as you see,

oooooooooooooooooooooooooooooooooooooooo

That done, begin to count from the first, and mark the ninth (or what other shall be assigned

Chap.  
by put  
ward  
another  
to do t  
at the  
Line a  
already  
ber requ  
til you  
marked  
and the  
it is evi  
phers, v  
those w  
a Rule f  
By th  
famous  
Life ver  
Forty of  
ous and  
pata: F  
wickedl  
yield to  
Argumen  
from so h  
ecute the  
help of t  
himself  
alive, af  
they agr  
by save t  
large in  
Book of  
the Jews.



by putting a point or cross over it; then count forward from that which you have marked, and place another point over the next ninth; and continue to do the same, beginning again when you shall be at the end (if the Cyphers are placed in a right Line and passing over those, which you shall have already marked until you have marked the number required, as in the Example propounded, until you have marked 15, for then all the Cyphers marked shall be those which must be cast away, and the others those which shall remain. Hence it is evident, that if you observe how those Cyphers which are marked, are disposed amongst those which are not marked, you will easily make a Rule for any number whatsoever.

By this Invention (as some do conjecture) the famous Historian *Josephus* the Jew, preserved his Life very subtilty in the Cave, to which himself and Forty of his Country Men had fled from the furious and conquering *Romans* at the Siege of *Jotapata*: For his said Country Men having most wickedly resolved to kill one another, rather than yield to their Enemies, he at length (when no Arguments that he could use would dissuade them from so horrid an Act) prevailed with them to execute their tragical Design by Lot; and so by the help of the aforesaid Artifice, as we may suppose, himself with one other Person only remaining alive, after the rest were inhumanely murdered, they agreed to put an end to the Lot, and thereby save their Lives. This Story you may see at large in the Fourteenth Chapter of the Third Book of the History of *Josephus* of the Wars of the Jews.

Probl.

## Probl. 4.

Many numbers which proceed from 1 or unity in a progression, according to the natural order of numbers (such as these, 1, 2, 3, 4, 5, 6, &c.) being placed in a round form like a Ring; to discover which of those numbers any one shall have thought upon.

Let any multitude of numbers in the aforesaid progression, suppose these 10, to wit, 1. 2. 3. 4. 5. 6. 7. 8. 9. 10 to be written upon 10 Ivory counters (or for want thereof upon ten small pieces of Paper which may be represented by these 10 Letters, A. B. C. D. E. F. G. H. K. L. viz, suppose 1 to be written upon the counter A, 2 upon B, 3 upon C. &c. Then having placed those Counters circularly as you see (with their blank faces uppermost, and the Figures underneath, that the subtilty of the Sport

|   |    |   |   |
|---|----|---|---|
| L | 10 | 2 | B |
| K | 9  | 3 | C |
| H | 8  | 4 | D |
| G | 7  | 5 | E |
|   |    | 6 |   |
|   |    | F |   |

may the better be concealed) let any one think upon any number of unites which doth not exceed 10: that done bid him touch one of those Counters at pleasure, and to the number on the back side of the counter touched (which you cannot be ignorant of, having noted well the place of 1 or

A) ad all the him be the count made will h ter up ten.

For and the ber of be 12 at B g number 9 upon upon shew 7

The prehen one is 1 soever in one cular, some n to coun of num falleth or nam unites a nue to c say, th of both 9 Letter

A) add secretly in your mind, the just number of all the counters, and reserve the sum; then bid him imagine in his mind the counter touched to be the number which he thought, and from that counter to count backwards, until he shall have made up the aforesaid sum, which you reserved, so will his computation infallibly end upon the counter upon which the number thought upon is written.

For Example, suppose that he thought 7 or G, and that he touched B, to wit, 2. add to 2 the number of all the counters, to wit, 10, so the sum will be 12; then bid him to count unto 12 beginning at B going backwards, and esteeming B to be the number thought, to wit 7, so will 8 fall upon A, 9 upon L, 10 upon K, 11 upon H, and lastly, 12 upon the counter G, which being turned up will shew 7 the number thought.

The reason of this Rule is not difficult to be apprehended, two principles being presupposed, the one is this, to wit, many counters or things whatsoever being disposed orderly one after the other, in one continued line, whether it be right or circular, if you value or name the first counter to be some number of unites at pleasure, and continue to count forward according to the natural order of numbers until another number be named which falleth upon the last counter; or if you imagine or name the last counter, to be the same number of unites as before you put upon the first, and continue to count backwards unto the first counter; I say, that the same number will be named at the end of both those computations: For Example, in these 9 Letters A.B.C.D.E.F.G.H.K. if the Letter A be esteemed



esteemed to be 4, and from thence you count forwards unto K, according to the natural order of numbers the Letter K will fall upon the number 12. In like manner, if you esteem K to be 4, and count backwards from K to A, the Letter A will likewise fall upon 12.

4. 5. 6. 7. 8. 9. 10. 11. 12.  
A. B. C. D. E. F. G. H. K.  
12. 11. 10. 9. 8. 7. 6. 5. 4

The other principal is this, to wit, many counters being disposed in a round manner like a Ring, if you esteem any one of those counters to be some number at pleasure, and then from that counter if you count circularly, until you end upon the counter where you began, the number last named will be equal to the sum of the number of all the counters, and of the number which you put upon the first counter; for Example, If D be one of 10 Letters placed in a circumference, and that imagining D to be 7, you begin with it, and count round the whole circumference, according to the natural progression of numbers till

A  
I  
L 10 2 B  
K 9 3 C  
H 8 4 D  
G 7 5 E  
6  
F

you

you end with D where you began; the number 17 which is composed of 10 and 7 will necessarily fall upon D; for 9 (which is the number of Letters in the circumference besides D) being added to 7 (which was first put upon D) makes 16, to which 1 being added (because D doth end as well as begin the circumference) the sum is 17.

Now these two Principles being presupposed, it will not be difficult to apprehend the reason of the aforesaid Rule in all cases that can happen; for imagine that one hath thought upon 7, or the counter G, then that counter which he shall touch must either be the same counter G or some other that proceedeth or followeth G.

First, therefore supposing the counter or number touched to be the same with the number thought, the truth of the Rule will be then evident, for by the Rule given, he shall begin to count from the same G unto 17, putting 7 upon G, therefore by the second presupposition the number 17 will fall upon G.

Secondly, imagine that he touched a counter or number following G the number thought, as L or 10, then according to the Rule adding to (the multitude of all the counters placed circularly) unto 10, L (the counter touched) bid him count backwards unto 20 by beginning at L, and esteem L to be 7. Now because by beginning to count at G which is 7, and proceeding to count forward, the number 10 will fall upon L; therefore by the first presupposed principle, if we esteem L to be 7 and count backwards, the number 10 will infallible fall upon G, and then the number 20 shall also fall upon the same G by the second presupposed principle.

Ll

Lastly,

Lastly, imagine he touched some number or counter which precedeth 7 the number sought, as B or 2; then adding 10 to 2, you are to bid him count unto 12, he having first imagined B to be the number thought 7, and going backwards to A, L, K, &c. Now because by proceeding to count at B, which is 2, and beginning to count forward to C, D, &c. the number 7 falleth upon G; therefore if one imagine that G is 2, and from thence count backwards towards F, E, &c. the number 7 will fall upon B (by the first presupposed principle;) therefore when one assumeth B to be 7, and counteth towards A, L, &c. to any assigned number, it is in effect as much as when one imagineth G to be 2, and counteth towards F, E, &c. unto the said assigned number, for each of those computations will end in the same point; but it is manifest (by the second presupposed principle) that esteeming G to be 2, and counting towards F, E, D, &c. round the whole circumference, the number 12 will fall upon the same G. And because G being supposed to be 2, and counting on the same coast as before the number 7 falls upon B; therefore if the computation be continued on the same coast from B 7 unto 12, the number 12 will fall upon the same G. So that the practice of this sport in all its Cases is fully demonstrated.

*Note*, that to the number of the counter touched you may not only add the number of all the counters once (as the rule directs) but twice, thrice or more times: For Example, B being touched you may cause him to count unto 12, or unto 22, or to 32, 42, &c. the reason whereof is evident from the second presupposed principle.

Probl

pair, E  
which  
tribut  
of a  
length.



## Probl. 5.

Many numbers being shewed by pairs, to wit, two by two, into any one, that he may think upon any one of those pairs at pleasure; to discover the pair that was thought upon.

Let 20 numbers, suppose these, 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. be written upon Ivory counters (or for want thereof upon small pieces of Paper) to wit, 1 upon one counter, 2 upon another, 3 upon a third, &c. Then dispose then into pairs as you see, viz. Suppose 1 and 2 to be one pair, 3 and 4 to be another

|     |    |
|-----|----|
| 1.  | 2  |
| 3.  | 4  |
| 5.  | 6  |
| 7.  | 8  |
| 9.  | 10 |
| 11. | 12 |
| 13. | 14 |
| 15. | 16 |
| 17. | 18 |
| 19. | 20 |

pair, &c. and of these pairs let any one think upon which pair he pleaseth. That done, you are to distribute the said 20 numbers in ranks, into the form of a long square, until there be 5 numbers in length, and 4 in breadth, after this manner, viz.

L 1 2

Lay

Lay the three first number, 1, 2 and 3 in a rank (as you see in the second figure) from A towards B; then place 4 underneath 1, and 5 after 3 (in the said rank AB.) Again place 6 under 4, and 7 after 5 (in the said rank AB.) Then place 8 under 6, also 9, 10, 11, on the right hand of 4 in the rank CD. Again place 12 under 9, and 13 on the right hand of 11 in the rank CD. and 14 under 12. Moreover place 15, 16, 17 on the right hand of 12 in the rank EF. Lastly, place 18, 19, 20, on the right hand of 14 in the rank GH, so will all the numbers be ranked as you see in the Table. That done, you are to demand of him that thought upon two numbers as aforesaid, in what rank or ranks the said numbers do happen to be found, viz.

|   |   |    |    |    |    |   |
|---|---|----|----|----|----|---|
| A | 1 | 2  | 3  | 5  | 7  | B |
| C | 4 | 9  | 10 | 11 | 13 | D |
| E | 6 | 12 | 15 | 16 | 17 | F |
| G | 8 | 14 | 18 | 19 | 20 | H |

in which of the ranks AB, CD, EF, GH, or in which two of the said ranks: Now if he answer, that the two numbers which he first thought upon are in the first rank AB, then 1 and 2 shall be the numbers thought upon; if in the second CD, then 9 and 10 shall be the numbers thought; if in the third rank EF, then 15 and 16 shall be the numbers thought: If they are in the fourth rank GH, then 19 and 20 shall be the numbers thought; but if he shall say that the numbers thought are in different ranks, then you are heedfully to mark the said numbers 1 and 2, 9 and 10, 15 and 16, 19 and 20, which

which may be called the keys of the Sport, in regard they serve not only to discover the two numbers thought, when they are both in one and the same rank (as aforesaid) but also when they are in two different ranks; for in this latter case as soon as it hath been declared to you in which two ranks the two numbers thought are placed, you must take the key of the highest of those two ranks, and descending in a down right line from the first number of that key unto the lower of the said two ranks, you shall there find one of the two numbers thought, and upon the right hand of the second number of the said key, at the same distant sidewise from the second number of the key, as one of the numbers thought was distant from the first number of the key, you shall find the other number thought.

For Example, suppose the two numbers thought are 7 and 8, and that it shall be declared unto you that they are in the first and fourth ranks; take then the key of the highest of these two ranks, to wit, of the first, which is 1 and 2, and descending down right from 1 unto the fourth rank, you shall there find 8 one of the numbers thought; then seek sidewise on the right hand of 2 the second number of the key, a number as far separated from 2, as 8 is distant from 1, and you will find 7 the other number thought.

Again, suppose he said that the numbers thought are in the second and third ranks; take then the key of the second rank which is 9 and 10. and descending down-right from 9 to the third rank, you shall there find 12, which is one of the numbers thought; then seek sidewise on the right  
 hand



hand of 10 (the second number of the key) a number as far distant from 10 as 12 is from 9, and you shall find 11, which is the other number thought.

The reason of this will be apparent from a serious consideration of the placing of the numbers according to the Rules before given, for it is thereby evident that of the numbers coupled two by two, there can never be found more than one pair in one and the same rank, and of all the other pairs one number is always found in one rank, and the other number in another rank.

Note also, that this sport may be practised with divers persons at once, and not only with 20 numbers, but with any such multitude of numbers which is produced by the multiplication of any two numbers which differ by 1, or unity; as 30, which is the product of 5 multiplied by 6, and 42 which is the product of the multiplication of 6 and 7. That which is chiefly to be regarded is the placing of the numbers in ranks according to the directions before given: And for the more easie comprehending of that Order, I have in the following Table ranked 30 numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer Illustration than can be express'd by many words.

|    |    |    |    |    |    |
|----|----|----|----|----|----|
| 1  | 2  | 3  | 5  | 7  | 9  |
| 4  | 11 | 12 | 13 | 15 | 17 |
| 6  | 14 | 19 | 20 | 21 | 23 |
| 8  | 16 | 22 | 25 | 26 | 27 |
| 10 | 18 | 24 | 28 | 29 | 30 |

Probl.

## Probl. 6.

Three jealous Husbands with their Wives, being ready to pass by Night over a River, do find at the River side a Boat which can carry but two Persons at once, and for want of a Boatman they are necessitated to row themselves over the River at several times: The question is, how these 6 Persons shall pass 2 by 2, so that none of the 3 Wives, may be found in the Company of 1 or of 2 Men unless her Husband be present?

They must pass in this manner, viz. First two Women pass, then one of them bringeth back the Boat and repasseth with the third Women; that done, one of the three Women bringeth back the Boat, and sitting down upon the Ground with her Husband permitteth the other two Men to pass over to find their Wives; then one of the said Men with his Wife bringeth back the Boat, and placing her upon the Ground he taketh the other Man, and repasseth with him: Lastly, the Woman which is found with the three Men entreth into the Boat, and at twice goeth to fetch over the other two Women.

## Probl. 7.

Two merry Companions are to have equal shares of 8 Gallons of Wine, which are in a Vessel containing exactly 8 Gallons, now to make this equal Partition they have only two other empty Vessels, whereof one containeth 5 Gallons, and the other 2; the question is, how they shall exactly divide the Wine by the help of those three Vessels?

First, from the Vessel which containeth 8 Gallons

and is full of Wine, let 5 Gallons be poured into the empty Vessel of 5, and from this Vessel so filled let 3 be poured into the empty Vessel of three, so there will remain 2 Gallons within the Vessel of 5. Then let the three Gallons which are within the Vessel of 3 be poured into the Vessel of 8, which will now have 6 Gallons within it, that done let the two Gallons which are in the Vessel of 5, be put into the empty Vessel of 3, then of the 6 Gallons of Wine which are within the Vessel of 8 fill again the five, and from those 5 pour out 1 Gallon into the Vessel of 3, which wanted only 1 Gallon to fill it, so there will remain exactly 4 Gallons within the Vessel of 5 and 4 within the other two Vessels. This question may be resolved in another way, but I leave that as an Exercise to the Wit of the ingenious Reader.

Now albeit at first sight it may be thought by some, that the two last mentioned *Problems* cannot be resolved by any certain Rule, but only by many Trials, yet by infallible Argumentation and Discourse, the solution of those Questions may be found out, or else the impossibility of them, if by chance they should have been propounded, impossible; as the most ingenious *Gasper Bachet* hath manifested in a little Book in the French Tongue, Entituled, *Problemes plaisans & delectables qui se font par les nombres*, from which Book I have extracted the Contents of this Chapter.



A Supplement



A  
SUPPLEMENT.

CONTAINING

Several Easy Contractions, in the  
Necessary and Useful Parts of  
**Arithmetick**, conducing to a Ready  
and Expedition way of Accounting.

WITH

Decimal Tables of *Flemish Exchanges*,  
at the most Usual and Probable Rates  
of Exchange.

AS ALSO

Tables of **Interest**, serving at any Rate, for  
any Time.

To which is added,

The **Mensuration** of Superficies and Solids,  
in Various and Practical Instances.

---

BY

GEORGE SHELLEY, Writing-Master, at the  
Hand and Pen in *Warwick-Lane*, near *St. Pauls*.

---

L O N D O N,

Printed for *J. Philips*, *J. Taylor*, and *J. Knapton*,  
in *St. Paul's Church-Yard*, MDCCVIII.

---

# TO THE READER.

**T**IS very well known, That the Ingenious Mr. Wingate hath excellently Explained the Principles of that most useful Science, Arithmetick; which together with the Incomparable Additions of Mr. Kerfy, the Theoretical Part is so largely insisted on, that it is in a manner incapable of any further Improvement.

But tho' a Theory may be so settled as to have no Alteration, yet the Practices deduced, are often changed and much Improv'd.

In this Edition of Mr. Wingate's Arithmetick, I was prevail'd upon, not only to Correct it, but also to Add as a Supplement, some Practical, Compendious, and Easy Rules, not inserted by the Authors; or whatever else might tend to the making it more generally Useful. And which I have endeavoured chiefly in the most necessary Rules of Arithmetick.

The Tables of Simple Interest and of Exchanges, were never before Published, and are such as I dare recommend for Practice, beyond any I know extant.



G. SHELLEY.

## SECTION I.

## MULTIPLICATION.

**H**OW to Multiply by any Number under 20, so as to bring the whole Operation into one Line.

Rule (I.) Multiply each Figure in the Multiplicand by the unite Figure in the Multiplier, adding unto each its back Figure; as in the following Examples.

$$\begin{array}{r} 47947 \\ 13 \\ \hline \end{array}$$

$$623311$$

$$\begin{array}{r} 27942 \\ 17 \\ \hline \end{array}$$

$$475014$$

which are done thus, viz. 3 times 7 is 21, set down 1 and carry 2; then 3 times 4 is 12, and 2 is 14, and 7 (which is the back Figure to 4) is 21, 1 and carry 2; 3 times 9 is 27, and 2 is 29, and 4 (the back Figure) is 33, 3 and carry 3; 3 times 7 is 21, and 3 is 24, and 9 (the back Figure) is 33, 3 and carry 3; 3 times 4 is 12, and 3 is 15, and 7 is 22, 2 and carry 2, which added to 4 makes 6.

In like manner, the whole Operation of the Multiplication of Numbers between 20 and 30, are brought into one Line, by taking in the Double of the back Figure, and between 30 and 40, the Treble of it, &c.

By which Rule, in Multiplying by Numbers compounded of those under 20, the Product is readily found thus;

By



By 112, either *Multiply* by 2 and 11, or by 11 and 1; as,

|                                                                                                  |                                                                                                   |
|--------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 94267 \\ \times 112 \\ \hline 188534 \\ 942670 \\ \hline 10557904 \end{array}$ | $\begin{array}{r} 94267 \\ \times 112 \\ \hline 1131204 \\ 942670 \\ \hline 10557904 \end{array}$ |
|--------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|

Also in *Multiplying* by 1614, and 1715; in the first I *Multiply* by 14 in one Line, and by 16 in the other: And in the latter by 15 in one Line and 17 also in the other, as above directed; observing always, To place the first Figure of Line under its own *Multiplier*; as,

|                                                                                                       |                                                                                                       |
|-------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 479479 \\ \times 1614 \\ \hline 6712706 \\ 7671664 \\ \hline 773879106 \end{array}$ | $\begin{array}{r} 479479 \\ \times 1715 \\ \hline 7192185 \\ 8151143 \\ \hline 822306485 \end{array}$ |
|-------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|

otherwise for any Number under 20, a way not so troublesome to the Memory.

*Rule (2.) Multiply by the unite Figure, setting the whole Product one Figure back on the right hand which added to the Multiplicand, gives the Product viz,*

But for two Figures *Multiplication*

*Note,* Figures were the

II. How a Number That is divided by 36, the

# Multiplication.

549

$$\begin{array}{r} 47947 \\ 13 \\ \hline \end{array}$$

$$\begin{array}{r} 143841 \\ \hline \end{array}$$

$$\begin{array}{r} 623311 \\ \hline \end{array}$$

$$\begin{array}{r} 27942 \\ 17 \\ \hline \end{array}$$

$$\begin{array}{r} 195594 \\ \hline \end{array}$$

$$\begin{array}{r} 475014 \\ \hline \end{array}$$

But for 112, Multiply by 12, setting the *Product* two Figures back, which likewise added to the *Multiplicand*, gives the *Product*; as,

$$\begin{array}{r} 94267 \\ 112 \\ \hline \end{array}$$

$$\begin{array}{r} 1131204 \\ \hline \end{array}$$

$$\begin{array}{r} 10557904 \\ \hline \end{array}$$

Note, If the *Multiplicand* hath less Significant Figures in it, then the *Multiplier*, work as if it were the *Multiplier*; as in Examples,

$$\begin{array}{r} 13000 \\ 874 \\ \hline \end{array}$$

$$\begin{array}{r} 11362000 \\ \hline \end{array}$$

$$\begin{array}{r} 11362000 \\ \hline \end{array}$$

$$\begin{array}{r} 1107 \\ 3758 \\ \hline \end{array}$$

$$\begin{array}{r} 26306 \\ \hline \end{array}$$

$$\begin{array}{r} 413380 \\ \hline \end{array}$$

$$\begin{array}{r} 4160106 \\ \hline \end{array}$$

II. How to Multiply by the component *Parts* of a Number instead of the *whole*.

That is to Multiply 42291 by 7, and that *Product* by 8, is the same as if it were Multiplied by 56, the *Product* of 7 by 8. Example—

$$42291$$

42991

7

300937

8

2407496

*Note.* If any *odd Numbers* are given that cannot be produced by the *Multiplication* of any two *Figures*, then find out two *Figures* whose *Product* comes nearest, either under or above it, to which add what is wanting, or from which Subtract what is over the given Number ; as,

479 by 47

6

2874

8

22992

479

22513

Subtracted

927 by 29

4

3708

7

25956

927

26883

Added

III. To Multiply Numbers of several Denominations.

(1.) By whole Numbers.



$$\begin{array}{r} l. \quad s. \quad d. \\ 47 : 18 : 11 \\ \hline \end{array}$$

$$\hline 383 : 11 : 04$$

$$\begin{array}{r} C. \quad q. \quad l. \\ 9 : 3 : 15 \\ \hline \end{array}$$

$$\hline 59 : 1 : 06$$

(2.) *By the parts instead of the whole, viz.*

$$Ta. \quad q. \quad n.$$

$$\begin{array}{r} 47 : 3 : 3 \text{ by } 45 \\ \hline \end{array}$$

$$\begin{array}{r} 239 : 2 : 3 \\ \hline \end{array}$$

$$\hline 2157 : 0 : 3$$

$$C. \quad q. \quad l.$$

$$\begin{array}{r} 21 : 3 : 08 \text{ by } 36 \\ \hline \end{array}$$

$$\begin{array}{r} 130 : 3 : 20 \\ \hline \end{array}$$

$$\hline 785 : 2 : 08$$

(3.) *By mixt Numbers.*

$$C. \quad q. \quad l.$$

$$\begin{array}{r} 9 \quad 3 \quad 15 \text{ by } 11 \frac{1}{2} \\ \hline \end{array}$$

$$\hline 108 : 2 : 25$$

$$\hline 4 : 3 : 21 : \frac{1}{2}$$

$$\hline 113 : 2 : 18 : \frac{1}{2}$$

$$l. \quad s. \quad d.$$

$$\begin{array}{r} 39 : 18 : 11 \quad 9 : \frac{1}{2} \\ \hline \end{array}$$

$$\hline 359 : 10 : 03$$

$$\hline 9 : 19 : 08 : \frac{3}{4}$$

$$\hline 369 : 09 : 11 : \frac{3}{4}$$

l. s. d.

47 . 18 . 11 . by 27  $\frac{3}{4}$ 


---

 4

191 . 15 . 8

---

 7

1342 . 09 . 8

11 . 19 . 8 .  $\frac{3}{4}$ 


---

 1330 . 09 . 11 .  $\frac{1}{4}$ 

In the first of these last examples, I *Multiply* by 11 adding one half of the *Multiplicand* for the  $\frac{1}{2}$ . In the 2d. I *Multiply* by 9, adding a fourth for  $\frac{1}{4}$ . In the 3d. I *Multiply* by 4 and that *Product* by 7, gives 28 times the sum, from which *Subtracting*  $\frac{1}{4}$  of the first sum, the *Remainder* is the *Product* of the given sum, *Multiply'd*. by 27  $\frac{3}{4}$ .

(4) *By Numbers of Several Denominations.*(1) *By Aliquot Parts.* (2) *By Cross Multiplication*

F. i. p.

47 . 08 . 06

by 9 . 04 . 04

---

 429 . 04 . 06
 $\frac{1}{3}$  15 . 10 . 10 $\frac{1}{12}$  1 . 03 . 10

---

 446 . 07 . 02 Answer

F. i.

47 . 08

9 . 04

---

 423 . 00

06 . 00

15 . 06

---

 02

---

 444 . 10 . 8

Which

Which are done thus, viz. *EXAMPLE (c)*

In the first, the several *Denominations* in the *Multiplicand*, are *Multiply'd* by the *Integers* in the *Multiplier*, unto which *Product* add the one third of the *Multiplicand*, for 4 inches (being one  $\frac{1}{3}$  of one foot) and the  $\frac{1}{3}$  of that  $\frac{1}{3}$  for the 4 parts (being the  $\frac{1}{3}$  of 4 inches) gives the Answer.

In the second called *Cross Multiplication*, the *Feet* are *Multiply'd* by the *Feet*, then (*Cross-wise*) viz. the 9 *Feet* in the *Multiplier* by the 8 *Inches* in the *Multiplicand*, gives 72 *Inches* or 6 *Feet*, and the 47 *Feet* in the *Multiplicand* being *Multiply'd* by 4 *Inches* in the *Multiplier*, gives 15 *Foot* 8 *Inches*. Lastly the *Inches* in both the *Factors*, being *Multiply'd* one into the other, make 32 parts or 2 *Inches*, 8 parts the *Total* of which is the *Product* of 47 *Feet*, 8 *Inches* by 9 *Feet* 4 *Inches*:

## Section II.

# DIVISION.

TO Divide whole Numbers by the Component parts of a Number.

(i) Examples having no Remainders.

Divide 22992 by 48 Divide 25956 by 28,  
that is by 6 and that or by 4, and that Quo-  
Quotient by 8. tient by 7:

$$\begin{array}{r} 6 \overline{) 22992} \\ 3832 \end{array}$$

$$\begin{array}{r} 8 \overline{) 3832} \\ 479 \end{array}$$

Quot.

M m

$$\begin{array}{r} 4 \overline{) 25956} \\ 6489 \end{array}$$

$$\begin{array}{r} 7 \overline{) 6489} \\ 927 \end{array}$$

Quot.

(2) Ex.



(2) Examples having Remainders

Divide 47953 by 49

7) 47953

Divide 47991 into 3

parts

7) 6850.3

4)

978.4

1333.0

The Quot. 978.  $\frac{4}{9}$

Ans. 1333.36

If 21 pieces of Baggs Holland contain 471 Ells how much is contain'd in one piece?

3 471

Ell. 9 3

7 157

Answer

22.21 7

22.3

The true remainders of which Divisions found by the following Rule.

Multiply the first Diviser by the last remainder, adding thereto the first remainder, (if any be) give Numerator, whose Denominator is the whole Number composing the parts which Fraction so arising is a Remainder, and the very same that would be found Dividing by the Whole.

II. Of Dividing Numbers of Several Denominations, by the Parts of the Whole.

(1) among

(2) when 2

III. T Numbers.

Rule, into the sa

(1) Ex Damask to do this into thir reduce the being to b foregoing

(1)

(1) Example, If 19 . 14 . 10 be Divided among 16 Men, what is each Mans Share?

$$\begin{array}{r}
 4 \overline{) 19 \text{ . } 14 \text{ . } 10} \\
 \underline{4 \text{ . } 18 \text{ . } 08} \phantom{. 1} \\
 1 \text{ . } 04 \text{ . } 08 \text{ . } \frac{1}{4}
 \end{array}$$

Answer

(2) Example, What is the weight of one Bale, when 25 weigh 93 . 1 . 01

$$\begin{array}{r}
 C. \quad q. \quad l. \\
 5 \overline{) 93 \text{ . } 1 \text{ . } 01} \\
 \underline{18 \text{ . } 2 \text{ . } 17 \text{ oz.}} \\
 3 \text{ . } 2 \text{ . } 25 \text{ . } 12 \text{ . } \frac{1}{4}
 \end{array}$$

### III. To Divide Numbers by Broken or Mixt Numbers.

Rule, Reduce both the Dividend and the Divisor, into the same Denominator.

(1) Example, What comes 1 Yard of Flowred Damask to, when  $1\frac{1}{3}$  Yards cost 21 l. 15 s. 11 d. To do this, reduce the Yards by Multiplying by 3 into thirds, which Product will be 40, likewise reduce the Sum into thirds, is 65 l. 7 s. 9 d. which being to be Divided by 40, work according to the foregoing Rule taking  $\frac{1}{3}$  of  $\frac{1}{3}$  for the Answer.

$$\begin{array}{r}
 21 \cdot 15 \cdot 11 \\
 \underline{\phantom{00} 3} \\
 5) 65 \cdot 07 \cdot 09 \\
 \underline{\phantom{00} 13} \\
 8) 13 \cdot 01 \cdot 06 \cdot \frac{2}{5}
 \end{array}$$

Answer  $1 \cdot 12 \cdot 08 \cdot \frac{1}{5}$

(2) Ex. What comes one piece of Cloth to when 17 $\frac{1}{2}$  pieces cost:

$$\begin{array}{r}
 1 \cdot 5 \cdot 4 \\
 57 \cdot 07 \cdot 09 \\
 \underline{\phantom{00} 2} \\
 5 \quad 114 \cdot 15 \cdot 06 \\
 \underline{\phantom{00} 7} \quad 22 \cdot 19 \cdot 01 \cdot \frac{1}{5}
 \end{array}$$

Answer  $3 \cdot 05 \cdot 07 \cdot \frac{1}{5}$

3. Ex. Divide 8792  $\frac{1}{2}$  by 27  $\frac{1}{2}$

$$\begin{array}{r}
 8792 \cdot \frac{1}{2} \\
 \underline{\phantom{00} 27} \\
 55
 \end{array}$$

The Quot. is 319  $\frac{1}{5}$

$$\begin{array}{r}
 17585 \\
 \underline{\phantom{00} 35} \\
 319 \cdot \frac{1}{5}
 \end{array}$$

IV. How to Divide Numbers of Several Denominations by those of Several Denominations.

Example

Soct.  
Exa  
valued  
10. A  
lowi  
To  
2. 11  
then se  
Divisor  
and Su  
Multipl  
the Div  
you rep  
Observ



Sec. 2.

# Division.

8557

*Example, (1.)* How many pieces of Gold, each valued at 2. 11. 8. are contained in 478. 19. 10. Answer 185 pieces, as appears by the following operation.

$$\begin{array}{r} 2. 11. 8 \overline{) 478. 19. 10} \\ 2. 11. 08 \end{array}$$

$$\begin{array}{r} 1. 08. 4 \\ 10 \end{array}$$

$$\begin{array}{r} 21. 03. 4 \\ 20. 13. 4 \end{array}$$

$$\begin{array}{r} 13. 00. 0 \\ 12. 18. 4 \end{array}$$

$$\begin{array}{r} 12. 18. 4 \\ 10. 10. 8 \end{array}$$

$$\begin{array}{r} 19. 10. 8 \\ 19. 10. 8 \end{array}$$

$$\begin{array}{r} 21. 6 \text{ Remainder} \end{array}$$

To do which, First seek how many times 2. 11. 8 is contained in 1. 4, which is once, then setting one in the Quotient, Multiply the Divisor by it, the Product of which setting under, and Subtracting from 4 l. rest 1 l. 8. 4, which Multiply by 10, taking in 7 l. the next Figure of the Dividend, then taking a new Quotient Figure, you repeat the same method as above, until finisht. Observing at the last, to add the odd Money in

M m 3

the

the Dividend, to the Money Remaining, as in the Example.

Note, That in Adding, Multiplying, and Dividing of Money, work the second place in the Shillings, as Angels.

### Section III.

## Of REDUCTION.

### I. In Money.

**P**Ounds into Guineas, Multiply by 40, which Product divide by 43.

Marks into Pounds, Subtract  $\frac{1}{2}$

Pounds into Marks, Add  $\frac{1}{2}$

### II. In Avoirdupois.

To Reduce Hundreds, &c. into Pounds, Multiply by 112 into two lines, as Taught in Multiplication, taking in the Pounds contain'd in the odd Weight.

By this Weight Silk is Weigh'd, in which there is a great Pound of 24 Oz. and by which is Weigh'd Raw, Long, Short, China, Morea Silk &c.

And a small one of 16 Oz. by which Ferret Fillozel, Sleeve Silk, Fine and Course, &c. are Weigh'd.

To Reduce one into the other.

Note, Great Pounds of 24 Oz. into Pounds of 16 Oz. Add:  
Small Pounds of 16 Oz. into Pounds of 24, Subtract.

### III. Liquid Measure.

Of Beer { Firkins into Hogheads ——— Take  $\frac{1}{2}$   
Hogheads into Barrels ——— Add  $\frac{1}{2}$   
Kilderkins into Hogheads ——— Take  $\frac{1}{3}$   
Barrels into Tunns ——— Take  $\frac{1}{7}$

54 Gallons one Hoghead of Beer.

Of Wine { Hogheads into Tierce ——— Add  $\frac{1}{2}$   
Tierce into Hogheads ——— Sub.  $\frac{1}{2}$   
Tierce into Pipes ——— Take  $\frac{1}{3}$   
Pipes into Hogheads ——— Double it

63 Gallons one Hoghead of Wine.

### IV. Cloth Measure.]

Of which there are 2 Sorts most in use among us, a *Tard* and an *Ell*, both Subdivided into 4 *Quarters*, and each *Quarter* into 4 *Nails*. But the *Nail* of an *Ell* is  $\frac{1}{4}$  greater than the *Nail* of a *Tard*; whence it is, That 20 *Nails* or 5 *Quarters* of a *Tard* are contain'd in one *Ell* English.



Tards into Ells English

Ells English into Tards

Tards into Flemish

Ells Flemish into Tards

Ells English into Flemish

Ells Flemish into English

Subtract  $\frac{1}{2}$  partAdd  $\frac{1}{4}$  partAdd  $\frac{1}{8}$  partSubtract  $\frac{1}{4}$  part

Add 0 and Divide by 6

Multiply by 6, and cut off the last Figure.

## Of Tare and Tret, &amp;c.

Call'd Beyond Sea, *The Courtesies of London*, because not Practised in any other place; concerning which, observe, That

Gross is the Weight of the Commodity, and that which contains it without any Deduction.

Tare is an Allowance for that which contains the Commodity, whether it be Bag, Barrel, Sack, Frail, &c. and is either,

I. At so much per Bag, Barrel, Frail, &c.

II. At so much per Cent: or,

III. By so much of the Gross Weight, called, *Invoice-Tare*.

I. When the Tare is so much per Bag, as Almonds; per Barrel, as Oyl in Candy Barrels; per Frail, as Raisins, &c.

Rule, Multiply the Number of the said Bags, Barrels, Frails, Hogsheads, &c. by the Allowance of Tare, which Product Subtract from the Gross, and (when no Tret is mention'd) the Remainder is Neat.

Ex-

## Sect. 3.

## Of Reduction.

561

Ex. What is the Neat Weight of 38 Hogsheds of Tobac<sup>co</sup>  
 weighing Gross 102 . 2 . 17, Tare per Hoghead 70 l.

$$\begin{array}{r}
 102 \quad 2 \quad 17 \\
 112 \\
 \hline
 1297
 \end{array}$$

11497 Gross  
 2660 Tare

8837 Neat

Or which is more short, tho' not according to  
 Rule, Multiply the Hogsheds by 5, Dividing the  
 Product by 8, will give the Tare of the whole in  
 Hundreds, Quarters, and Pounds.

|                                    |     |     |                            |
|------------------------------------|-----|-----|----------------------------|
| 38                                 | C.  | q.  | l.                         |
| <u>5</u>                           | 102 | : 2 | : 17 Gross                 |
| 8) 190                             | 23  | : 3 | : 00 Tare                  |
| <u>23 <math>\frac{3}{4}</math></u> | 78  | : 3 | : 17 Neat Equal to 8837 l. |

II. When the Tare is at so much per Cent, and  
 that,

(I.) If an Aliquot part, or parts of the Hundred  
 Weight;

as Figs, Almonds, Argol, &c. 14 }  
 Caroteels, and Buts of Currants, &c. 16 } per Cent.  
 Oyl in uncertain Casks, &c. 18 }

Rule,





*Tare*  
For  $\left\{ \begin{smallmatrix} 2 \\ 4 \\ 7 \\ 8 \end{smallmatrix} \right\}$  Divide the *Gross* by  $\left\{ \begin{smallmatrix} 7 \\ 4 \\ 4 \\ 2 \end{smallmatrix} \right\}$  and that quote by  $\left\{ \begin{smallmatrix} 8 \\ 7 \\ 4 \\ 7 \end{smallmatrix} \right\}$  gives the *Tare*.

2. But if the *Tare* is not readily found to be an Aliquot part, or parts of a Hundred.

Rule, Multiply the Pounds *Gross* by the Allowance of *Tare*, Dividing the Product by 112, the Quotient is the *Tare* of the whole.

Example, What is the Neat Produce of 12 Barrels of Potashes, each *Gross* 203 l. *Tare* 10 l. per Cent.

|            |       |                           |
|------------|-------|---------------------------|
| 203        | 2436  | Gross                     |
| 12         | 217   | . $\frac{1}{2}$ Tare      |
| -----      | ----- |                           |
| 112) 24360 | (217  | 2218 . $\frac{1}{2}$ Neat |
| 196        |       |                           |
| 840        |       |                           |
| -----      |       |                           |
| 56         |       |                           |
| -----      |       |                           |

III. If the Allowance for *Tare* is not by the Hundred, but according to the *Gross* Weight as Sumack in Bags, Bales or Fangots of Aleppo, Cyprus, and Smyrna Silk, &c. such *Tare* is called, Invoice-Tare; of which the Book of Rates, in the Table of Allowances for *Tare*, says, viz.

Cyprus

## Cyprus, and Smyrna Silk.

|       |                        |                       |                        |
|-------|------------------------|-----------------------|------------------------|
| Bales | About, or above 300 l. | { allowed<br>for Tare | { L.<br>16<br>14<br>12 |
|       | from 300 to 200        |                       |                        |
|       | from 200 downwards     |                       |                        |

## Tobacco from Virginia.

|          |                  |                    |                               |
|----------|------------------|--------------------|-------------------------------|
| Hogheads | 5 C. and upwards | { allow'd<br>Tare, | { L.<br>100<br>90<br>80<br>70 |
|          | From 4 to 5,     |                    |                               |
|          | From 3 to 4,     |                    |                               |
|          | under 3 Hundred  |                    |                               |

*Example, What is the Neat Weight of 4 Hogheads of Virginia Tobacco, weighing Gross and Numbred? viz.*

| No.   | C. | q. | l. |                          | l.     |
|-------|----|----|----|--------------------------|--------|
| 27    | 2  | 2  | 19 | { Tare found<br>as above | 70     |
| 9     | 5  | 0  | 15 |                          | 100    |
| 4     | 3  | 2  | 11 |                          | 80     |
| 19    | 4  | 2  | 19 |                          | 90     |
| <hr/> |    |    |    |                          |        |
|       | 16 | 0  | 08 | Gross                    | 112    |
|       | 3  | 0  | 04 | Tare                     | 4      |
| <hr/> |    |    |    |                          |        |
|       | 13 | 0  | 04 | Neat and so of Silk, &c. | 340 (3 |

## Also Sugar from India.

|                                  |        |   |
|----------------------------------|--------|---|
| In Casks and Canisters,          | { Tare | { |
| In Chests, and Casks, St. Thome. |        |   |

Sect.

Tr  
sort of  
the Dr  
after y  
the res

Rul  
(becau  
and the  
the Sut

Exa  
Gross,

Othe  
Rule  
Number  
Pounds.

**Tret**, Is an Allowance of 4 l. per 104, on some sort of Goods, (to the Buyer by the Merchant) for the *Dust and Waste* in them (which when allow'd) after you have subtracted the Tare from the Gross, the remainder is *Suttle*.

**Rule**, Which *Suttle* take out, and divide by 26 (because the allowance of 4 l. per 104, is  $\frac{1}{26}$  part) and the *Quote* is the *Tret*, which *Tret* Subtract from the *Suttle*, and the *Neat* Remains.

**Example**, How many l. *Neat* in 12 C. 3 q. 12 l. Gross, Tare 2 l. per Cent. Tret 4 l. per 104?

26) 1415 (54      C.    q.    l.

115      12 . 3 . 12  
112

11      8) 1440    Gross

180

25 + Tare

1415    Suttle

54    Tret

1361    Neat

**Otherwise to find and deduct Tret**,

**Rule**. Add 2 Cyphers to the *Suttle* Pounds, which Number divide by 104, and the *Quote* is the *Neat* Pounds.

Ex.



*Example, What is the Neat Weight of 4 Par. of Spanish Tobacco, containing? viz.*

| No | C. | q. | l. |                                          |
|----|----|----|----|------------------------------------------|
| 4  | 1  | 0  | 04 | } Tare 14 per Cent.<br>Tret 4 per 104 l. |
| 7  | 0  | 3  | 25 |                                          |
| 9  | 1  | 0  | 05 |                                          |
| 3  | 0  | 2  | 26 |                                          |

3 . 3 . 04

112

8) 424 Gross  
53 Tare

104) 37100 (356 + Neat

590

700

76

Cloff, Is another Allowance of 2 l. weight, to Citizens of London, on every Draught above 3 Hundred Weight, on some Goods, as Galls, Madder, Sumack, Argol, &c.

Neat, Is the pure Weight of the Commodity, when all given Allowances are deducted.

As for other Allowances not so common, as Break, Damage, &c. the first being generally at so much per Cask, Bale, Bag, &c. the other so much in the whole, can have no difficulty.

*Example,*

*Example*  
Gross 2  
per Hog

(1.)

(2.)

The  
foregoi  
Gallons.

Rule  
duct of  
Neat G

Ex  
per Cen

*Example, What is the Neat Weight of 7 Hogsheads  
Gross 28 C. 1 q. 19 l. Tare 16 per Cent. Break 8  
per Hogshead Damage 98  $\frac{1}{2}$ ?*

|    | C.    | q. | l. |         |
|----|-------|----|----|---------|
| 7) | 28    | 1  | 19 | Gross   |
|    | <hr/> |    |    |         |
|    | 4     | 0  | 06 | Tare    |
|    | <hr/> |    |    |         |
|    | 24    | 1  | 12 | 5       |
|    | <hr/> |    |    |         |
|    | —     | 2  | 00 | 0 Break |
|    | <hr/> |    |    |         |
|    | 23    | 3  | 12 | 5       |
|    | <hr/> |    |    |         |
|    | 3     | 14 | 8  | Damage  |
|    | <hr/> |    |    |         |
|    | 22    | 3  | 25 | 13 Neat |

*In Oyl the Tare is of two Sorts.*

- (1.) On Candy Barrels — at 29 l. per Barrel.
- (2.) On uncertain Casks — at 18 l. per Cent.

The Neat Pounds of which are found by the foregoing Rules. But to reduce those Pounds into Gallons.

Rule, Multiply those Pounds Neat by 2, the Product of which Divide by 3, and that Quote by 5, give Neat Gallons, 7 l.  $\frac{1}{2}$ , making a Gallon.

*Example, Oyl, Gross 127 C. 3 q. 10 l. Tare 18 l. per Cent, how many Neat Gallons?*

| C.       | q.          | l. |
|----------|-------------|----|
| 127      | 3           | 7  |
| 112      |             |    |
| <hr/>    |             |    |
| 1615     |             |    |
| <hr/>    |             |    |
| 7) 14315 | Gross       |    |
| <hr/>    |             |    |
| 8) 2045  |             |    |
| 255      |             |    |
| <hr/>    |             |    |
| 2300     | Tare        |    |
| <hr/>    |             |    |
| 12015    | Neat Pounds |    |
| 2        |             |    |
| <hr/>    |             |    |
| 3) 24030 |             |    |
| 5) 8010  |             |    |
| <hr/>    |             |    |
| 1602     | Gallons     |    |

## Section IV.

## Of PRACTICE.

THE variety of this Rule may be reduc'd to these two Heads, viz.

- (1.) In respect to the *Rate* or *Price* of *Goods*.
- (2.) The *Given Number*, or *Quantity* of such *Goods*.

Sect. 4.

Of the F  
whenOf the C  
ber, whoMr. A  
pendix T  
rife is pe  
said.A Tab  
a Pound.

Of i. Wh



Of the Price,  
when

a Fraction,  
Farthings,

\* An Aliquot part of a s. } whether d.

\* Not an Aliquot part } or d. & far.

\* Between one s. and two

\* An Aliquot part of a l. } whether s.

\* Not an Aliquot part } or s. and d.

\* Between one Pound and two,  
having a Fraction annex.

Of the Given Num-  
ber, when

\* It hath odd W. or M. in it.

\* It hath a Fraction annex.

\* It is not of the same name with  
that on which the price is set.

\* It is of less Denomination than it.

Mr. Kerfy having in the 2d. Chapter of his *Appendix* Treated of those Heads, to which an *Affix* is perfixt, I shall therefore omit what he hath said.

A Table of such Pence as are an Aliquot part of  
a Pound.

$$\begin{array}{c} d \\ \left. \begin{array}{c} 2 \\ 3 \\ 4 \\ 6 \\ 8 \end{array} \right\} \text{ is } \left\{ \begin{array}{c} \frac{1}{2} s. \\ \frac{1}{3} s. \\ \frac{1}{4} s. \\ \frac{1}{6} s. \\ \frac{1}{8} s. \end{array} \right\} \text{ of a Pound.} \end{array}$$

### I. Of the Given Price.

i. When it is a Fraction,

N n

Rule.

**Rule.** Multiply the Given Number, by the Numerator, the Product of which Divide by the Denominator, gives the Answer in the same Denomination with the Integer, of which the Price is a Fraction.

**Example,** What comes the Whitening of 479 Ells of Osnabrigs to, at  $\frac{1}{4}$  of a Penny an Ell.

$$\begin{array}{r}
 479 \\
 5 \overline{) 2395} \\
 \underline{2395} \\
 0 \\
 12 \overline{) 299. \frac{3}{4}} \\
 \underline{299} \\
 0 \\
 \text{Answ. } 24. 11. \frac{3}{4}
 \end{array}$$

**II. When the Price is Farthings only.**

**Rule.** Reduce the Farthings into 3 Pences, which Divided by 80, produce Pounds.

**Example,** What come 6472 Yards of Tape to at 1 Farthing, or 3 Farthings per Yard?

$$\begin{array}{r}
 6472 \text{ at } \frac{1}{4} \text{ per Yard} \\
 12 \overline{) 5319.4} \\
 \underline{5319} \\
 4 \\
 80 \overline{) 6.14.10} \\
 \underline{6.14.10}
 \end{array}
 \qquad
 \begin{array}{r}
 6472 \text{ at } \frac{3}{4} \text{ per Yard} \\
 4 \overline{) 16118} \\
 \underline{16118} \\
 80 \overline{) 20.04.6} \\
 \underline{20.04.6}
 \end{array}$$

To value the Remainder in Dividing by any Number consisting of a Diget and a Cypher, 80 . 60 . 40 &c.

Rule

**Rule.** Taking

the Odd

are like

Digit, gi

Otherwi

What

add to th

half the

hence, tu

half, give

III. W

hilling,

the sam

gs. Bu

the Aliquo

Rule, L

Pound, th

inder bein

Example.

60) 47

Answ. 7

IV. If th

as 5, 7

**Rule.** What Pounds remain, turn into Shillings (taking in the double of the Figure cut off) which Shillings being divided by the Digit in the Divisor, gives the Odd Shillings; what Shillings remain from thence, turn likewise into Pence, which Dividing also by the Digit, gives the Odd Pence; and so in the Farthings. Otherwise,

What Remains in the Pounds, reckon as Tens, which add to the Unite cut off, the Sum of which Divide by half the Digit, gives Shillings; what Remains from thence, turn into Pence, which Dividing also by the half, gives Pence, &c.

III. When the Price is an Aliquot part of a Shilling, the Rule is, To Divide the Given Number by the same part, and the Quote is the Answer in Shillings. But since those parts of a Shilling are also the Aliquot parts of a Pound,

**Rule,** Divide the Given Number by the said Part of Pound, the Quote is the Answer in Pounds, the Remainder being valued as above.

**Example.** What come 472 Oz. to, at 4d. or 8d. per Oz.

610) 4712 at 4d. per Oz.      310) 4712 at 8d. per Oz.

Answ.  $\overline{7} \cdot 17 \cdot 4$       Answ.  $\overline{15} \cdot 14 \cdot 8$

IV. If the Price is not an Aliquot Part of a Shilling, as 5, 7, 9, 10, and 11 Pence,



**Rule.** Take the greatest even Part of a Pound, and for what is wanting, take Parts out of that Part, which added to the other, gives the Facit.

*Example,* What come 645 l. of Sugar to, at 7 d. or 9 d. per Pound?

40) 645 at 7 d. per Pound

$\frac{1}{2}$ ) 16.02.6

2.13.9

18.16.3 Facit

40) 645 at 9 d. per Pound

$\frac{1}{2}$ ) 21.10

2.13.9

An. 24.03.9

**V.** When any *Fartbings* are added to any of the fore-mention'd Prices, the

*Rule* is the same with the foregoing.

*Example,* What come 735 Dozen of Buttons to at 7 d.  $\frac{1}{2}$  or 6 d.  $\frac{3}{4}$  per Dozen?

40) 735 at 6 d.  $\frac{3}{4}$  per Dozen

$\frac{1}{4}$ ) 18.07.06

2.05.11.  $\frac{1}{4}$

20.13.05.  $\frac{1}{4}$

40) 735 at 7 d.  $\frac{1}{2}$  per Dozen

$\frac{1}{4}$ ) 18.07.06

4.11.10.  $\frac{1}{2}$

22.19.04.  $\frac{1}{2}$

**VI.** If it were here required at such a Profit in the Shilling, to know what Profit per Cent.

**Rule,** Divide 100 l. by the Parts that the said Profit is of a Shilling, the total of which is the Answer.

*Exan*

Example, What Profit per Cent. When I Gain  
4d.  $\frac{1}{2}$  or 7d. per Shilling?

$$4 \frac{1}{2} \quad 100$$

$$\frac{1}{2} \quad \frac{1}{2} \quad 33 : 06 : 08$$

$$4 : 03 : 04$$

$$37 : 10 : 00$$

$$4 \frac{1}{2} \quad 100 \text{ at } 7$$

$$3 \frac{1}{4} \quad 33 : 06 : 08$$

$$25 : 00 : 00$$

$$58 : 06 : 08$$

Otherwise 4d.  $\frac{1}{2}$  being the  $\frac{1}{2}$ , and 7d. the  $\frac{7}{2}$  of a s.

Multiply a 100 by the Numerator, and Divide that Product by the Denominator, according to the first Rule, gives the Facit, as

$$100$$

$$3$$

$$8) 300$$

$$\text{Answer } 37 . 10$$

$$100$$

$$7$$

$$12) 700$$

$$\text{Facit } 58 . 06 . 08$$

VII. When the Price is any Number of Pence, under two Shillings, as, 14, 15, 16, 18, 21, 22, Pence.

Rule. Reduce the said Prices, either into 2d. 3d. 4d. 6d. or 8 pences, and those into Pounds at one Operation, by the third Rule.

Gallons

Bushels

2751 at 14 d. per Gal. 375 at 15 d. per Bu

7

5

12lo) 192517 2 Pences 8lo 18715 3 Pences

Facit 160 . 09 . 06 Facit 23 . 08 . 09

Dozen

Pair

297 at 16 d. per Dozen 736 at 18 d. per Pair

2

3

3lo) 5914

4lo) 22018

Answ. 19 . 16

Facit 55 . 04

L.

Ells at 22 d. per Ell

715 at 21 d. per l.

917

7

11

8lo) 50015

12lo) 100817

Facit 62 . 11 . 3 Answ. 84 . 01 . 02

For 13, 17, and 19 Pence.

Rule. Multiply the given Number by the said Pence (as taught in Multiplication) which Product dividing by 12 and 20, gives the Facit.

Examples

IX.

25, 2,



Sect. 4.

# Of Practice.

## Examples.

463 l. of Copper, at 17 d. l. 479 Ells of Long Cloth, at 19 d.

$$\begin{array}{r} 463 \\ 17 \overline{) 7871} \\ \underline{6515} \quad .11 \end{array}$$

Ans. 32 . 15 . 11

$$\begin{array}{r} 479 \\ 19 \overline{) 9101} \\ \underline{7518} \quad .59 \end{array}$$

37 . 18 . 58

VIII. If any of the last foregoing Prices had Farthings annexed,

Rule. Then take Parts for the Pence and Farthings above a Shilling, out of a Shilling, which added to the Given Number, the total is the Answer in Shillings; as

Example, What do I spend per Annum, at 19 d.  $\frac{1}{2}$  or 18 d.  $\frac{3}{4}$  per Day?

365 Days at 19 d.  $\frac{1}{2}$  each.

$$\begin{array}{r} 6 \quad \frac{1}{2} \quad 365 \\ 1 \quad \frac{1}{2} \quad \frac{1}{4} \quad 182 \quad .06 \\ \quad \quad 45 \quad .07 \quad .\frac{1}{2} \\ \underline{210) 5913 \quad .01 \quad .\frac{1}{2}} \\ \quad \quad 29 \quad .13 \quad .01 \quad .\frac{1}{2} \end{array}$$

365 Days at 18 d.  $\frac{3}{4}$  per Day.

$$\begin{array}{r} 6 \quad \frac{1}{2} \quad 365 \\ \frac{3}{4} \quad \frac{1}{8} \quad 182 \quad .06 \\ \quad \quad 22 \quad .09 \quad .\frac{3}{4} \\ \underline{210) 5710 \quad .03 \quad .\frac{3}{4}} \\ \quad \quad 28 \quad .10 \quad .03 \quad .\frac{3}{4} \end{array}$$

IX. If the Price is any Even Number of Shillings, as, 2, 4, 6, 8, 12, 14, 16, 18, 22, &c.

N n 4

Rule,

Rule. Multiply the Given Number by the half of the Price, setting the Double of the Unite Figure for Shillings.

Example, What come 379 Yards of Broad-Cloth to, at 18s. or 22s. per Yard?

379 at 18s.

$$\begin{array}{r} 379 \\ 9 \\ \hline 341.82 \end{array}$$

379 at 22s

$$\begin{array}{r} 379 \\ 11 \\ \hline 416.18 \end{array}$$

See Mr. Kersey's 5th & 6th Rule of the second Chapter.

X. When the Price is an Odd Number of Shillings,

Rule, Multiply the given Number by the said Shillings, Dividing the Product by 20.

Example, What come 796 Small Grofs to, at 13s. or 17s. per Grofs?

796 at 13s.

$$\begin{array}{r} 796 \\ 13 \\ \hline 210) 103418 \\ \hline 517.08 \end{array}$$

796 at 17s.

$$\begin{array}{r} 796 \\ 17 \\ \hline 210) 135312 \\ \hline 644.12 \end{array}$$

Or rather according to Mr. Kersey's 8th Head, in the Rule of Practice.

XI. When the Price is Shillings and Pence, and such Shillings and Pence, are the same Figure,

Rule. Multiply the Given Quantity by the Shillings, taking always the  $\frac{1}{12}$  of the Product for the Pence, the total

## Sect. 4.

## Of Practice.

577

total of which dividing by 20, gives the Answer in Pounds.

Example, What come 731 Ells of Holland to, at 7s. 7d. or 11s. 11d. per Ell?

731 Ells at 7s. 7d.

731 Ells at 11s. 11d.

7

11

$$\begin{array}{r} \frac{1}{2}) \quad 5117 \\ \underline{426 \text{ . } 05} \end{array}$$

$$\begin{array}{r} \frac{1}{2}) \quad 8041 \\ \underline{670 \text{ . } 01} \end{array}$$

$$20) \quad 55413 \text{ . } 05$$

$$20) \quad 87111 \text{ . } 01$$

$$\underline{277 \text{ . } 03 \text{ . } 05}$$

$$\underline{435 \text{ . } 11 \text{ . } 01}$$

XII. When the Price is between one Pound and two,

Rule, Work after the manner of the 8th Rule, i. e. only for the Money above a Pound, adding thereto the Given Quantity.

Example, What come 224 l. of Cochensal to, at 27s. 6d. per l.

1. But the Price being  $\frac{1}{4}$  of a l. work, viz,

$$\frac{1}{4}) \quad 224$$

$$\underline{224}$$

$$2 \cdot 6 \frac{1}{2}) \quad 56$$

$$\underline{28}$$

$$\underline{308}$$

$$8) \quad 2464$$

$$\underline{308} \quad \text{Answer}$$

XIII. When the Price hath a Fraction annexed,  
Rule,



Rule, *Work for the Pence according to the foregoing Rules, and for the Fraction, either value it as directed in the first Rule: Or if an Aliquot part of the Money foregoing, take such part for it.*

Example, (1.) What come 479 l. of Tobacco to, at 22 d.  $\frac{3}{4}$  per l.

$$\begin{array}{r}
 479 \\
 11 \\
 \hline
 1210) 52619 \\
 \hline
 43 \cdot 18 \cdot 2 \\
 14 \cdot 11 \cdot \frac{5}{8} \quad 12) 179 \cdot \frac{5}{8} \\
 \hline
 \text{Answer } 44 \cdot 13 \cdot 01 \cdot \frac{5}{8} \quad 14 \cdot 11 \cdot \frac{5}{8}
 \end{array}$$

Example, (2.) How much Sterling for 7439 French Crowns Exchange, at 55 Pence  $\frac{5}{8}$  per Crown?

$$\begin{array}{r}
 s. \quad d. \quad 7439 \\
 \hline
 2 \quad 6 \quad \frac{1}{8} \quad 929 \cdot 17 \cdot 06 \\
 1 \quad 8 \quad \frac{1}{8} \quad 619 \cdot 18 \cdot 04 \\
 \quad 5 \quad \frac{1}{4} \quad 154 \cdot 19 \cdot 07 \\
 \quad \frac{5}{8} \quad \frac{1}{8} \quad 19 \cdot 07 \cdot 05 \cdot \frac{3}{8} \\
 \hline
 \text{Answer } 1724 \cdot 02 \cdot 10 \cdot \frac{3}{8} \quad \text{Sterling}
 \end{array}$$

## II. Of the Given Quantity.

1. When it hath *Odd Weight* or *Measure* added,

Rule,

Rule, Value the whole Number as before Directed, then take Parts for such Odd Weight or Measure out of an Integer; Dividing the Given Price by such Parts, the total of which added to the value of the whole Number gives the amount.

Example, What come 294 C. 2 q. 18 l. to, at 10s. 4 d. Hundred?

| $  \begin{array}{r}  294 \\  \underline{5} \\  \frac{1}{3} 147 . 00 \\  \quad 4 . 18 \\  \quad \quad 06 . 09 . \frac{1}{2} \\  \hline  152 . 04 . 09 . \frac{1}{2}  \end{array}  $ | $  \begin{array}{r}  \text{s.} \quad \text{d.} \\  \frac{1}{2} \quad 10 . 4 \\  \hline  \frac{1}{7} \quad 5 . 2 \\  \frac{1}{8} \quad 1 . 5 . \frac{3}{4} \\  \quad 0 . 2 . 0 \\  \hline  6 . 9 . \frac{1}{2}  \end{array}  $ |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

II. When the Given Number is not of the same Name with the Integer on which the Price is,

Rule, It must be reduc'd into the same according to the Rules of Reduction.

Ex. What come 49 Thousand 10 l. 11 oz. of Silk to, at 17 s. 4 d. per Hundred? 2 s. 8 d. per Oz.

|                                                                                                                         |                                                                                                                                                        |
|-------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| $  \begin{array}{r}  490 \\  \underline{9} \\  441 . 00 \\  \quad 16 . 06 . 8 \\  \hline  424 . 13 . 04  \end{array}  $ | $  \begin{array}{r}  10 . 11 \\  \underline{16} \\  3) 171 \\  \quad 57 \text{ Answ. } 22 \text{ l. } 16 \text{ s.} \\  \hline  22 . 8  \end{array}  $ |
|-------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|

III. When

## III. When it hath a Fraction annex,

Rule, Value the whole Number according to the Rules afore going, and for the Fraction, Multiply the Price by the Numerator, and that Product divided by the Denominator, gives the value of the Fraction, which added to the value of the whole Number, gives the Answer.

Example. What come 372 Ells  $\frac{1}{4}$  of Bag Holland to, at 5 s. 10 d.

$$\begin{array}{r}
 \frac{1}{4} \ 372 \\
 \hline
 93 \\
 15 \ . \ 10 \\
 \hline
 3 \ . \ 6 \\
 \hline
 108 \ . \ 13 \ . \ 6
 \end{array}
 \qquad
 \begin{array}{r}
 5 \ . \ 10 \\
 3 \\
 \hline
 5) 17 \ . \ 06 \\
 \hline
 3 \ . \ 06
 \end{array}$$

## IV. When the Given Quantity is of Less Denomination than that on which the Price is Given,

Rule, Take parts for the Less Name out of the Greater, dividing the Price by such parts.

Ex. At 7 l. 10 s. per Tun, what per Barrel? How much a Gallon, at 38 s d. per Hoghead?

$$\begin{array}{r}
 7) \ 7 \ . \ 10 \\
 \hline
 \text{Answ. } 1 \ . \ 01 \ . \ 5 \ . \ \frac{1}{7}
 \end{array}
 \qquad
 \begin{array}{r}
 9) \ 38 \ . \ 3 \\
 \hline
 7) \ 4 \ . \ 3 \\
 \hline
 \text{Answ. } 0 \ . \ 7 \ . \ \frac{1}{7}
 \end{array}$$

What

Sect.  
wha  
weight

Answ.

The C  
sal  
mit  
the

I. C

Lar  
and  
Staves  
Cod,  
with m  
cia, Ir

To

Rul  
which  
lings,

.Exa  
s d.  $\frac{1}{7}$



4.

Sect. 4.

Of Practice.

581

What come a *Cheshire Cheese* to weight 44 l. at 37 s. 9 d. Hundred? At 19 l. 7 s. 8 d. Hundred, how much a Pound?

|               |       |    |    |               |    |               |    |    |    |
|---------------|-------|----|----|---------------|----|---------------|----|----|----|
| $\frac{1}{4}$ | 37    | 19 | 12 | 10            | 11 | 12            | 1  | s. | d. |
| $\frac{1}{7}$ | 9     | 5  | .  | $\frac{1}{4}$ | 16 | $\frac{1}{7}$ | 19 | 07 | 08 |
|               | 4     | 8  | .  | $\frac{1}{4}$ | 2  | $\frac{1}{4}$ | 2  | 15 | 04 |
|               | <hr/> |    |    |               | 12 | $\frac{1}{2}$ | 0  | 06 | 11 |
|               | <hr/> |    |    |               |    |               |    |    | 0  |

Ans. 14 . 9 .  $\frac{3}{4}$

Ans. 0 . 03 . 05 .  $\frac{1}{2}$

The Quantities of particular Goods in Wholesale Trade, found in the Book of Rates, with Concise and Ready ways of Casting up the same.

I. Goods Sold by Six-Score to the Hundred.

Lamb-Skins, Barlings, Balks, Great, Middle, and Small; Clap-Boards, Pipe-Boards, Bow-Staves, Capravens, Deal-Boards, Spars of all Sorts, Cod, Cole, Ling; and all Sorts of Stock Fish, with many Sorts of Linen, viz. Hamborough, Slescia, Irish, Muscovy, Westphalia, Hannover, &c.

To cast up Goods of this Quantity,

Rule, Take half the Pence of the Price of one, which gives you the Value, of 120 in Pounds, Shillings, &c.

120  


---

 being the  $\frac{1}{2}$  of a l. Sterling.

240

Example, What come 120 Ells of Canvas to, at s.d.  $\frac{1}{2}$  per Ell?

$$2) 5 \frac{1}{2}$$

$$\underline{2 \text{ l. } 15 \text{ s.}} \text{ Answer } 2 \text{ l. } 15 \text{ s.}$$

*Ex. (2.) At 7 . 15 s. per Hundred of Deals, how much each?*

$$\underline{15 \text{ s. } 10 \text{ d.}} \text{ Answer } 15 \text{ s. } \frac{1}{2} \text{ d.}$$

*N. B.* That you esteem every Penny that Remains (in the First Example) as 20 s. and every 20 s. as one Penny, (in the Second.) As

In Example, the First there Remains one Penny, which reckoning 20 s. and accordingly the Half-penny in the Given Price for 10 s. both which making 30 s. which Dividing by 2, gives 15 s. and so the Answer is found to be 2 l. 15 s.

## II. Goods Sold by the Thousand, are

Paste-Boards for Books, Tennis Balls, Limons, Oranges, Teazels, Flanders Paving and Pantyles, Lantern-Horns, Barrel-Hoops, and Boards, Quills, Lamperns, Squirrel-Skins, Ox-Bones, Yards of Lift, &c.

To cast up Goods of this Quantity,

Rule, Multiply the Pence that one costs by 50, and the Product divided by 12, gives the Value of a Thousand.

1000

being the  $\frac{1}{2}$  or  $\frac{1}{4}$  of 20s.

240

Example, (1.) What come 1000 of *Flanders Tyles*  
to, at 3 d. each?

$$\begin{array}{r} 3 \\ 12 \overline{) 150} \end{array}$$

Answer l. 12 . 10

Example, (2.) If a 1000 of *Squirrel-Skins* cost  
11 l. 05 s. what per Skin?

$$\begin{array}{r} 11 \quad . \quad 05 \\ 12 \overline{) 1315 \quad . \quad 00} \\ 510 \overline{) 1315 \quad . \quad 00} \end{array}$$

2 .  $\frac{1}{2}$  per Skin.

III. Goods Sold by the *Great Gross*, con-  
taining 12 *Small Gross*, or 144 *Dozen*, viz.

Buttons of Metal, Glass, Thread, Silk, or  
Hair, Beads of Bone, Box, Glass, or Wood,  
Cap-Hooks, Chess-men, Tobacco-Pipes, Combs  
of Light, or Box-Wood, Thread or Silk Points,  
Playing-Cards, &c.

Goods of this Quantity are cast up, viz.

Rule,

Rule, *Multiply the Pence of the Price of one Dozen by 3, and Divide the Product by 5, gives the Value of a Great Gross in Pounds, &c.*

$$\begin{array}{r} 144 \\ \hline \end{array}$$

240 being the  $\frac{1}{5}$  of a Pound Sterling.

Example, (1.) What come a Great Gross of Silk Buttons to, at 8 d.  $\frac{1}{2}$  per Dozen.

$$\begin{array}{r} 8 \cdot \frac{1}{2} \\ \hline 3 \\ \hline 5) 25 \cdot \frac{1}{2} \quad \text{Answer } 5 \text{ l. } 2 \text{ s.} \\ \hline 5 \cdot 2 \\ \hline \end{array}$$

Example, (2.) What come  $\frac{1}{2}$  Dozen to, when a Great Gross cost 7 l. 8 s.

$$\begin{array}{r} 7 \cdot 8 \\ \hline 5 \\ \hline 3) 37 \cdot 0 \quad \text{Answer } 12 \text{ d. } \frac{1}{4} \\ \hline 12 \cdot \frac{1}{4} \end{array}$$

IV. Goods Sold by the *Small Gross* of 12 Dozen, viz.

Tobacco and Pepper-Boxes, Nest and Tinder-Boxes, Buckles of some Sorts, Wash-Balls, Ink-horns,



**Sec. 4**      *Of Practice*      **585**  
 horns, Bodkins, several Sorts of Knives, Comb-  
 Cases, &c.

Goods of this Quantity may be cast up by the  
 last preceding Rule, seeing there are as many par-  
 ticulars in a Small Gross, as Dozens in a Great  
 one.

**V. Goods Sold by the Five-Score to the  
 Hundred.**

Annotto, Capers, Saflower, Thrums, Ginger,  
 Cloves, Indico, Cross-Bow-Thread, Pack-Thread,  
 Kids or Goats-Hair, Quick-Silver, Cotton-Wool,  
 English Hard Wax, Brass and Latin Work, as  
 Chaffing-Dishes, And Irons, Laver-Cocks, &c.

To cast up which Goods,  
 Rule, *Multiply the Pence of the Price of one Pound  
 Weight by 5, dividing the Product by 12.*

100

240 being the  $\frac{4}{3}$  of a l.

*Example, (1.) What come 100 Weight of China  
 Roots to, at 15 d. per l.*

15

5

12 75 Answer 6 l. 5 s.

6 . 5

*Example 2. What comes 1 Pound to, when 100  
 Pound cost 14 l. 15 s.*

60

14

Goods of this Quantity may be cast up by the  
 preceding Rule, seeing there are many per-  
 centages in a small Grose, as 100, 110, 120, &c.

Ans. 35.  $\frac{1}{2}$  or 2 s. 11 d.  $\frac{1}{2}$

To Value Goods sold by the common Hundred  
 of 112 Pound to the Hundred.

Rule. Multiply the Pence that one Pound costs by  
 and the Product divide by 14, or multiply by 24 and  
 divide by 30 gives the Value, as being the  $\frac{1}{3}$  of  
 of a Pound.

Example, 1. What comes a Hundred Weight to  
 at 8 d.  $\frac{1}{2}$  per Pound?

15) 61.  $\frac{1}{4}$  d.

4 1. 8.

Example 2. At 9 l. per Hundred what comes  
 Pound?

9  
 15  
 7) 135

19.  $\frac{1}{4}$

To cast up Goods of this Quantity in mind,  
 Mr. Kersey's 18th Rule of the 2d. Chapter.

Now if it be further required to know what an  
 Number (not exceeding 5 Score) of the aforefaid  
 Quantities in wholesale Trade (or any other)

Sec. 2  
 moun  
 fiest W  
 Rul  
 the gro

What  
 7

91  
 3  
 94

What

2  
 8

81  
 2  
 79

If inf  
 Goods at  
 what Qu  
 Sum of  
 If the

mount to at any Rate, the most practical and easiest Way is by the following Rule.

Rule, Multiply the Price of one, by the Parts of the given Quantity (as taught in Multiplication.)

## Examples.

What comes 32 C.  $\frac{1}{2}$  at  
7l. 11s. 8d. per Hundred?

7 . 11 . 8  
12 .  $\frac{1}{2}$

91 . 00 . 0

3 . 15 . 10

94 . 15 . 10

At 4s. 7d.  $\frac{1}{2}$  per Gross,  
what comes 46 Gross?

4 . 7 .  $\frac{1}{2}$   
5

23 . 01 .  $\frac{1}{2}$

9

10 . 08 . 01 .  $\frac{1}{2}$

04 . 07 .  $\frac{1}{2}$

10 . 12 . 09 . 0

What comes 2900 to, at  
2l. 14s. 7d. per Hundred?

2 . 14 . 07  
3

8 . 03 . 09  
10

81 . 17 . 06

2 . 14 . 07

79 . 02 . 11

33 Hundred at 19s. 7d.  
per Hundred.

19 . 07  
11

10 . 15 . 05  
3

32 . 06 . 03

If instead of knowing the Value or Worth of Goods at a certain Price, It be required to know what Quantity of Goods can be bought for a certain Sum of Money.

If the Price be even Shillings:

002

Rule,

Rule, Add a Cypher to the Pounds, and divide by the Price.

Example, 1. How many lb. oz. and drams, can I buy for 479l. at 6s. per lb?

$$3) 4790 \text{ oz. dra.}$$

Answer, lb. 1596 . 10 . 10 .  $\frac{2}{3}$

Example 2. At 8s. per Yard, how many Yards, Quarters and Nails, can I buy for 315l.

$$4) 3150 \text{ Yards Quarters.}$$

$$787 . \frac{1}{2} \text{ Ans. } 787 . 2$$

At 14s. . 3d. per Yard, how much per Ell? Add  $\frac{1}{4}$  part.

$$4) 14 . 3 \\ 3 . 6 . \frac{3}{4}$$

Ans. 17 . 9 .  $\frac{3}{4}$

What cost 1 Pound when  $\frac{3}{4}$  cost 16s . 8d. Add  $\frac{1}{4}$  the price.

$$16 . 8 \\ 8 . 4$$

Ans. 25 . 0

What comes 47 Pound to, when  $\frac{3}{4}$  cost 6d. i. 8d. per Pound?

$$310) 417$$

Ans. 1 . 11 . 4

How much can I buy for 11d. when 24 lb. oz. cost 11s.

$$\frac{1}{12}) 24 . 11 \text{ dr.}$$

Ans. 2 . 00 . 14  $\frac{2}{3}$

Secd  
II

He  
like  
have  
57 by

Of S

To

In th

of. 30  
and the  
by this  
Yet b  
so little  
ration,  
ming it.

The F

Rule,

per Cent

two Figu

Year : F

quired.

Examp

2 Tears a



If 220 Yards cost 57 l. what is that per Yard?

11) 57

d.

Ans. 5 . 2 .  $\frac{1}{11}$ .

Here I divide 57 l. by 11, seeing there is the like reason between 11 and 57, as there would have been between 220 and 1140, the Product of 57 by 20.

## SECTION V.

### Of Simple INTEREST.

**T**O find the Interest of any Sum at any Rate, for any Time given.

#### I. By Months.

In this the Year is taken as containing 12 Months of 30 Days each; which wants  $\frac{1}{12}$  of a Day, and therefore the precise Interest cannot be found by this method.

Yet because 'tis so much in use among Traders, so little differing from truth, and so ready in operation, I shall give three several ways of performing it.

*The First and usual Way.*

**Rule,** Multiply the Principle by the Rate of Interest per Cent. per Annum, and the Product (casting off two Figures on the right Hand) is the Interest for a Year: From whence 'tis found for any Time required.

**Example 1.** What is the Interest of 479 l. 18 s. for 2 Years at 4 l.  $\frac{1}{2}$  per Cent. per Annum?

O a 3

479

$$\begin{array}{r} 479 \cdot 182 \\ \underline{4 \frac{1}{2}} \end{array}$$

$$1919 \cdot 12$$

$$\underline{239 \cdot 19}$$

$$2159 \cdot 11$$

$$\underline{20}$$

$$1191$$

$$\underline{12}$$

$$1092$$

$$\underline{4}$$

$$368$$

$$\text{Answ. } 43 \cdot 03 \cdot 09 \cdot \frac{1}{2}$$

**Example 2.** At 6 per Cent. per Annum, what is the Interest of 571 l. 15 s. for 8 Months?

$$\begin{array}{r} 571 \cdot 15 \\ \underline{6} \end{array}$$

$$\begin{array}{r} 3430 \cdot 10 \\ \underline{20} \end{array}$$

$$610$$

$$\underline{12}$$

$$120$$

$$\begin{array}{r} \text{l. s. d.} \\ 34 \cdot 06 \cdot 01 \end{array}$$

$$\begin{array}{r} 17 \cdot 03 \cdot 00 \cdot \frac{1}{2} \\ \underline{5 \cdot 14 \cdot 04 \cdot 0} \end{array}$$

$$\text{Answ. } 22 \cdot 17 \cdot 04 \cdot \frac{1}{2}$$

**Note,** By this Rule, Broakage, Commission, Provision, Storage, Insurance, &c. are wrought, having no respect to Time.

**Example 1.** Unto what comes the Commission of 259 l. 17 s. 11 d.  $\frac{1}{2}$ , at 2  $\frac{1}{2}$  per Cent.

But  
and re

Rule, Divide the Sum of the Rates of the Commission or Brokerage,  
 by 100. As for instance, in the last Example,  
 Example 1. The Commission being 5 per  
 cent, which is the 4th part of 100. Therefore  

$$\frac{649}{20} = 32 \frac{9}{20}$$

6/49: 141 107 1/2 1/2 (04  
20

120

994  
8311

994

12

1/38

14

155

**Example 2.** At  $1\frac{1}{2}$  per Cent. what comes the Brokerage of 1984  $\text{£}115.7d$ .

41 i

to

2/2

12

9.6

**I**

7.7

7

219

Ans<sup>r</sup>, 2l. 9s. 7d.  $\frac{1}{2}$

But Questions of this nature are more concisely and readily wrought by this

Rule, Divide the given Sum by the Aliquot part that the Rate of the Commission or Broakage, &c. is of 100 l. As for instance, in the last Examples.

Example 1. The Commission being  $2\frac{1}{2}$  per Cent. which is the 40th part of 100 l. Therefore

$$40 \overline{) 2519 \text{ l. } 17 \text{ s. } 11 \text{ d. } \frac{1}{2}}$$

$$\text{Answ. } 634 \text{ l. } 9 \text{ s. } 11 \text{ d. } \frac{1}{2}$$

Example 2. The Broakage being  $1\frac{1}{8}$  per Cent. which is the 80th part of 100 l. Therefore

$$80 \overline{) 1918 \text{ l. } 11 \text{ s. } 7 \text{ d.}}$$

$$\text{Answ. } 239 \text{ l. } 7 \text{ s. } 7 \text{ d.}$$

The Second way.

Rule, Multiply the Principle, by the Interest of 100 l. for the Time required, and the Product (striking off two Figures on the Right-hand) is the Answer.

Example 1. What is the Interest of 479 l. 18 s. for two Years, at  $4\frac{1}{2}$  per Cent. per Annum.

The Interest of 100 l. for two Years, at  $4\frac{1}{2}$  per Cent. is 9 l. Therefore

$$479 \cdot 18$$

$$\underline{9}$$

$$43 \overline{) 19 \cdot 02}$$

$$382$$

$$\underline{12}$$

$$984$$

$$\underline{14}$$

$$3136$$

$$\text{Answ. } 43 \text{ l. } 03 \text{ s. } 09 \text{ d. } \frac{1}{2}$$

Example,



**Sect. 5. Simple Interest. 593**

**Example,** What is the Interest of 571*l.* 15*s.* for 8 Months, at 6 per Cent. per Annum?

The Interest of 100*l.* for 8 Months, at 6 per Cent. is 4*l.* Therefore

571*l.* 15*s.*

4

2287 00

20

1740

12

480

4

320

**Ans.** 22*l.* 17*s.* 04*d.*  $\frac{1}{4}$ .

**Note,** If it be for 3 Months, at 5 per Cent. per Annum, then you need only add  $\frac{1}{4}$  of the Principle to it self, working as before directed, &c.

**The Third way.**

**Rule,** Since the Interest of any Principle,

at  $\left\{ \begin{matrix} 5 \\ 6 \\ 4 \\ 3 \end{matrix} \right\}$  per Cent, for  $\left\{ \begin{matrix} 12 \\ 10 \\ 15 \\ 20 \end{matrix} \right\}$  Months.

is just so many Shillings as it hath Pounds. Therefore

(1) What is 379*l.* 15*s.* for 12 Months, at 5 per Cent?

210) 3719 . 15

**Ans.** 18.19 . 9

(2) What is 416*l.* 12*s.* 6*d.* for 10 Months, at 6 per Cent?

210) 4116 . 12 . 6*d.*

**Ans.** 20.16 . 7 $\frac{1}{2}$ .

(3) What

(3) What is 427*l*. 13*s*. 9*d*. for 20 Months at 3 per Cent?

(4) What is 927*l*. 11*s*. 3*d*. for 15 Months at 4 per Cent?

$$\begin{array}{r} 210) 4217 \cdot 13 \cdot 9 \\ \hline \end{array} \quad \begin{array}{r} 210) 9217 \cdot 11 \cdot 3 \\ \hline \end{array}$$

Ans. 21 . 7 . 8 $\frac{1}{2}$       Ans. 46 . 7 . 6 $\frac{1}{2}$

Hence, to find the *Interest* of a given *Principle*, for any other *Time*.

Rule, Take parts, for the *Time* required, out of the *Time* affixed

Thus, If the *Interest* of 379*l*. 15*s*. for 12 Months, at 5 per Cent. is so many *Shillings*, then for 6 Months it is the  $\frac{1}{2}$  of so many *Shillings*, or so many *Six-pences*; and for 4 Months the  $\frac{2}{3}$  of it, or so many *Groats*, as the *Principle* hath *Pounds*.

Example 1. What is the *Interest* of 279*l*. 11*s*. for 4 Months at 5 per Cent.

$$\begin{array}{r} 210) 2719 \cdot 11 \\ \hline 3) 13 \cdot 19 \cdot 16 \cdot 1 \\ \hline \end{array}$$

Ans. 4 . 13 . 2

Example 2. What is the *Interest* of 197*l*. 11*s*. for 2 Months at 6 per Cent.

$$\begin{array}{r} 210) 1917 \cdot 11 \\ \hline 3) 9 \cdot 17 \cdot 16 \cdot 1 \\ \hline \end{array}$$

Ans. 1 . 12 . 6

Examp

To find  
two give  
This  
Diffich,  
Thirt  
November  
Febru  
reft hath  
And a  
tains 36

**Seck. 3.**

**Simple Interest.**

395

**Example 3.** What is 49*l.* 17*s.* 10*d.* for 5 Months, at 4 per Cent?

$$\begin{array}{r} 210) 4911 \cdot 17 \cdot 10 \\ \hline \end{array}$$

$$\begin{array}{r} 3) 24 \cdot 11 \cdot 10 \frac{3}{4} \\ \hline \end{array}$$

*Ans.* 8 . 3 . 11  $\frac{1}{4}$

**Example 4.** What is 279*l.* 11*s.* 8*d.* for 7 Months, at 4 per Cent?

$$\begin{array}{r} 210) 279 \cdot 11 \cdot 8 \\ \hline \end{array}$$

$$\begin{array}{r} 2) 13 \cdot 19 \cdot 7 \\ \hline \end{array}$$

*Ans.* 6 . 19 . 9  $\frac{1}{2}$

For what odd Money is adjoyn'd to the *Principle*, value every 20*s.* as 12*d.* and lesser Sums in proportion.

**SECTION VI.**

**II. Interest for Days.**

To find the exact number of *Days*, between any two given *Times*, without any *Book* or *Table*.

This is done, by carrying in Memory that old Distich, viz.

Thirty Days hath September, April, June and November.

February hath twenty eight alone; each of the rest hath Thirty one.

And according to this reckoning the Year contains 365 Days; therefore, for Instance,

To

To find the number of Days between the 7th of June 1701, and the 15th of September 1703. Say thus,

365

2

From the 7th of June 1701 to the 7th of June 1703 is 730 or two Years.

From the 7th of June 1703 to the end of the same Month is

Then in { July ——— 31  
August ——— 31

From which, to the 15th of September, is — 15

So that from the 7th of June 1701, } Days 830  
to the 15th of September 1703, is }

Note, That 1 Day must be added to every 4 Years contain'd in the Time. For every fourth Year, being Leap-year, February hath then 29 Days.

After this manner may any number of Days, even to  $\frac{1}{2}$  or  $\frac{1}{4}$  of a Day, be exactly and readily found. Now

To find the Interest of any Sum, at any Rate, for any Number of Days, observe these two General ways.

First Rule, Multiply the Pence of the Principle by the number of Days, and the Product by the Rate of Interest for a Dividend, which divide by 36500, the Quotient is the Answer in Pence.

Otherwise, Take the Product of the Pence of the Principle, Multiply'd by the Days, for a Dividend, and the Quotient of 36500, divided by the Rate of Interest, for a Divisor, and the Quotient of that Division is the Answer.

Example



## Sect. 62

## Simple Interest.

597

Ex. 1. What comes the Interest of 192 l. 14 s. 10 d. for 830 Days, at 5 per Cent. per Ann.

$$\begin{array}{r}
 20 \\
 \hline
 3854 \\
 12 \\
 \hline
 46258 \\
 830 \\
 \hline
 1387740 \\
 370064 \\
 \hline
 38394140
 \end{array}$$

365100) 1919707100 (5259

$$\begin{array}{r}
 947 \quad 4318 \cdot 3 \\
 2170 \quad 21.18 \cdot 3 \text{ Ans.} \\
 \hline
 3457 \\
 17200
 \end{array}$$

Ex. 2. What comes the Interest of 311 l. 13 s. 4 d. for 315 Days at 6 per Cent. per Annum.

$$\begin{array}{r}
 20 \\
 \hline
 6233 \\
 12 \\
 \hline
 74800 \\
 315
 \end{array}$$

$$\begin{array}{r}
 1122000 \\
 224400
 \end{array}$$

6083) 23562000 (3873

$$\begin{array}{r}
 53130 \quad 3212 \cdot 9 \\
 44660 \quad 16.2.9 \\
 20790 \\
 \hline
 2541
 \end{array}$$

The

The second, which I received from my Friend Mr. Ralph Snow, is.

Rule, Multiply the Pence in the given Principle, by the number of Days, divide the Product always by 7300, and the Quotient is the Interest, in Pence, of that Principle, for that Time, at 5 per Cent. per Annum.

Thus the Interest of 192*l.* 14*s.* 10*d.* for 830*d.* at 5 per Cent.

$$\begin{array}{r}
 20 \overline{) 19214510} \\
 \underline{40000} \phantom{00} \\
 3854 \phantom{00} \\
 \underline{120000} \phantom{00} \\
 46258 \phantom{00} \\
 \underline{830000} \phantom{00} \\
 1387740 \phantom{00} \\
 \underline{3700640} \phantom{00} \\
 52394140
 \end{array}$$

52394140 (5239 Interest in Pence at 5 per Cent.

$$\begin{array}{r}
 189 \overline{) 52394140} \\
 \underline{4340000} \phantom{00} \\
 691 \phantom{00} \\
 \underline{3400000} \phantom{00} \\
 3400000
 \end{array}$$

Whence, to find the Interest of the same Principle, for the same Time, at the Rate of 3, 4, 6 7 &c. per Cent.

Add so many Fifths as the Rate of Interest exceeds 5; Or Subtract so many Fifths as it is less than 5,

The Fifths of any Number is readily found, by Multiplying double the Numerator, cutting off the unit Figure of the Product.

Because

Because so many *Fifths* of any Number is the same with twice so many *Tenths* of that Number.

Now 5259*d.* being the Interest of 192*l.* 14*s.* 10*d.* for 830 Days, (at 5 per Cent, as in the last Precedent) let it be required, to know the Interest of the same Principle, for the same time, at any other Rate, suppose 3, 4, or 6 per Cent, per Annum, then, viz.

|          |      |
|----------|------|
| 977 1/2  | 5259 |
| 01 . 982 | 4    |
|          |      |

Ex. 1.  $\frac{4}{100}$ , or  $\frac{2}{5}$ , being subtracted rests 2103 . 6  
The Interest likewise in Pence, at 3 per Cent. 3155 . 4

Ex. 2. One Tenth subtracted, rests the 525 . 9  
Interest at  $4\frac{1}{2}$  per Cent. 4733 . 1

Ex. 3.  $\frac{2}{100}$ , or  $\frac{1}{5}$ , being added, gives 1051 . 8  
the Interest at 6 per Cent. 6311 .

And the like for any other Rate whatever.

*Of Exchange Notes and Tallies.*

Chequer Notes of five and twenty Pound did formerly bear five Farthings, but now a Penny per Day Interest.

Tallies have been at 6, but now at 5 per Cent. per Annum, the Interest payable every 3 Months.

Example, Unto what comes the Interest of two 25*l.* Notes from the 27th of November 1702 to this Day, being the 4th of January 170 $\frac{3}{4}$ ?

The Days found, according to the direction given in the beginning of this Section, are 403, which

600

## Simple Interest.

Sect. 6.

which at 2d. per Day Interest, is 3l. 7s. 2d. as,  
viz.

1210) 4013

3. 7. 2

**Example 1.** What is the quarterly Interest of a  
Tally, viz. Numb. 2704, struck June 11, 1702  
for 579l. at 6 per Cent?

$\frac{1}{4}$ ) 579  
289 . 10

868110

20

1370

12

840

4

1160

Answ. 8l. 13s. 8d.  $\frac{1}{4}$ 

**Example 2.** What comes the Interest, until the  
10th of January 170 $\frac{1}{4}$ , of two Tallies, the Num-  
ber, Principle and Time, of whose Orders are, viz.

N<sup>o</sup>. 3081,

Sect.

N<sup>o</sup> 3081.  
per Annu

Answ.

N<sup>o</sup> 5792,  
per An



Sect. 6.

Simple Interest.

601

N<sup>o</sup> 3081, from August 5, 1702, for 914 l. 5 s. at 6 per Cent.  
per Annum.

20  
18285  
12

219420  
523 Days

658260

Ans<sup>r</sup>. 78 l. 12 s. 438840  
1097100

73) 1147566160 (15720

417 3144

525 12) 18864

146 19712

6 78. 12

N<sup>o</sup> 5792, struck May 22. 1703, for 379 l. 10 s. at 5 per  
Cent. per Annum.

20  
7590  
12

91080  
233

273240  
273240  
182160

73) 212516140 (2907

662 242.3

516 12.2.3

(5

Pp

TABLE

# TABLES OF

*Simple Interest,*

Serving at any Rate for any  
Time.

## SECTION VII.

| Days | At 3 per<br>Cent. | At 4 per<br>Cent. | At 5 per<br>Cent. |
|------|-------------------|-------------------|-------------------|
| 1    | .000082192        | .000109589        | .000136986        |
| 2    | .000164383        | .000219178        | .000273972        |
| 3    | .000246575        | .000328767        | .000410959        |
| 4    | .000328767        | .000438356        | .000547945        |
| 5    | .000410959        | .000547945        | .000684931        |
| 6    | .000493150        | .000657534        | .000821918        |
| 7    | .000575342        | .000767123        | .000958904        |
| 8    | .000657534        | .000876712        | .001095890        |
| 9    | .000739726        | .000986301        | .001232877        |
| 10   | .000821918        | .001095890        | .001369863        |
| 11   | .000904109        | .001205479        | .001506849        |
| 12   | .000986301        | .001315068        | .001643835        |
| 13   | .001068493        | .001424657        | .001780822        |
| 14   | .001150685        | .001534246        | .001917808        |
| 15   | .001232876        | .001643835        | .002054794        |
| 16   | .001315068        | .001753424        | .002191781        |
| 17   | .001397260        | .001863013        | .002328767        |
| 18   | .001479452        | .001972602        | .002465753        |
| 19   | .001561644        | .002082192        | .002602740        |
| 20   | .001643835        | .002191781        | .002739726        |
| 21   | .001726027        | .002301370        | .002876712        |
| 22   | .001808219        | .002410959        | .003013698        |
| 23   | .001890412        | .002520548        | .003150685        |
| 24   | .001972602        | .002630137        | .003287671        |
| 25   | .002054794        | .002739726        | .003424657        |
| 26   | .002136986        | .002849315        | .003561644        |
| 27   | .002219178        | .002958904        | .003698630        |
| 28   | .002301369        | .003068493        | .003835616        |
| 29   | .002383561        | .003178082        | .003972603        |
| 30   | .002465753        | .003287671        | .004109589        |
| 31   | .002547945        | .003397260        | .004246575        |
| 32   | .002630137        | .003506849        | .004383561        |
| 33   | .002712328        | .003616438        | .004520548        |
| 34   | .002794520        | .003726027        | .004657534        |
| 35   | .002876712        | .003835616        | .004794520        |
| 36   | .002958904        | .003945205        | .004931507        |

| Days | At 6 per Cent. | At 7 per Cent. | At 8 per Cent. |
|------|----------------|----------------|----------------|
| 1    | .000164383     | .000191781     | .000219178     |
| 2    | .000328767     | .000383561     | .000438356     |
| 3    | .000493150     | .000575342     | .000657534     |
| 4    | .000657534     | .000767123     | .000876712     |
| 5    | .000821918     | .000958904     | .001095890     |
| 6    | .000986301     | .001150685     | .001315068     |
| 7    | .001150685     | .001342466     | .001534246     |
| 8    | .001315068     | .001534246     | .001753424     |
| 9    | .001479452     | .001726027     | .001972603     |
| 10   | .001643835     | .001917808     | .002191781     |
| 11   | .001808219     | .002109589     | .002410959     |
| 12   | .001972602     | .002301370     | .002630137     |
| 13   | .002136986     | .002493150     | .002849315     |
| 14   | .002301370     | .002684931     | .003068493     |
| 15   | .002465753     | .002876712     | .003287671     |
| 16   | .002630137     | .003068493     | .003506849     |
| 17   | .002794520     | .003260274     | .003726027     |
| 18   | .002958904     | .003452054     | .003945205     |
| 19   | .003123287     | .003643835     | .004164383     |
| 20   | .003287671     | .003835616     | .004383561     |
| 21   | .003452055     | .004027397     | .004602739     |
| 22   | .003616438     | .004219178     | .004821917     |
| 23   | .003780822     | .004410959     | .005041095     |
| 24   | .003945205     | .004602739     | .005260273     |
| 25   | .004109589     | .004794520     | .005479452     |
| 26   | .004273972     | .004986301     | .005698630     |
| 27   | .004438356     | .005178082     | .005917808     |
| 28   | .004602739     | .005369863     | .006136986     |
| 29   | .004767123     | .005561643     | .006356164     |
| 30   | .004931506     | .005753424     | .006575342     |
| 31   | .005095890     | .005945205     | .006794520     |
| 32   | .005260273     | .006136986     | .007013698     |
| 33   | .005424657     | .006328767     | .007232876     |
| 34   | .005589041     | .006520547     | .007452054     |
| 35   | .005753424     | .006712328     | .007671233     |
| 36   | .005917808     | .006904109     | .007890411     |

At 3



| Days | At 3 per<br>Cent. | At 4 per<br>Cent. | At 5 per<br>Cent. |
|------|-------------------|-------------------|-------------------|
| 37   | .003041096        | .004054794        | .005068493        |
| 38   | .003123287        | .004164383        | .005205479        |
| 39   | .003205479        | .004273972        | .005342466        |
| 40   | .003287671        | .004383561        | .005479452        |
| 41   | .003369863        | .004493150        | .005616438        |
| 42   | .003452054        | .004602739        | .005753424        |
| 43   | .003534246        | .004712328        | .005890411        |
| 44   | .003616438        | .004821917        | .006027397        |
| 45   | .003698629        | .004931506        | .006164383        |
| 46   | .003780821        | .005041095        | .006301369        |
| 47   | .003863013        | .005150684        | .006438356        |
| 48   | .003945205        | .005260273        | .006575342        |
| 49   | .004027396        | .005309862        | .006712329        |
| 50   | .004109589        | .005479452        | .006849315        |
| 51   | .004191780        | .005589041        | .006986301        |
| 52   | .004273972        | .005698629        | .007123287        |
| 53   | .004356164        | .005808218        | .007260274        |
| 54   | .004438355        | .005917807        | .007397260        |
| 55   | .004520547        | .006027396        | .007534246        |
| 56   | .004602739        | .006136985        | .007671233        |
| 57   | .004684931        | .006246575        | .007808219        |
| 58   | .004767123        | .006356164        | .007945205        |
| 59   | .004849314        | .006465753        | .008082192        |
| 60   | .004931506        | .006575342        | .008219178        |
| 61   | .005013698        | .006684931        | .008356164        |
| 62   | .005095890        | .006794520        | .008493150        |
| 63   | .005178082        | .006904109        | .008630137        |
| 64   | .005260273        | .007013698        | .008767123        |
| 65   | .005342465        | .007123287        | .008904109        |
| 66   | .005424657        | .007232876        | .009041096        |
| 67   | .005506849        | .007342465        | .009178082        |
| 68   | .005589041        | .007452054        | .009315068        |
| 69   | .005671232        | .007561643        | .009452055        |
| 70   | .005753424        | .007671232        | .009589041        |
| 71   | .005835616        | .007780821        | .009726027        |

| Days | At 6 per<br>Cent. | At 7 per<br>Cent. | At 8 per<br>Cent. |
|------|-------------------|-------------------|-------------------|
| 37   | .005082191        | .007095890        | .008109589        |
| 38   | .006246575        | .007287671        | .008328767        |
| 39   | .006410959        | .007479452        | .008547945        |
| 40   | .006575342        | .007671233        | .008767123        |
| 41   | .006739726        | .007863013        | .008986301        |
| 42   | .006904109        | .008054794        | .009205479        |
| 43   | .007068493        | .008246575        | .009424657        |
| 44   | .007232876        | .008438356        | .009643835        |
| 45   | .007397259        | .008630137        | .009863013        |
| 46   | .007561643        | .008821917        | .010082191        |
| 47   | .007726027        | .009013698        | .010301369        |
| 48   | .007890410        | .009205479        | .010520547        |
| 49   | .008054794        | .009397260        | .010739725        |
| 50   | .008219178        | .009589041        | .010958904        |
| 51   | .008383561        | .009780821        | .011178082        |
| 52   | .008547945        | .009972602        | .011397259        |
| 53   | .008712328        | .010164383        | .011616438        |
| 54   | .008876712        | .010356164        | .011835616        |
| 55   | .009041095        | .010547945        | .012054794        |
| 56   | .009205479        | .010739725        | .012273972        |
| 57   | .009369863        | .010931506        | .012493150        |
| 58   | .009534246        | .011123287        | .012712328        |
| 59   | .009698629        | .011315068        | .012931506        |
| 60   | .009863013        | .011506849        | .013150684        |
| 61   | .010027397        | .011698629        | .013369862        |
| 62   | .010191780        | .011890410        | .013589040        |
| 63   | .010356164        | .012082191        | .013808219        |
| 64   | .010520547        | .012273972        | .014027397        |
| 65   | .010684931        | .012465753        | .014246575        |
| 66   | .010849314        | .012657533        | .014465753        |
| 67   | .011013698        | .012849314        | .014684931        |
| 68   | .011178081        | .013041095        | .014904109        |
| 69   | .011342465        | .013232876        | .015123287        |
| 70   | .011506849        | .013424657        | .015342465        |
| 71   | .011671232        | .013616437        | .015561643        |

At

| Days            | At 3 per Cent. | At 4 per Cent. | At 5 per Cent. |
|-----------------|----------------|----------------|----------------|
| 72              | .005917808     | .007890410     | .009863013     |
| 73              | .006000000     | .008000000     | .010000000     |
| 74              | .026082192     | .008109589     | .010136986     |
| 75              | .006164383     | .008219178     | .010273972     |
| 76              | .006246575     | .008328767     | .010410959     |
| 77              | .006328767     | .008438356     | .010547945     |
| 78              | .006410959     | .008547945     | .010684931     |
| 79              | .006493150     | .008657534     | .010821918     |
| 80              | .006575342     | .008767123     | .010958904     |
| 81              | .006657534     | .008876712     | .011095890     |
| 82              | .006739726     | .008986301     | .011232877     |
| 83              | .006821918     | .009095890     | .011369863     |
| 84              | .006904109     | .009205479     | .011506849     |
| 85              | .006986301     | .009315068     | .011643835     |
| 86              | .007068493     | .009424657     | .011780822     |
| 87              | .007150685     | .009534246     | .011917808     |
| 88              | .007232876     | .009643835     | .012054794     |
| 89              | .007315068     | .009753424     | .012191781     |
| 90              | .007397260     | .009863013     | .012328767     |
| 91              | .007479452     | .009972602     | .012465753     |
| —               | — Months. —    | — Months. —    | — Months. —    |
| 3               | .007500000     | .010000000     | .012500000     |
| 6               | .015000000     | .020000000     | .025000000     |
| 9               | .022500000     | .030000000     | .037500000     |
| —               | — Years, &c. — | — Years, &c. — | — Years, &c. — |
| 1               | .030000000     | .040000000     | .050000000     |
| 1 $\frac{1}{4}$ | .037500000     | .050000000     | .062500000     |
| 1 $\frac{1}{2}$ | .045000000     | .060000000     | .075000000     |
| 1 $\frac{3}{4}$ | .052500000     | .070000000     | .087500000     |
| 2               | .060000000     | .080000000     | .100000000     |
| 2 $\frac{1}{4}$ | .067500000     | .090000000     | .112500000     |
| 2 $\frac{1}{2}$ | .075000000     | .100000000     | .125000000     |
| 2 $\frac{3}{4}$ | .082500000     | .110000000     | .137500000     |
| 3               | .090000000     | .120000000     | .150000000     |
| 3 $\frac{1}{4}$ | .097500000     | .130000000     | .162500000     |
| 3 $\frac{1}{2}$ | .105000000     | .140000000     | .175000000     |
| 3 $\frac{3}{4}$ | .112500000     | .150000000     | .187500000     |

At

| Days | At 6 per<br>Cent. | At 7 per<br>Cent. | At 8 per<br>Cent. |
|------|-------------------|-------------------|-------------------|
| 72   | .011835616        | .013808218        | .015780821        |
| 73   | .012000000        | .014000000        | .016000000        |
| 74   | .012164383        | .014191781        | .016219178        |
| 75   | .012328767        | .014383561        | .016438356        |
| 76   | .012493150        | .014575342        | .016657534        |
| 77   | .012657534        | .014767123        | .016876712        |
| 78   | .012821918        | .014958904        | .017095890        |
| 79   | .012986301        | .015150685        | .017315068        |
| 80   | .013150685        | .015342466        | .017534246        |
| 81   | .013315068        | .015534246        | .017753424        |
| 82   | .013479452        | .015726027        | .017972603        |
| 83   | .013643835        | .015917808        | .018191781        |
| 84   | .013808219        | .016109589        | .018410959        |
| 85   | .013972602        | .016301370        | .018630137        |
| 86   | .014136986        | .016493150        | .018849315        |
| 87   | .014301370        | .016684931        | .019068493        |
| 88   | .014465753        | .016876712        | .019287671        |
| 89   | .014630137        | .017068493        | .019506849        |
| 90   | .014794520        | .017260274        | .019726027        |
| 91   | .014958904        | .017452054        | .019945205        |
| --   | --Months--        | --Months--        | --Months--        |
| 3    | .015000000        | .017500000        | .020000000        |
| 6    | .030000000        | .035000000        | .040000000        |
| 9    | .045000000        | .052500000        | .060000000        |
| --   | --Years,&c.--     | --Years,&c.--     | --Years,&c.--     |
| 1    | .060000000        | .070000000        | .080000000        |
| 1½   | .075000000        | .087500000        | .100000000        |
| 1¾   | .090000000        | .105000000        | .120000000        |
| 1¾   | .105000000        | .122500000        | .140000000        |
| 2    | .120000000        | .140000000        | .160000000        |
| 2¼   | .135000000        | .157500000        | .180000000        |
| 2½   | .150000000        | .175000000        | .200000000        |
| 2¾   | .165000000        | .192500000        | .220000000        |
| 3    | .180000000        | .210000000        | .240000000        |
| 3¼   | .195000000        | .227500000        | .260000000        |
| 3½   | .210000000        | .245000000        | .280000000        |
| 3¾   | .225000000        | .262500000        | .300000000        |

Exph.



## Explication.

The first Column of every Page being Time, against it stands the Interest of one Pound for that Time, at the Rate per Cent. specify'd on the top of the Column, and at the same View you have the Interest of 10*l.* 100*l.* 1000*l.* and all in one Line. Let the Interest of 500*l.* for 73 Days at 5 per Cent. be required; turn to 73 Days, and against that, under the Rate of 5 per Cent, it appears that

| <i>l.</i> |    |      |    | <i>l. . s. . d.</i> |    |    |                 |
|-----------|----|------|----|---------------------|----|----|-----------------|
| 1         | is | .010 | or |                     |    | 2  | $\frac{1}{4}$   |
| 10        | is | .10  | or |                     | 02 | 0  | 0               |
| 100       | is | 1.00 | or |                     | 1  | 00 | 0               |
| <hr/>     |    |      |    | <hr/>               |    |    |                 |
|           |    | 1.11 | or |                     | 1  | 02 | $2 \frac{1}{4}$ |

So that one Pound after that manner being found to be the Interest of an 100*l.* at 5 per Cent. per *Annun* for 73 Days, 5 times that Money, to wit, 5, must therefore be the Interest of 500*l.* Again, If the Interest of 111*l.* at the same Rate and for the same Time were required; take the total of the above Sums for an Answer, being 1.11 or 1*l.* 2*s.* 2*d.*  $\frac{1}{4}$ . So that let the given Principle be what number of Pounds, Shillings and Pence it will, it may be either taken in Parts as 111*l.* above, or by multiplying the Interest of one Pound by the given Principle; so that no Sum at any of the inserted Rates or Time can require more than one Multiplication. And although this is Calculated only for 3 Years  $\frac{3}{4}$ . beyond which time simple Interest seldom happens, yet the Interest from thence

thence for any longer Time may as easily and readily be found, viz.

*Example 1.* Unto what amounts the Interest of 79*l.* 15*s.* for 6 Years  $\frac{1}{2}$  and 73 Days, at 6 per Cent. per *Annum*.

|      |   |                                                                                            |
|------|---|--------------------------------------------------------------------------------------------|
|      |   | Tears.                                                                                     |
| .180 | } | the Interest of $\left\{ \begin{array}{l} 3 \\ 3 \frac{1}{2} \end{array} \right.$ 73 Days. |
| .210 |   |                                                                                            |
| .012 |   |                                                                                            |

.402 the Interest of one Pound at the given Rate and Time, which multiply'd by the Principle, produces the Answer.

|        |
|--------|
| 79 .75 |
| .402   |
| -----  |
| 31950  |
| 31900  |
| -----  |

32,05950 *Ans.* 32*l.* 01*s.* 2*d.* +

Likewise may the Interest of any Sum at any other Rate per Cent. whatsoever, be readily found according to the Directions in the last Head in the Rule of Interest for Days.

*Example 2.* What comes the Interest of 432*l.* 10*s.* for 1 Year  $\frac{1}{4}$  at 4*l.*  $\frac{1}{2}$  per Cent. per *Annum*.

First

Sect. 8.

Exchange.

611

First take the Interest of one Pound for the }  
 given Time at 5 per Cent. per Annum, which is } .0625  
 From which Subtracting one Tenth 625

gives the Interest of one Pound at  $4\frac{1}{2}$  per Cent. .5625  
 Multiply'd by the given Principle 432,5

28125

11250

16875

22500

Answer 24 l. 06 s. 06 d.  $\frac{3}{4}$  24,328125

## SECTION VIII.

## EXCHANGE.

I. IN London, and throughout England, Accompts are kept in Pounds, Shillings and Pence Sterling, and are cast up as *Flemish* and *French* Money are, by 12 and 20, and it Exchanges with all places by Pence, i. e. giving so many pence Sterling for the Pieces on which the Exchange is made, except

1. Portugal, and with it in Shillings and Pence, on their *Milrea*.

2. Antwerp, Hamborough, &c. Countries bordering upon *Flanders* and *Holland*, and with them by the Pound of 20 s. Sterling.

3. Ireland, and with it by the hundred Pound.

II. The principal places where the course of Exchange is settled, and in what Denomination they Exchange with London, you may find in the

3d.

3d. Chapter of Mr. Kerfy's Appendix; but the places to which there is a more constant course of Exchange from London, are

*In Italy.*

*Genoa, Legorn, Venice, &c.*

*Genoa* and *Legorn* exchange with *London* by the Dollar, or piece of eight, *Venice* by the Ducat.

*Genoa* and *Legorn* keep their Accompts in Livres, Solz and Deniers, 5 Livres a piece of Eight at *Genoa*, and 6 at *Legorn*.

The Par of the Dollar, or piece of Eight, with *London* is 54d. Sterling.

*In Spain.*

*Madrid, Cales, Bilbo, &c.*

Exchange also by the piece of Eight.

The Par being the same as *Genoa* and *Legorn*.

*Portugal.*

*Lisbon* and *Oporto, &c.*

Exchange on the *Milrea*.

The Par about 6s. 8d.  $\frac{1}{2}$  Sterling.

*In France.*

*Paris, Lyons* and *Roven*.

Exchange by the Crown.

But keep their Accompts in Livres, Solz and Deniers,

|           |         |         |
|-----------|---------|---------|
| 12 Denier | } makes | 1 Sol.  |
| 20 Solz   |         | 1 Livre |
| 3 Livres  |         | 1 Crown |

The

Sect.

TI

4s. 6

An

boroug

Ex

Flemi

Flemi

Guild

Poun

Mone

ling i

16 P

6 S

20 S

6 G

30 S

50 S

63 S

The

Sterlin

may,

In th

change

and FL

by the

and It

Brussels

Holland

Pound

Pence.

The r

two, viz



The Par of the *French Crown* with *London* is  
4s. 6d. Sterling.

In the *Netherlands*.

*Antwerp, Brussels, Amsterdam, Rotterdam, Ham-  
borough, &c.*

Exchange with *London* by Shillings and Pence  
*Flemish*. Accompts are in these places kept in  
*Flemish* Pounds, Shillings and Pence, by some in  
Guilders, Stivers and Pennicks. The *Flemish*  
Pounds, Shillings and Pence, are divided as our  
Money, viz. one Pound into 20s. and one Shil-  
ling into 12d. But,

|                   |                  |                   |                         |
|-------------------|------------------|-------------------|-------------------------|
| 16 Pennicks,      | 1 Stivre         | The par with Lon. | 0 s. 1 d. $\frac{1}{2}$ |
| 6 Stivres,        | Flemish Shilling |                   | 0 : 7 : $\frac{1}{2}$   |
| 20 Stivres,       | Guilder          |                   | 2 : 0 : 0               |
| 6 Guilders, } one | Flemish Pound    |                   | 12 : 0 : 0              |
| 30 Stivres,       | Common Dol.      |                   | 3 : 0 : 0               |
| 50 Stivres,       | Specie Dollar    |                   | 5 : 0 : 0               |
| 63 Stivres,       | Ducatoon         |                   | 6 : 3 : $\frac{1}{2}$   |

The Par between the *Flemish* Pound and Pound  
Sterling, is 32s. 4d. But the course of Exchange  
may, and doth vary some Shillings.

In the Arithmetical part, or casting up of Ex-  
changes, I shall only instance in those with *France*  
and *Flanders*; for as *London* exchanges with *France*  
by the Pence, so it doth with all places in *Spain*  
and *Italy*; and as it exchanges with *Flanders*, at  
*Brussels* and *Antwerp*, so it doth with *Germany* and  
*Holland*, at *Hamborough* and *Amsterdam*, &c. by the  
Pound Sterling, for their *Flemish* Shillings and  
Pence.

### Of French Exchange.

The most common ways of casting these up are  
two, viz. Rules,

Rules, Either multiply the given Crowns by the Pence of Exchange, taking parts for the Farthings; (if any be) otherwise,

Take the Aliquot Parts for the Pence of the Exchange out of a Pound, dividing the given Crowns by those Parts, and the Total of which is the Answer.

Example. How much Sterling must I pay here to receive in France 479 Crowns, Exchange at 52d Sterling per Crown.

|                                                                                                                                                                   |                                                                                                                                                                                             |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $  \begin{array}{r}  4s. \frac{1}{2} \quad 479 \text{ Crowns} \\  4 \frac{1}{2} \quad 95 : 16 \\  \quad \quad 7 : 19 : 8 \\  \hline  103 : 15 : 8  \end{array}  $ | $  \begin{array}{r}  479 \text{ Crowns} \\  \quad \quad 52 \\  \hline  958 \\  \quad \quad 2395 \\  \hline  12) 24908 \\  \hline  210 \quad 20715 : 8 \\  \hline  10315 : 8  \end{array}  $ |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

So that all Exchanges made with London, upon the Ducat, Dollar, Duccatoon, Florin, Milrea, &c. are wrought after the same manner, and are no more than so many several Questions in the Rule of Three, or of Practice; for if the above Crowns were Ells, &c. you must pay as much for them at such a price, as Sterling, for so many Crowns at such Exchange. If the Exchange is at so many Pence and a Fraction of a Penny, see my second Example of the 13th Rule in Practice.

### Of Flemish Exchange.

First, To reduce Sterling into Flemish.

Rule, Take the Aliquot parts for what the Rate of the Exchange is above a Pound, dividing the given Sterling

Sterling by the said Parts, the total of which, with the given Sterling, is the Flemish Money.

*Example 1.* One in *Antwerp* delivering Money by Exchange for *London* at 35 s. 6 d. *Flemish* per pound Sterling, how much must he pay there to receive here 597 l. Sterling?

$$\begin{array}{rcl}
 10s. \frac{1}{2} & 597 & \\
 5 \frac{1}{2} & 298 : 10 & \\
 6 \frac{1}{2} & 149 : 05 & \\
 & 14 : 18 : 6 &
 \end{array}$$

*Answer 1.* 1059 : 13 : 6 *Flemish*.

*Example 2.* A Merchant at *Rotterdam* hath a Bill drawn on him for the value of 673 l. 16 s. 8 d. Sterling, Exchange at 33 s. 4 d. *Flemish* per Pound Sterling, how much must he pay there?

$$\begin{array}{rcl}
 10s. \frac{1}{2} & 673 : 16 : 8 & \\
 3 : 4 & 336 : 18 : 4 & \\
 & 112 : 06 : 1 &
 \end{array}$$

*Answ.* 1123 l. 1 s. 1 d.  $\frac{1}{4}$  *Flemish* 1123 : 01 :  $\frac{1}{4}$

*Secondly, To reduce Sterling into Guilders.*

*Rule, Reduce the Sterling into Flemish, by the foregoing Rule, which Flemish Money multiply by 6.*

*Example, If 397 l. 15 s. Sterling Exchange at 34 s. 8 d. Flemish per Pound Sterling, were remitted to Amsterdam, how many Guilders, Stivers, &c. may be received at the said Rate of Exchange?*

10s.

|      |               |               |
|------|---------------|---------------|
| 10s. | $\frac{1}{2}$ | 397 : 15      |
| 4    | $\frac{1}{8}$ | 198 : 17 : 06 |
| 8    | $\frac{1}{4}$ | 79 : 11 : 00  |
|      |               | 13 : 05 : 02  |

---

689 : 08 : 08

Guilders. Stivers

---

6

Ans. 4136 : 12      4136 : 12 : 00

Thirdly, To Reduce *Flemish* into Sterling.

Rule, Reduce the given *Flemish* Sum, either into Pence, two Pences, three Pences, Groats or Six-pences, &c. which make a Dividend, then Reduce the Rate of Exchange into the same, which take for a Divisor, and the Quote of that Division will be the Answer in Pounds Sterling, the Remainder being valued according to Mr. Wingate's 5th Rule of the 7th Chapter.

Example 1. A Merchant at *Brussels* delivers 579 l. 10 s. *Flemish*, Exchange at 35 s. 6 d. to receive at *London* one Pound Sterling, How much Sterling must he receive?

|        |                 |
|--------|-----------------|
| 35 : 6 | 579 l. 10 s.    |
| 2      | 40              |
| 711    | 23180           |
| 22180  | (226 : 09 : 06) |
| 888    |                 |
| 460    |                 |

Ans. 226 : 09 : 06      34

Example 2. How much Sterling must be here paid to receive in *Hamborough* 836 l. 8 s. 4 d. *Flemish*, Exchange at 36 s. 8 d. *Flemish* per Pound Sterling.

36 : 8.



When the Pence of the Exchange is many, that do (which is the Flemish Pence in one Guilders) is an Aliquot Part of the Guilders, &c. by that Rule, Divide the Guilders, &c. by the Pence, &c. and the Quotient will be the Pence of the Exchange.

**Fourthly, To reduce Guilders, Stivers and Pennicks into Sterling.**  
**Rule, Reduce the said Guilders, Stivers, and Pennicks into Pence, two Pence, or Groats, which divide by the given Rate of Exchange (when reduced into the same Name) and the Quotient will be Pounds Sterling.**

**Note, That 40 Flemish Pence makes one Guilders.**

**Example 1.** A Merchant at Rotterdam delivering 1489 Guild. 10 Str. 8 Pen. for London, Exchange at 35 s. 7 d. I demand how much Sterling he may draw the Bill for?

$$1489 : 10 : 8 :: 40 : 10 : 8$$

$$427) 59581 (139 : 10 : 8$$

$$4074$$

$$228 \text{ Ans. } 139 : 10 : 8$$

Guilders, Str.

**Ex. 2.** How much Sterling is equivalent to 4372 : 10, Exchange 36 s. 4 d. Flemish per Pound Sterling?

$$3$$

$$109 \text{ Groats } 109) 43725 (401 : 2 : 11$$

$$125$$

$$16$$

Or more short.

When the Pence of the Exchange is so many, as that 40 (which is the *Flemish* Pence in one Guilder) is an Aliquot part thereof.

Rule, Divide the Guilders, Stivers, &c. by that Aliquot part, and the Quotient is the Sterling Money in Pounds, Shillings, and Pence, but to reduce Sterling into Guilders, Stivers, &c. multiply by the aforesaid part.

Example 1. How much Sterling, shall 36 l. 18 d. *Flemish* per pound Sterling must be paid for 1587 to Stivers?

Ans. 1587 : 10 :: 144 : 06 : 04

Example 2. Exchange at 33 s. 4 d. *Flemish* per Pound Sterling, how many Guilders, Stivers, &c. may I draw for 179 l. 17 s. 10 d. Sterling?

Ans. 1798 : 18 : 05

Here note, That when the Rate of Exchange is at Par, then the Proportion between their Money and Ours, is the same as between their Cloth Measure and Ours; for as five of their Ells make three of Ours, so, at such Exchange, five of their *Flemish* Pounds make three Pound Sterling, or 500 l. *Flemish* 300 Sterling.

Of Gain or Loss by Exchange.

Note, That when Exchange is made with Italy, France, Spain, &c. with which, as aforesaid, London Exchanges for so many Pence for their Peice

Sect. 8. XI V. *Exchange.* 619

The gain thereby is so much the more, by how much the Course of Exchange runs low; because it is evident that I can receive more Ducats for 500 l. when the Exchange is at 53 d. than at 56 per Piece.

But when I remit for *Flanders*, or *Holland*, with which *London* Exchanges on the pound Sterling, the higher the Exchange is above Par, the more we have the advantage of Gain; for according to reason, I may receive more *Flemish* Money for 1000 l. Sterling, when the Exchange is at 36 s. 8 d. than at 35 s. 4 d. for at 35 s. 4 d. I gain but 2 s. per Pound, but at 36 s. 8 d. *Flemish*, I gain by Exchange 3 s. 4 d. for every pound Sterling I remit.

Decimal Tables for *F L E-  
M I S H* Exchanges, at the  
most usual Rates of Ex-  
change.

Q q 2



# A DECIMAL TABLE

| Rate of Exchan. | The Value of Pounds Sterling in Pounds Flemish. | The Value of Pounds Flemish in Pounds Sterling. |
|-----------------|-------------------------------------------------|-------------------------------------------------|
| 1               | .6333333                                        | .6122449                                        |
| 2               | .6375...                                        | .6106870                                        |
| 3               | .6416666                                        | .6091370                                        |
| 4               | .6458333                                        | .6075949                                        |
| 5               | .65.....                                        | .6060606                                        |
| 6               | .6541666                                        | .6045343                                        |
| 7               | .6583333                                        | .6030150                                        |
| 8               | .6625...                                        | .6015037                                        |
| 9               | .6666666                                        | .6.....                                         |
| 10              | .6708333                                        | .5985937                                        |
| 11              | .675...                                         | .5970729                                        |
| 12              | .6791666                                        | .5955525                                        |
| 13              | .6833333                                        | .5940594                                        |
| 14              | .6875...                                        | .5925925                                        |
| 15              | .6916666                                        | .5911330                                        |
| 16              | .6958333                                        | .5896806                                        |
| 17              | .7.....                                         | .5882352                                        |
| 18              | .7041666                                        | .5867970                                        |
| 19              | .7083333                                        | .5853658                                        |
| 20              | .7125...                                        | .5839416                                        |
| 21              | .7166666                                        | .5825242                                        |



Boek 29. *for Flemish Exchanges.* 2621

To shew the Value of the Value of  
Rate of Exchange of Pounds Sterling in Guilders and in  
Pounds Sterling.

| s.      |              |   |          |
|---------|--------------|---|----------|
| 882127  | 9.68888957   | I | 10486666 |
| 101707  | 9.625...757  | I | 10389610 |
| 221827  | 9.63001987   | I | 10362694 |
| 088097  | 9.65378887   | I | 10335916 |
| 708247  | 9.7...2787   | I | 10309278 |
| 120157  | 9.72300147   | I | 10287775 |
| 280767  | 9.78888747   | I | 10256410 |
| 782477  | 9.775...27   | I | 10226178 |
| 217087  | 9.80001477   | I | 10204081 |
| 008797  | 9.82378887   | I | 10178116 |
| 877107  | 9.85...2787  | I | 10152283 |
| 770117  | 9.87300007   | I | 10126581 |
| 833717  | 9.98888077   | I | 10101010 |
| 208817  | 9.925...27   | I | 10075571 |
| 000027  | 9.95001077   | I | 10050250 |
| 074707  | 9.97378887   | I | 10025061 |
| 704407  | 10.0...2787  | I | IX.....  |
| 708187  | 10.02500107  | I | 09975061 |
| 744807  | 10.05378887  | I | 09950248 |
| 277777  | 10.075...8   | I | 09929558 |
| 2078187 | 10.10001408  | I | 09900990 |
| 470097  | 10.125...08  | I | 09876541 |
| 142710  | 10.150...78  | I | 09852216 |
| 787111  | 10.175...018 | I | 09828011 |
| 0034—   | 10.2...8058  | I | 09803921 |
| 274071  | 10.225...78  | I | 09779951 |
| 070027  | 10.25001058  | I | 09756006 |
| 247473  | 10.275...88  | I | 09732360 |
| 071074  | 10.3...2788  | I | 09708736 |
| 1080047 | Q 7 3        | I | 01       |
| 7007147 | 2888718      | I | 21       |

Rate

The Value of  
Rate of Exchange  
The Value of  
Pounds Sterling  
The Value of  
Pounds Flemish  
in Pounds Ster-  
ling.

| s. d. |   |          |          |
|-------|---|----------|----------|
| 34 5  | I | .7208333 | .5811388 |
| 34 6  | I | .725     | .5797101 |
| 34 7  | I | .7291666 | .5783132 |
| 34 8  | I | .7333333 | .5769230 |
| 34 9  | I | .7375    | .5755395 |
| 34 10 | I | .7416666 | .5741621 |
| 34 11 | I | .7458333 | .5727923 |
| 34 12 | I | .75      | .5714285 |
| 34 13 | I | .7541666 | .5700712 |
| 34 14 | I | .7583333 | .5687209 |
| 34 15 | I | .7625    | .5673758 |
| 34 16 | I | .7666666 | .5660377 |
| 34 17 | I | .7708333 | .5647058 |
| 34 18 | I | .775     | .5633802 |
| 34 19 | I | .7791666 | .5620609 |
| 34 20 | I | .7833333 | .5607476 |
| 34 21 | I | .7875    | .5594405 |
| 34 22 | I | .7916666 | .5581395 |
| 34 23 | I | .7958333 | .5568445 |
| 34 24 | I | .8       | .5555555 |
| 34 25 | I | .8041666 | .5542725 |
| 34 26 | I | .8083333 | .5529954 |
| 34 27 | I | .8125    | .5517241 |
| 34 28 | I | .8166666 | .5504587 |
| 34 29 | I | .8208333 | .5491990 |
| 34 30 | I | .825     | .5479452 |
| 34 31 | I | .8291666 | .5466970 |
| 34 32 | I | .8333333 | .5454545 |
| 34 33 | I | .8375    | .5442176 |
| 34 34 | I | .8416666 | .5429864 |
| 34 35 | I | .8458333 | .5417607 |

# Sect. 9. for Flemish Exchanges.

653

Rate of Exchange    The value of pounds Sterling in Guilders    The value of Guilders in pounds Sterl.

| Rate of Exchange | The value of pounds Sterling in Guilders | The value of Guilders in pounds Sterl. |
|------------------|------------------------------------------|----------------------------------------|
| 34               | 10.325                                   | 09685231                               |
| 6                | 10.35                                    | 09661835                               |
| 7                | 10.375                                   | 09638533                               |
| 8                | 10.4                                     | 09615383                               |
| 9                | 10.425                                   | 09592325                               |
| 10               | 10.45                                    | 09569368                               |
| 11               | 10.475                                   | 09546538                               |
| 35               | 10.5                                     | 09523808                               |
| 1                | 10.525                                   | 09501186                               |
| 2                | 10.55                                    | 09478681                               |
| 3                | 10.575                                   | 09456263                               |
| 4                | 10.6                                     | 09433961                               |
| 5                | 10.625                                   | 09411763                               |
| 6                | 10.65                                    | 09389671                               |
| 7                | 10.675                                   | 09367681                               |
| 8                | 10.7                                     | 09345793                               |
| 9                | 10.725                                   | 09324008                               |
| 10               | 10.75                                    | 09302325                               |
| 11               | 10.775                                   | 09280741                               |
| 36               | 10.8                                     | 09259259                               |
| 1                | 10.825                                   | 09237875                               |
| 2                | 10.85                                    | 09216590                               |
| 3                | 10.875                                   | 09195402                               |
| 4                | 10.9                                     | 09174311                               |
| 5                | 10.925                                   | 09153316                               |
| 6                | 10.95                                    | 09132420                               |
| 7                | 10.975                                   | 09111616                               |
| 8                | 11.0                                     | 09090909                               |
| 9                | 11.025                                   | 09070293                               |
| 10               | 11.05                                    | 09049773                               |
| 11               | 11.075                                   | 09029345                               |

Q q 4

Rates

| Rate of<br>Exch. | The value of<br>Pounds Sterl.<br>in Pounds Fl. | The value of<br>Pounds Fl. in<br>Pounds Sterl. | Rate |
|------------------|------------------------------------------------|------------------------------------------------|------|
| 1.87             | 85                                             | 5405405                                        |      |
| 37               | 8541666                                        | 5393258                                        |      |
| 58               | 8583333                                        | 5381166                                        |      |
| 72               | 8625                                           | 5369127                                        |      |
| 86               | 8666666                                        | 5357143                                        |      |
| 80               | 8708333                                        | 5345211                                        |      |
| 88               | 875                                            | 5333333                                        |      |
| 18               | 8791666                                        | 5321508                                        |      |
| 88               | 8833333                                        | 5309734                                        |      |
| 70               | 8875                                           | 5298013                                        |      |
| 87               | 8916666                                        | 5286243                                        |      |
| 75               | 8958333                                        | 5274725                                        |      |
| 38               | 9                                              | 5263157                                        |      |
| 97               | 9041666                                        | 5251641                                        |      |
| 80               | 9083333                                        | 5240174                                        |      |
| 72               | 9125                                           | 5228758                                        |      |
| 47               | 9166666                                        | 5217391                                        |      |
| 97               | 8                                              | 5205975                                        |      |
| 77               | 728                                            | 5194559                                        |      |
| 09               | 728                                            | 5183143                                        |      |
| 30               | 728                                            | 5171727                                        |      |
| 11               | 728                                            | 5160311                                        |      |
| 01               | 728                                            | 5148895                                        |      |
| 24               | 728                                            | 5137479                                        |      |
| 11               | 728                                            | 5126063                                        |      |
| 00               | 728                                            | 5114647                                        |      |
| 99               | 728                                            | 5103231                                        |      |
| 90               | 728                                            | 5091815                                        |      |
| 80               | 728                                            | 5080399                                        |      |
| 70               | 728                                            | 5068983                                        |      |
| 60               | 728                                            | 5057567                                        |      |
| 50               | 728                                            | 5046151                                        |      |
| 40               | 728                                            | 5034735                                        |      |
| 30               | 728                                            | 5023319                                        |      |
| 20               | 728                                            | 5011903                                        |      |
| 10               | 728                                            | 5000487                                        |      |
| 00               | 728                                            | 4989071                                        |      |
| 90               | 728                                            | 4977655                                        |      |
| 80               | 728                                            | 4966239                                        |      |
| 70               | 728                                            | 4954823                                        |      |
| 60               | 728                                            | 4943407                                        |      |
| 50               | 728                                            | 4931991                                        |      |
| 40               | 728                                            | 4920575                                        |      |
| 30               | 728                                            | 4909159                                        |      |
| 20               | 728                                            | 4897743                                        |      |
| 10               | 728                                            | 4886327                                        |      |
| 00               | 728                                            | 4874911                                        |      |

Rate

The  
change  
no far  
Coin,  
lower t



| Rate of Exch. | The value of Pounds Sterl. in Guilders. | The value of Guilders in Pounds Sterl. |
|---------------|-----------------------------------------|----------------------------------------|
| 27. —         | 11. 1. . . . .                          | .09009009                              |
| 1             | 11. 1.25 . . . . .                      | .08988763                              |
| 2             | 11. 1.5 . . . . .                       | .08968610                              |
| 3             | 11. 1.75 . . . . .                      | .08948545                              |
| 4             | 11. 2. . . . .                          | .08928571                              |
| 5             | 11. 2.25 . . . . .                      | .08908685                              |
| 6             | 11. 2.5 . . . . .                       | .08888888                              |
| 7             | 11. 2.75 . . . . .                      | .08869180                              |
| 8             | 11. 3. . . . .                          | .08849556                              |
| 9             | 11. 3.25 . . . . .                      | .08830021                              |
| 10            | 11. 3.5 . . . . .                       | .08810571                              |
| 11            | 11. 3.75 . . . . .                      | .08791208                              |
| 38. —         | 11. 4. . . . .                          | .08771928                              |
| 1             | 11. 4.25 . . . . .                      | .08752735                              |
| 2             | 11. 4.5 . . . . .                       | .08733624                              |
| 3             | 11. 4.75 . . . . .                      | .08714596                              |
| 4             | 11. 5. . . . .                          | .08695651                              |

*The Explication.*

The first Column shews the Rate of the Exchange, from 32s. to 38s. 4d. and is extended no farther; for a regulation being made of the Coin, its not probable it will rise higher, or fall lower than those Limitations.

2. The

2. The second Column on the Left hand, shews the value of Pounds Sterling, exprest in Pounds *Flemish*, whereby, at the first view, you discern what is the amount of 1, 10, 100, 1000, &c. to ten Millions of Sterling, in Pounds *Flemish*, at any of the inserted Rates of Exchange: For Example, If the Exchange is at 35 s. 2 d. One Pound Sterling being Equivalent to one Pound fifteen Shillings and two Pence at one view; therefore it appears, that

|                      | l.   | s. | d.    |                |
|----------------------|------|----|-------|----------------|
| 10 l. Sterling makes | 17   | 11 | 8     | } <i>Flem.</i> |
| 100                  | 175  | 16 | 8     |                |
| 1000                 | 1758 | 6  | 8 &c. |                |

all which *Flemish* Moneys are found in the Line against the said given Rate of Exchange: But if the Sterling Money, for which the value in pounds *Flemish* is required, consists in odd Pounds, or have Shillings, &c. annexed; then multiply the *Flemish* Money, at the given Exchange, by the Sum in Sterling, and the Product answers the Question. Thus 67 l. 10 s. Sterling, multiply'd by 1, 75833, the *Flemish* Money at 35 : 2, makes 118 l. 13 s. 9 d. *Flemish*.

3. The third Column on the Left Page shews the value, or amount of pounds *Flemish* in Sterling, after the same manner as the second shews the value or amount of pounds Sterling in *Flemish* Money:

For Instance.

At the aforesaid Exchange of 35 s. 2 d. I see that one pound *Flemish* being Equivalent to, 568 7209, or 1. 11 s. 4 d. Therefore 10 *Flem.* makes 568 7209, or 5. 18 s. 9 d. 100 l. 568 7209, or 568 : 14 : 5: and 1000 568,7209, or 568 : 14 : 5:

B  
is re  
have  
by t  
duct  
so t  
*Flem*  
duce  
4.  
Righ  
Poun  
Guil  
of St  
chan

At  
that  
equiv

If  
have  
the G  
by y  
10 s.  
to ma

V.  
ders i  
ought  
ders,

But if your *Flemish* Money, the value of which is required in Sterling, consists of odd Pounds, or have Shillings, &c. annex'd, Multiply the same by the Sterling, as before directed, and the Product of such Multiplication answers the Question; so the Exchange being at 35 s. 2 d. 137 l. 10 s. *Flemish*, multiplied by 5687209 Sterling, produces 781. 19912375, or 781. 4 s. + Sterling.

4. The Fifth Column, or the second on the Right-hand Page, shews the value or amount of Pounds Sterling in Guilders, or what number of Guilders I ought to receive, or pay, for any Sum of Sterling, at any of the inserted Rates of Exchange;

*For Example.*

At the aforefaid Exchange of 35 s. 2 d. it appears that 10 Guilders, 10 Stivers, and 8 Pennicks, are equivalent to one pound Sterling, as likewise, that

Guild. Stivers

105 : 10

are to

10

1055

to

100

10550

to

1000

} Sterling.

If the Sterling Money is in odd Pounds, or have Shillings, &c. annexed, you are to multiply the Guilders corresponding to the Rate of Exchange by your Sterling Sum, as above directed; so 132 l. 10 s. Sterling, Exchange at 35 s. 2 d. will be found to make 1397 Guilders, 17 Stivers, 8 Pennicks.

V. The last Column shews the value of Guilders in pounds Sterling, or how much Sterling I ought to receive, or pay, for any number of Guilders, at any of the specify'd Rates of Exchange.







# SECTION X. OF MEASURING.

THE various kinds of which, are Three,  
viz.

I. *Lineal*, call'd (by Work-men) *Running Measure*, in that it respects *Length* only, the parts of which are

12 Inches, 1 Foot,

3 Feet, 1 Yard,

16½ Feet, 1 Rodd,

II. *Superficial*, or *Square Measure*, in that it respects *Length* and *Breadth*; the parts are, viz.

144 Inches, 1 Foot,

900 Inches, 1 Foot,

1296 Inches, 1 Foot,

48 Inches, 1 Foot,

8 Inches, 1 Foot,

272½ Feet, 1 Rodd,

136 Feet, 1 Rodd,

108 Feet, 1 Rodd,

296, or 19 Feet, 1 Yard,

Note, That in dividing by 272½, which reduces *Superficial Feet* into *Square Rodds*, Work-men take no notice of the ½ of the Foot, dividing only by 272, which gives the Content a little too much.

III. *Solid*, or *Cube Measure*, respecting *Length*, *Breadth* and *Thickness*; whose parts are,

Inches.

Inches.

1728,

1296,

864,

432, &amp;c.

27 Feet,

Foot,

 $\frac{1}{4}$  of a Foot, $\frac{1}{2}$  of a Foot, $\frac{3}{4}$  of a Foot,

1 Yard.

## Of Superficial Measure.

## A General Rule.

Multiply the Length by the Breadth, and the Product is the Superficial Content.

## Demonstration.

For if a Line, A B, being divided into any number of equal parts, be moved constantly at right Angles with another Line, B C, divided into any other number of such equal Parts, as are in, A B, it will, when it is advanc'd to any part of, B C, describe, thereby as many equal Squares as there are Parts in it; and if mov'd on further, to the next Part, it will describe as many more Squares, and so the same number of Squares will be as often described as there are Parts in the Line, B C, and consequently there must be as many Squares in the Surface, made by these Lines, as there are Units in the Product of the Parts of the two Sides.

Al  
lowin  
us'd  
Foot  
of the  
Ho  
Head  
No  
Foot

I.

Dim  
Content

A B

in Glaziers

ni erect is a Foot of Glaziers is there in  
ap Foot 4.

|   |  |  |  |  |     |
|---|--|--|--|--|-----|
|   |  |  |  |  |     |
|   |  |  |  |  | 44  |
|   |  |  |  |  | 72  |
| 4 |  |  |  |  | 312 |
|   |  |  |  |  | 88  |

e

Now the *Square* of any thing is found three several ways, viz.

1. By *Decimals*;
2. By *Aliquot Parts*;
3. By *cross Multiplication*.

All which I shall promiscuously use in the following Examples. But the latter of the three is us'd by most Workmen, except *Glaziers*, whose Foot being *Decimally* divided, cast up the Content of their Work by *Decimals*.

How to Multiply by the two last, see my 4th Head in Multiplication.

Note, The Content of Work, is given either by the Foot, the Yard, the Square, or the Rod, viz.

### I. By the Foot.

#### Glazing and Masoning.

Dimensions are taken in Feet and Inches, the Content given per Foot Square.



## In Glazing.

Example, How many Foot of Glazing is there in that Pane of Glass which is 4 Foot 5. and 2 Foot 42.

|        |  |  |  |  |
|--------|--|--|--|--|
| 2,42   |  |  |  |  |
| 4,5    |  |  |  |  |
| 1210   |  |  |  |  |
| 968    |  |  |  |  |
| 30,890 |  |  |  |  |

Answer 10 f. 890

## In Masoning.

Example, Suppose a Tard paved with Free stone, viz. Length 22 Feet 4 Inches. Breadth 19 Feet 7 Inches, how many Square Feet?

22.4

19.7

424

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

7

But if only the Breadth was given, and it were required how much in Length will make a Foot Square, see Mr. Kersey's 51<sup>st</sup> Question of his 10<sup>th</sup> Chapter.

## II. By the Yard.

Painting, Joynery, Plaistering, &c.

Dimensions are taken in Feet and Inches; but the Content given in Square Yards, found by dividing the Superficial Feet by 9.

li



In Painting.

*Example, The Height of a Room being 12 . 4, and 84 Feet 11 Inches about, how many Square Yards?*

$$\begin{array}{r}
 84 \text{ } 11 \text{ about} \\
 12 \text{ } 4 \text{ high} \\
 \hline
 1019 \text{ } 0 \\
 28 \text{ } 3 \text{ } 8 \\
 \hline
 9) 1047 \text{ } 3 \text{ } 8 \\
 116 \text{ } 3 \text{ } 3 \text{ } 8 \text{ } \text{Ans. } 116 \text{ } 3 \text{ } 3 \text{ } 8
 \end{array}$$

Joinery.

*Example. How many Yards of Wainscort doth that Room take up, whose Height is 12 Feet 3 Inches, and Compass 104 Feet 6 Inches?*

$$\begin{array}{r}
 \text{feet. } \text{Inches.} \\
 104 \text{ } 6 \\
 12 \text{ } 3 \\
 \hline
 1248 \text{ } 0 \\
 66 \\
 26 \text{ } 1 \\
 \hline
 9) 1280 \text{ } 1 \text{ } 6 \\
 142 \text{ } 2 \text{ } 1 \text{ } 6 \text{ } \text{Ans. } 142 \text{ } 2 \text{ } 1 \text{ } 6
 \end{array}$$

Plastering.

*Example, How many Yards in a Ceiling 47 Feet, 4 Inches, 7 Parts Long, and 18 Feet Broad?*

47 . 4 . 7 long  
6 broad

284 . 3 . 6

9) 284 . 10 . 6

94 . 6 . 10 . 6

Tar. feet. inch. parts.

Facit. 94 . 6 . 10 . 6

Under which Head you may further note, that if you multiply the  $\frac{1}{3}$  part of the *Length*, by the  $\frac{1}{3}$  of the *Breadth*, the *Product* will be the *Content* in *Square Yards*, it being the same to take  $\frac{1}{9}$  of *Li-*  
*neals* of *Superficial Measure*.

### III. By the Square.

#### Partitioning, Flooring, Roofing, Tiling.

Dimensions are taken in *Feet* and *Inches*, but the *Content* or *Value* is by the *Square* of 10 *Feet*, done by cutting off two *Figures* of the *Superficial Feet* towards the *Right-hand*.

#### Partitioning.

*Example*, How many *Squares* are contained in that *Work* that is 199 *Feet* 10 *Inches*, in *Length*, and 10 *Feet* 7 *Inches* high?

199 . 10 . 0  
10 . 7 . 0

1998 . 4

66 . 7 . 4

21 . 14 . 10 . 10

Square feet, Inch. parts

Ans. 21 . 14 . 10 . 10

From

Measuring.

Sec. 10.

From which Sometimes the Content of Doors and other Vacancies are deducted,

Flooring.

Example, A Floor being 49 Feet, 7 inch. 4 part. long, 26 Feet 6 Inches, broad, how many Squares.

$$\begin{array}{r}
 49 \cdot 7 \cdot 4 \\
 \times 26 \cdot 6 \\
 \hline
 1289 \cdot 10 \cdot 8
 \end{array}$$

Roofing.

Example, Suppose an House 18 feet 4 inches in Front, and 37 feet 10 inches in Depth, how many Square in the Roof?

$$\begin{array}{r}
 18 \cdot 4 \\
 \times 37 \cdot 10 \\
 \hline
 681 \cdot 0 \\
 127 \cdot 4 \\
 \hline
 689 \cdot 4
 \end{array}$$

Square feet, inch.

Ans. 10 . 40 . 5      10,40 . 5 . 0

Here note, after you have multiply'd the Depth the Front, and so found the Content of the Ground.



Ground-Plat in Feet, half of which, being added, gives the Content of a Pitch Roof, which reduce into Squares.

#### IV. By the Rodd, as in

#### Brick-Work.

Dimensions are taken in Feet and Inches, and the Content given per the Square of a Rodd of  $16 \frac{1}{2}$  Foot.

The Content of this Work is found in the same manner as all the other, to wit, by Multiplying the Length by the Breadth, and only differs from the former in two things.

1. When you have so found the Superficial Content in Feet, divide it by 272, the Square Feet in one Rodd, the Quote is Rodds, the Content being always so given.

2. That when it is more or less than a Brick and  $\frac{1}{2}$  thick, it must be reduced into that thickness, being always measured by the Square Rodd of a Brick and  $\frac{1}{2}$ . To do which observe,

Rule. Multiply the Product of the Length and Height of the Wall by as many half Bricks as it contains in Thickness, which Product divided by 3, gives the Content of the Wall, 1 Brick and  $\frac{1}{2}$  thick, which superficial Feet divide by 272, the Quote is Square Rodds reduced to 1 Brick and  $\frac{1}{2}$ , or more, shew by the following Table.



**Brick.**

For  $\left\{ \begin{array}{l} 1 \text{ Subtract } 2\frac{1}{2} \\ 2 \text{ Add } 7\frac{1}{2} \\ 3 \text{ Multiply by } 2 \\ 4\frac{1}{2} \text{ Multiply by } 3 \\ 6 \text{ Multiply by } 4 \end{array} \right\}$  Reduces the said Thickness into 1 Brick and  $\frac{1}{2}$ .

**Ex. 1.** Suppose a Wall of a Garden 213 feet about, and 13 feet 4 inches high, and 1 Brick and  $\frac{1}{2}$  thick, how many Rodd?

213

13.4

2769.0

Rodd, feet.

272

2840.0

(10.120

120

**Ex. 2.** How many Rodd of Brick Work in that Wall which is 40 feet 7 inches long, and 11 feet high, being 1 Brick thick?

feet inches

40.7

11.0

3) 446.5

148.9

Rodd, feet, inch. part.

272) 297.7.4

Ans. 1. 25. 7. 4

1.25.7.4

**Ex. 3.** If a Wall be 294 feet about, and 12 feet 7 inches high, and 3 Bricks thick, how many Rodd?

248

ABCD

Rodds, feet, inch. 3196.2  
 2729 6392 4 (23) 136  
 952  
 536

Of Solid Measure.

Which respects Length, Breadth and Thickness,  
 Rule, Multiply the length by the breadth, and that  
 Product by the depth.  
 Otherwise, Multiply the Area of the Base by the  
 height, the Product gives the solid Content, which if  
 in Feet, divide by 27; the solid Feet in a Yard, re-  
 duces them into solid Yards, if in Inches by 1728,  
 gives solid Feet, &c.

Demonstration.

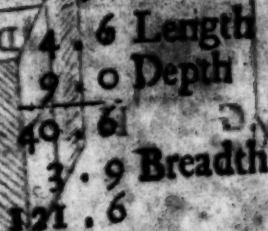
For a Surface ABCD being imagined to move,  
 suppose upwards or downwards, so that its Points  
 A, B, C, D, describe the Lines AE, BG, CF,  
 DE, and consequently, each Line therein describe  
 other Surfaces, &c. there will be generated by this  
 Motion, a Magnitude called a Solid, having three  
 Dimensions, viz. Length, Breadth and Thickness;  
 Therefore that Surface ABCD, being divided in-  
 to little Square Area's, and moving the height AH,  
 divided by a like measure for Length, may be  
 conceiv'd to produce as many little Cubes, as is  
 the number of the little Area's in the Surface,  
 ABCD,





**Digging.** A third and most

Ex. 3. A Vault dug 9 feet Deep, 4; Long, and 9 inches Broad, how many solid Yards?



27) 151.10.6

Ans. 16. 10. 6

and 7 feet  $\frac{1}{2}$  deep.  
 Answer 110 Yards, 6 feet, 8 inches.

*Soli Deo Glorie.*

Writing in all Hands, Arithmetick, and  
Modern, *Italian*, Book-keeping, in all their  
Parts, are Taught; and for more Expedition  
Youth are conveniently Boarded, by

George Shelley

